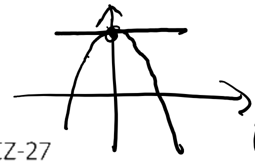


$\theta << 1$

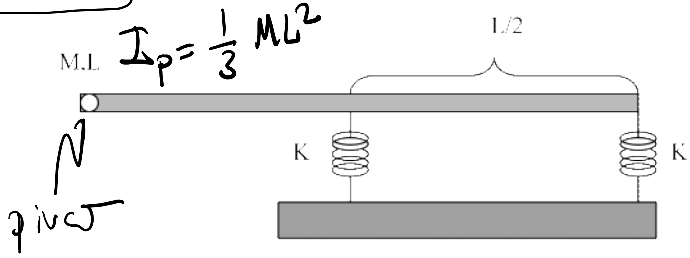
$\sin\theta \simeq \theta$

$\cos\theta \simeq 1$

Phys 101- Instructor: M. Özgür Oktel – 2016 QUIZ-27

A uniform rod of length L and mass M is fixed from its end on a frictionless horizontal table. The rod can rotate freely around its end point, but is attached to the side of table with two springs as shown in the figure. Both springs are at ~~their natural length~~, and have spring constant K. Find the angular frequency of small oscillations of the rod.

$I_p = \frac{1}{3} ML^2$

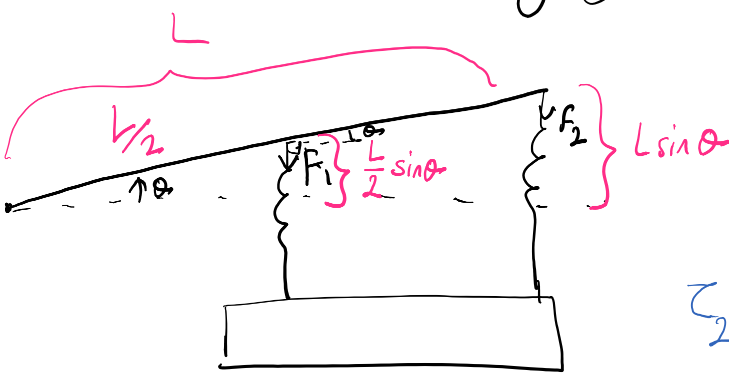


$\tau = I\alpha$

$$\frac{5}{4}\cancel{L^2} K \sin\theta\cos\theta = -\frac{1}{3} M\cancel{L^2} \frac{d^2\theta}{dt^2}$$

$$\boxed{\frac{5}{4} K\,\theta = -\frac{1}{3} M \frac{d^2\theta}{dt^2}}$$

θ ↻



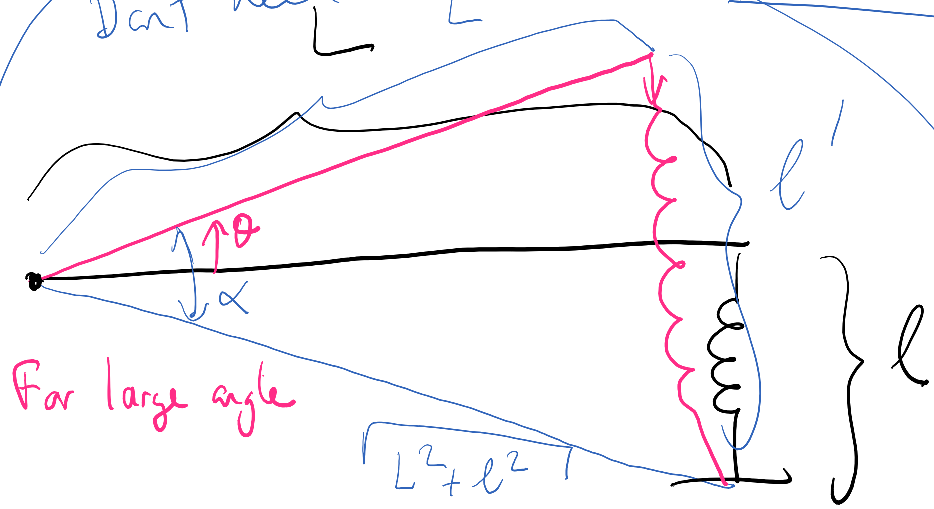
$\tau_1 = \frac{L}{2} F_1 \cos\theta$

$\tau_1 = \frac{L}{2} K \frac{L}{2} \sin\theta \cos\theta$

$\tau_2 = L^2 K \sin\theta\cos\theta$

$$\boxed{\tau = \frac{5}{4} L^2 K \sin\theta\cos\theta}$$

Don't need this!



$$l'^2 = L^2 + l^2 + L^2 - 2Ll\cos(\alpha)$$

$\theta << 1 \quad \sin\theta \simeq \theta$

$$\theta \ll 1 \Rightarrow \begin{array}{l} \sin\theta \simeq \theta \\ \tan\theta \simeq \theta \\ \boxed{\tan\theta \approx \sin\theta} \end{array}$$

$$\boxed{\frac{5}{4}k\,\theta = -\frac{1}{3}M\frac{d^2\theta}{dt^2}}$$

$$\frac{d^2\theta}{dt^2} = -\frac{15}{4}\frac{k}{M}\,\theta$$

$$\theta(t) = C_1\sin(\omega t) + C_2\cos(\omega t)$$

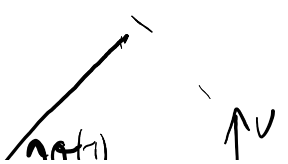
$$\frac{d^2\theta}{dt^2} = -\omega^2\,\theta$$

$$\boxed{\omega = \sqrt{\frac{15}{4}\frac{k}{M}}}$$

1°) Gal?

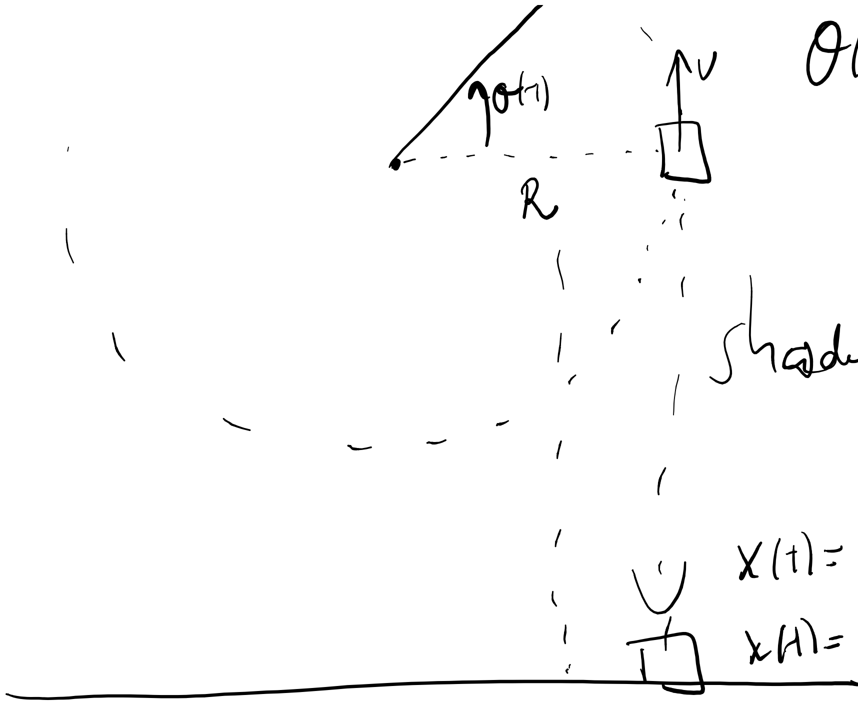
2°) Units $\quad [\omega] = \frac{1}{s} \quad \sqrt{\frac{N/m}{kg}} = \sqrt{\frac{\cancel{kg}\,m/s^2/\cancel{m}}{\cancel{kg}}} = \frac{1}{s}$ ✓

3°) Limits ??



$$\theta(t) = \frac{v}{R} + \ldots$$

$$\theta(t) = \frac{v}{R} t$$

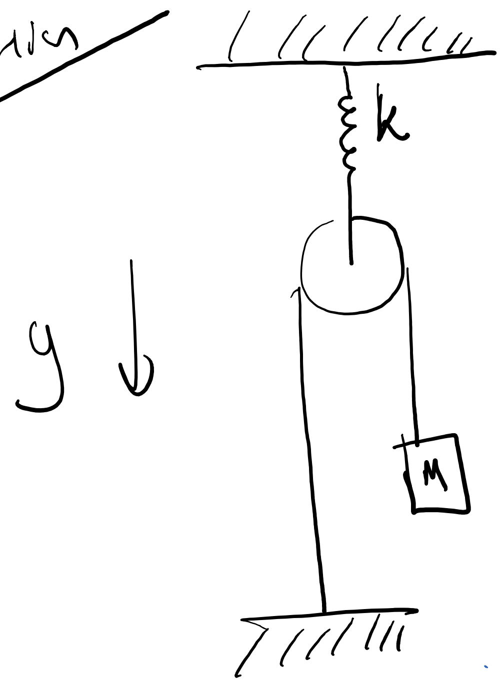


shadow ?

$$x(t) = R\cos(\theta(t))$$

$$x(t) = R\cos(\omega t)$$

Question

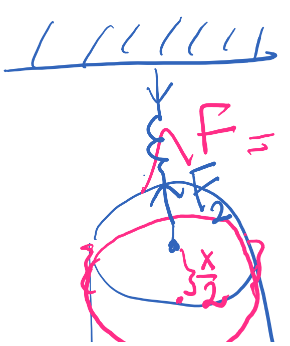


Ropes and pulley massless

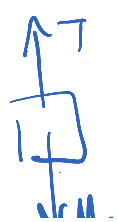
Find the frequency of small oscillations around the equilibrium point.

Solution

Find equilibrium point.

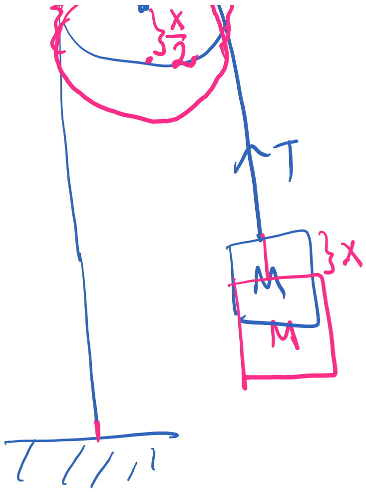


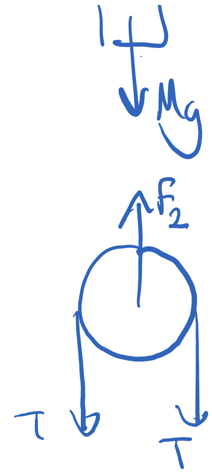
$$F = k\left(\Delta + \frac{x}{2}\right)$$



$a = 0$

$T = Mg$





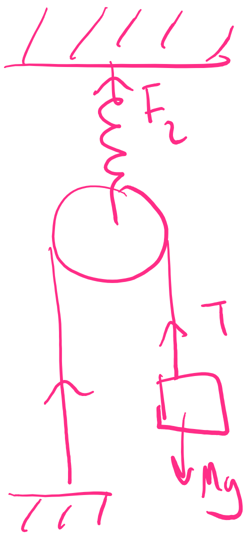
$T = Mg$

$a = 0$

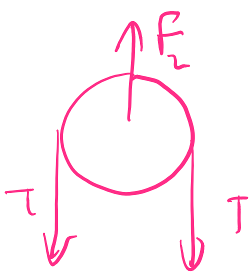
$F_2 = 2T$
$ = 2Mg$

Call the stretching of the spring Δ

$$k\Delta = 2Mg \implies \boxed{\Delta = \frac{2Mg}{k}}$$



$$F_2 = k\left(\Delta + \frac{x}{2}\right)$$



$\downarrow a$ $\quad M_p \to 0$

$$M_p a = 2T - F_2$$

$$\boxed{T = \frac{1}{2}F_2}$$

$\downarrow a$

$$Ma = Mg - T$$

$$Ma = Mg - \frac{1}{2}F_2$$

$$Ma = Mg - \frac{1}{2}k\left(\Delta + \frac{x}{2}\right)$$

$$M\frac{d^2x}{dt^2} = Mg - \frac{1}{\cancel{2}}\cancel{k}\Delta - \frac{1}{4}kx$$

$$\underbrace{M\frac{d^2x}{dt^2} = Mg - \frac{1}{\cancel{2}}\cancel{k}\underbrace{\Delta}_{\frac{\cancel{2}Mg}{\cancel{k}}}}_{0} - \frac{1}{4}kx$$

$$\frac{d^2x}{dt^2} = -\frac{1}{4}\frac{k}{M}x \Longrightarrow \omega^2 = \frac{k}{4M}$$

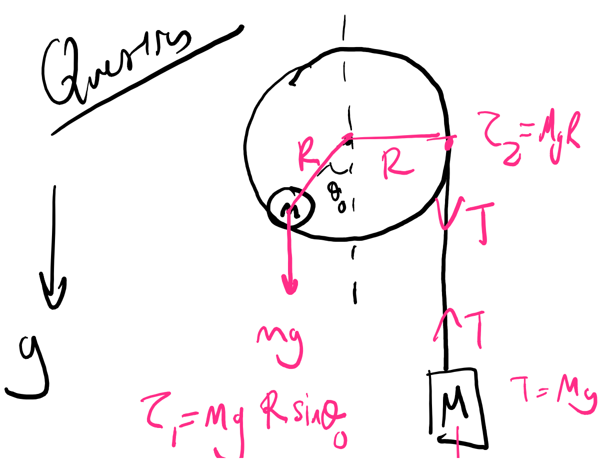
$$\boxed{\omega = \frac{1}{2}\sqrt{\frac{k}{M}}}$$

l_1 $\qquad$ l_2 $\qquad$ l_3

$$l_3 > l_1 > l_2$$

$$\boxed{\omega_3 = \omega_2 = \omega_1}$$

Question



Massless pulley with m attached to its edge

- Find θ_0 for equilibrium
- Find the frequency of small oscillations!

$\tau_1 = mgR\sin\theta_0$

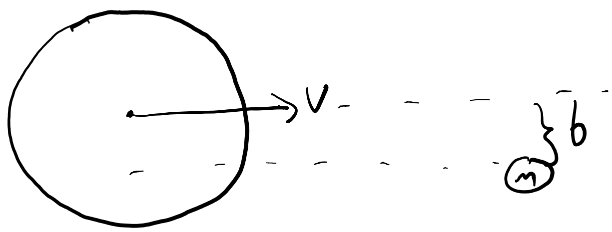
oscillations!

$$m\cancel{g}\cancel{R}\sin\theta_0 = M\cancel{g}\cancel{R} \Rightarrow \boxed{\sin\theta_0 = \frac{M}{m}}$$

$M > m \Rightarrow$ No equils

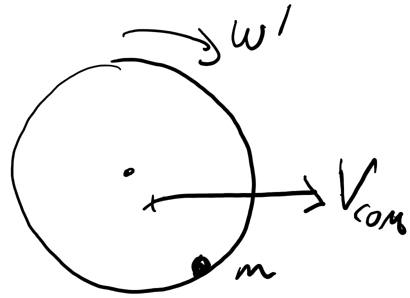
Ex $\omega = 0$ Hockey puck on (frictionless) ice → M, R

$I_{com} = \frac{MR^2}{2}$



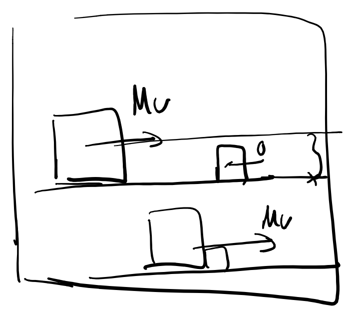
After the hit
m sticks to the puck.

$V_{com} = ?$
$\omega' = ?$

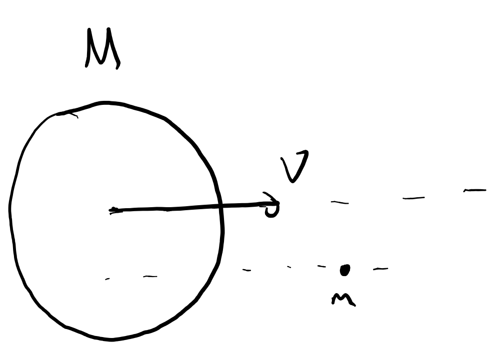


(1°) Momentum conservation.

(2°) Angular momentum conserved.

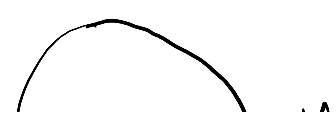


Initial



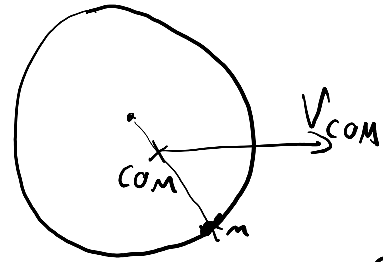
$P_i = Mv$

Final



$P_f = (M+m)V_{com}$

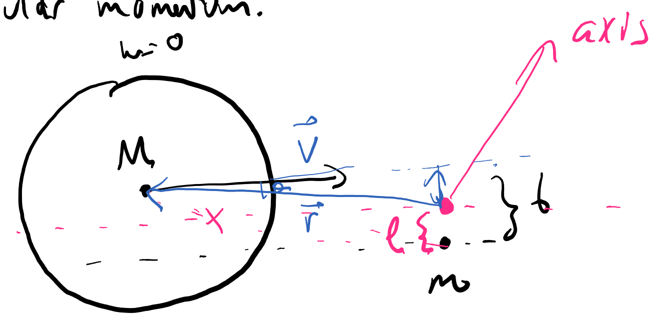
Final



$P_f = (M+m) V_{com}$

$P_i = P_f \Rightarrow \boxed{V_{com} = \frac{M}{(M+m)} v}$

Angular momentum.



$l\, m = M(b-l)$

com moves on the line

$\boxed{l = \frac{M}{M+m} b}$

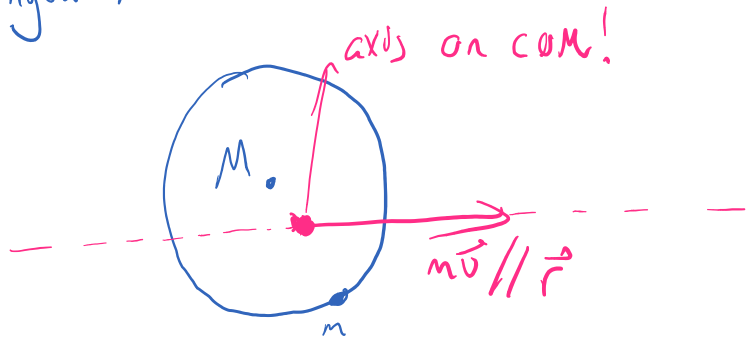
According to the point

$L_i = \vec{r} \times \vec{p} + \cancel{I\omega}^{\,0}$

$= \underbrace{|\vec{r}|\sin\theta}_{b-l}\, M v = (b-l) M v = \left(1 - \frac{M}{M+m}\right) M b v$

$\boxed{L_i = \frac{mM}{M+m} b v}$

Final Angular Momentum?



$L_f = \underbrace{\vec{r} \times \vec{p}}_{0} + I\omega$

Moment of inertia of puck + m

Moment of inertia of puck + m around the common $\boxed{COM}$

$$xM = (R-x)m$$

$$x = \frac{m}{M+m}R \qquad R-x = \left(1-\frac{m}{M+m}\right)R = \frac{M}{M+m}R$$

$$I_{COM} = m(R-x)^2 + \frac{MR^2}{2} + Mx^2$$

$$= m\frac{M^2}{(M+m)^2}R^2 + \frac{MR^2}{2} + M\frac{m^2}{(M+m)^2}R^2$$

$$= MR^2\left[\frac{Mm}{(M+m)^2} + \frac{1}{2} + \frac{m^2}{(M+m)^2}\right]$$

$$= MR^2\left[\frac{1}{2} + \frac{m\cancel{(M+m)}}{(M+m)^{\cancel{2}}}\right] = MR^2\left[\frac{M+3m}{2(M+m)}\right]$$

$$I\omega' = L_i = \frac{mM}{(m+M)}bv$$

$$\cancel{M}\frac{(M+3m)}{2\cancel{(M+m)}}R^2\omega' = \frac{m\cancel{M}}{\cancel{(m+M)}}bv$$

$$\boxed{\omega' = \frac{2m}{(M+3m)}\frac{bv}{R^2}}$$

$$\frac{\cancel{kg}}{\cancel{kg}}\ \frac{\cancel{m}\ m/s}{\cancel{m^2}}$$

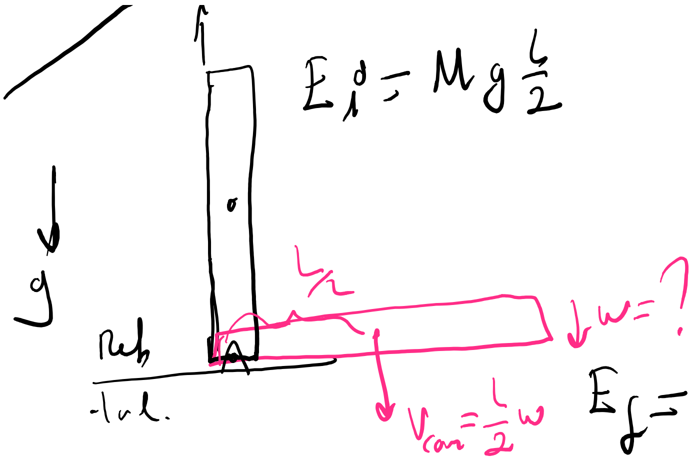
1°) Goal ✓
2°) Units ✓
3°) Limits
$b = 0 \Rightarrow \omega' = 0$ ✓
$m \to 0 \Rightarrow \omega' = 0$ ✓

Ex

M, L

$F_i d = Mg\frac{L}{2}$

$$E_i^0 = Mg\frac{L}{2}$$



Energy conservation

$$E_f = \frac{1}{2} I_{edge}\,\omega^2 \qquad I_{edge} = \frac{1}{3}ML^2$$

$$= \frac{1}{2} M v_{com}^2 + \frac{1}{2} I_{com}\,\omega^2 \qquad I_{com} = \frac{ML^2}{12}$$

$$E_i^0 = E_f$$

$$\cancel{M}g\frac{\cancel{L}}{\cancel{2}} = \frac{1}{\cancel{2}}\,\frac{1}{3}\cancel{M}L^{\cancel{2}}\,\omega^2 \Rightarrow \boxed{\omega = \sqrt{3\frac{g}{L}}}$$

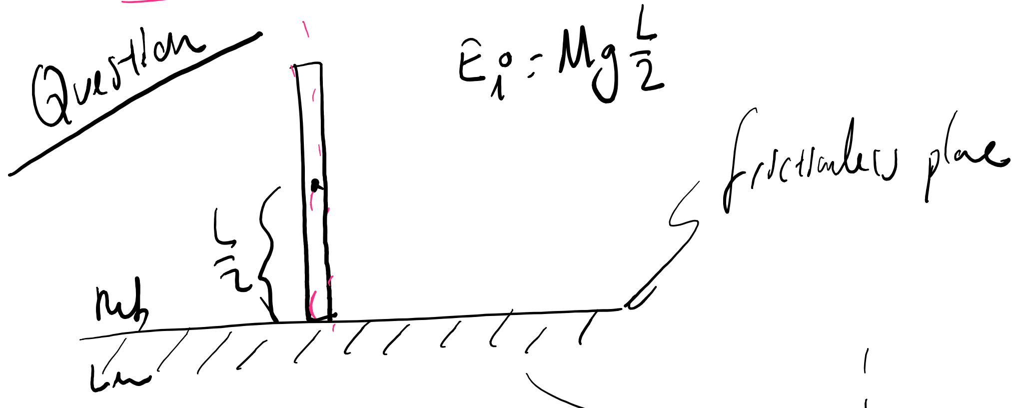
$$E_f = \frac{1}{2}M\left(\frac{L}{2}\right)^2\omega^2 + \frac{1}{2}\,\frac{1}{12}ML^2\omega^2$$

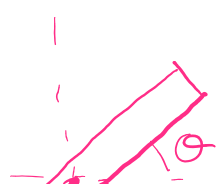
$$= \frac{1}{2}\underbrace{\left[\frac{M}{4} + \frac{M}{12}\right]}L^2\omega^2$$

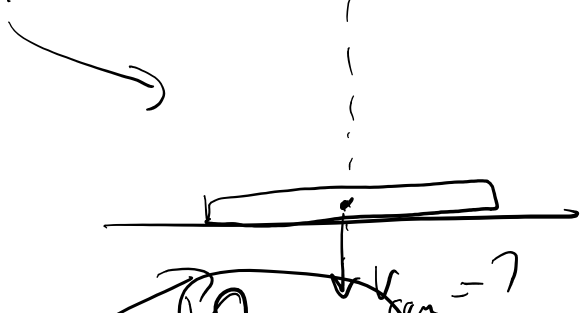
$$= \frac{1}{2}\left(\frac{M}{3}\right)L^2\omega^2$$

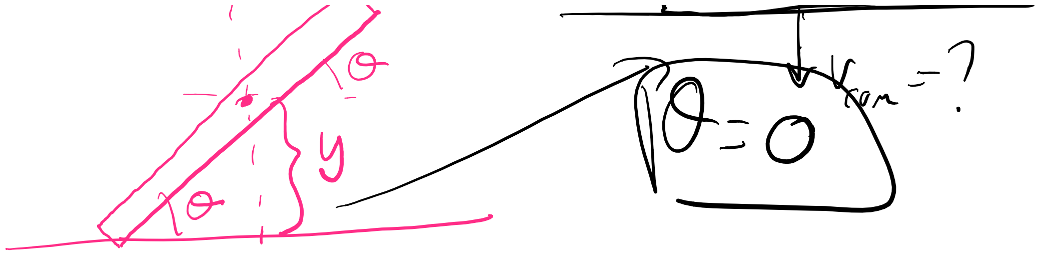
Question

$$E_i^0 = Mg\frac{L}{2}$$









$$\frac{L}{2}\sin\theta = y$$

[C.O.M does not move along x
(momentum conservation)]

$$\frac{d}{dt}\left(\frac{L}{2}\sin\theta = y\right)$$

$$\frac{L}{2}\cos\theta \underbrace{\frac{d\theta}{dt}}_{\omega} = \underbrace{\frac{dy}{dt}}_{v_y}$$

At any angle θ
v_y and ω
are related!

$$\boxed{\omega = \frac{2}{L}\frac{1}{\cos\theta}v_y}$$

$$v_{com} = v_y$$

$$E_i = E_f$$

$$\theta < \frac{\pi}{2}$$

$$Mg\frac{L}{2} = \frac{1}{2}m v_{com}^2 + \frac{1}{2}I_{com}\,\omega^2$$

$$= \frac{1}{2}M v_y^2 + \frac{1}{2}\frac{ML^2}{12}\left(\frac{2}{L}\frac{1}{\cos\theta}v_y\right)^2$$

At $\theta = 0$ $\cos\theta = 1$

$$Mg\frac{L}{2} = \frac{1}{2}M v_y^2 + \frac{1}{2}\frac{M\cancel{L^2}}{\underbrace{\cancel{12}}_{3}}\frac{\cancel{4}}{\cancel{L^2}}\underbrace{\frac{1}{\cos^2\theta}}_{1}v_y^2$$

$$\cancel{M}g\frac{L}{\cancel{2}} = \frac{1}{\cancel{2}}\cancel{M}\left(1+\frac{1}{3}\right)v_y^2$$

$$\boxed{v_y = \sqrt{\frac{3}{4} g L}}$$