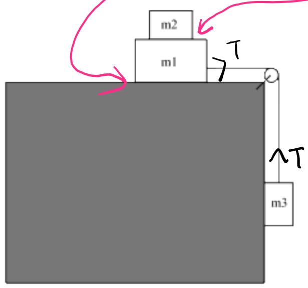
Week 05 (Oct 24 - Oct 28)

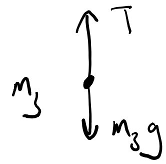
Friday, September 23, 2016 2:56 PM

QUIZ-9

Three masses m1=1kg, m2=1 kg and m3= 2 kg are arranged as shown in the figure. If the friction coefficient between m1 and m2 is $\mu_1 = 0.1$, and the friction coefficient between m1 and the table is $\boxed{\mu_2 = 0.65}$, find the acceleration of all masses and the tension of the cable. (Gravitational acceleration is g=10 m/s^2)



Acceleration of m1	Acceleration of m2	Tension of the cable
2 m/s²	1 m/s²	16

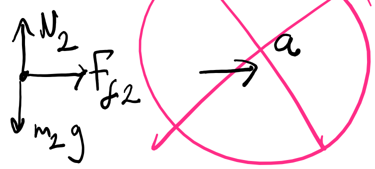


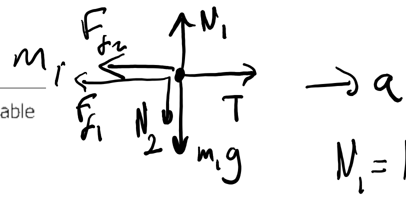
$\boxed{m_3 a = m_3 g - T}$

$N_2 = m_2 g$

$\boxed{m_2 a = F_{f2}}$

m2





$N_1 = N_2 + m_1 g$

$N_1 = (m_1 + m_2) g$

1°) Assume m_2 remains stationary w.r.t. m_1

$m_1 a = T - F_{f_1} - F_{f_2}$

kinetic friction

① $\boxed{m_3 a = m_3 g - T}$

② $\boxed{m_2 a = F_{f_2}}$

$\boxed{m_1 a = T - F_{f_1} - F_{f_2}}$

$F_{f_1} = \mu_2 N_1$

$= \mu_2 (m_1 + m_2) g$

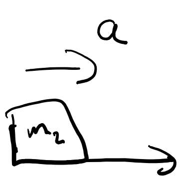
$(m_1 + m_2 + m_3) a = m_3 g - F_{f_1}$

$(m_1 + m_2 + m_3) a = m_3 g - \mu_2 (m_1 + m_2) g$

$a = \dfrac{m_3 - \mu_2 (m_1 + m_2)}{(m_1 + m_2 + m_3)} g = \dfrac{2 - (0.65) 2}{\cancel{4}\ 2} \cancel{10}\ 5$

$= (0.35) \times 5 = 1.75 \ m/s^2$

Assume



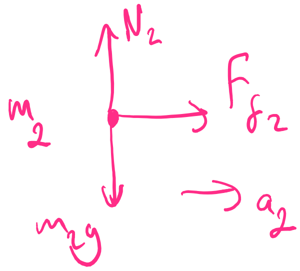
$F_{f_2} = m_2 a = 1.75 \ N$

$|F_{f_2}| \leq \mu_1 N_2 = 0.1 \, m_2 g = \boxed{1 \ N}$

$$|F_{f_2}| \leq \mu_1 N_2 = 0.1\, m_2 g = \boxed{1\,N}$$

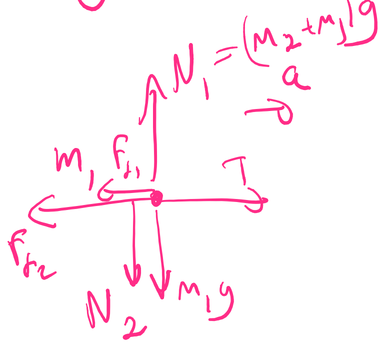
$$\cancel{1.75\,N \leq 1\,N}$$

m_2 is not static with respect to m_1!



$$|F_{f_2}| = \mu_1 N_2 = 0.1\, m_2 g = 1\,N$$

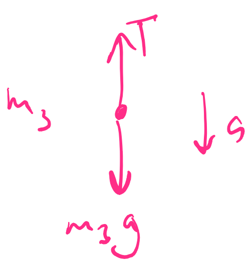
$$m_2 a_2 = F_{f_2} \Rightarrow \boxed{a_2 = 1\,m/s^2}$$



$$T - F_{f_1} - F_{f_2} = m_1 a$$

$$m_3 g - T = m_3 a$$

1 N



$$(m_1 + m_3) a = m_3 g - F_{f_1} - F_{f_2}$$

$$m_3 g - m_3 a = T$$

$$2(\underbrace{10-2}_{8}) = 16\,N$$

$$F_{f_1} = N_1 \mu_2$$

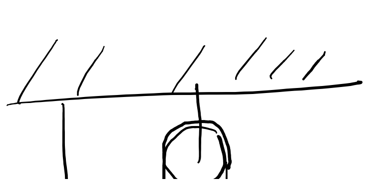
$$= 0.65 \cdot 2 \cdot 10$$

$$F_{f_1} = 13\,N$$

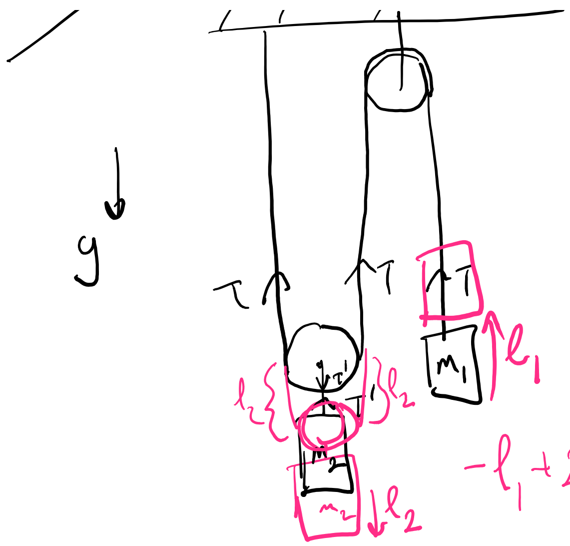
$$3a = 20 - 13 - 1$$

$$a = \frac{6}{3} = \boxed{2\,m/s^2}$$

EX



• No friction

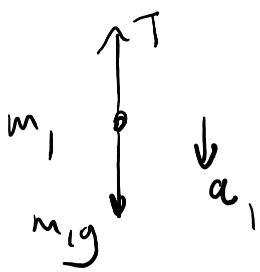


- No frictions
- inextendible string
- massless pulleys.

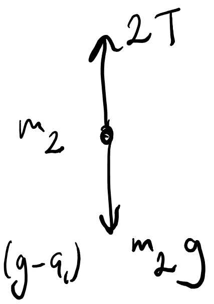
Find the acceleration of both masses and the tension in the rope.

$$-l_1+2l_2=0 \xrightarrow{\frac{d}{dt}} -v_1+2v_2=0$$

$$-a_1+2a_2=0$$



$T=m_1(g-a_1)$



$$\boxed{2a_2=a_1}$$ ③

① $m_1 g - T = m_1 a_1$

② $m_2 a_2 = 2T - m_2 g$

①, ③

$$2(m_1 g - T) = (m_1 2a_2)2$$

$$2T - m_2 g = m_2 a_2$$

$+$

$$2m_1 g \cancel{-2T} \cancel{+2T} - m_2 g = 4m_1 a_2 + m_2 a_2$$

$$(2m_1 - m_2)g = (4m_1+m_2)a_2$$

$$\boxed{a_2 = \frac{(2m_1-m_2)}{(4m_1+m_2)}g} \qquad \boxed{a_1 = 2\frac{(2m_1-m_2)}{(4m_1+m_2)}g}$$

$$T = m_1(g-a_1) = m_1 g\left(1-\frac{4m_1-2m_2}{\ldots}\right)$$

$$4m_1+m_2$$

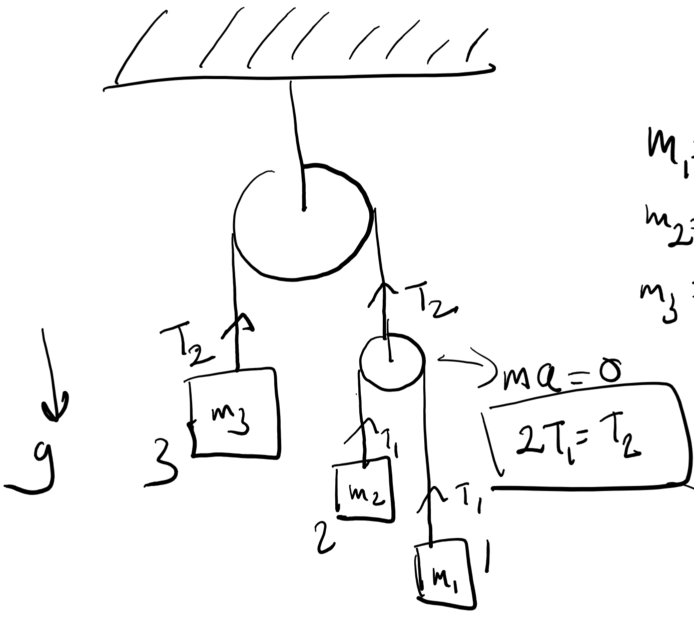
$$= m_1 g \, \frac{\cancel{4m_1} + m_2 - \cancel{4m_1} + 2m_2}{4m_1+m_2}$$

$$\boxed{T = 3\,\frac{m_1 m_2}{4m_1+m_2}\, g}$$

$$[T] = \frac{kg\,\cancel{kg}}{\cancel{kg}}\, m/s^2 = N$$

If $m_1 = 2m_2$ $\quad a_1 = a_2 = 0$

$T = ?$ $\quad \underline{T = m_1 g}$



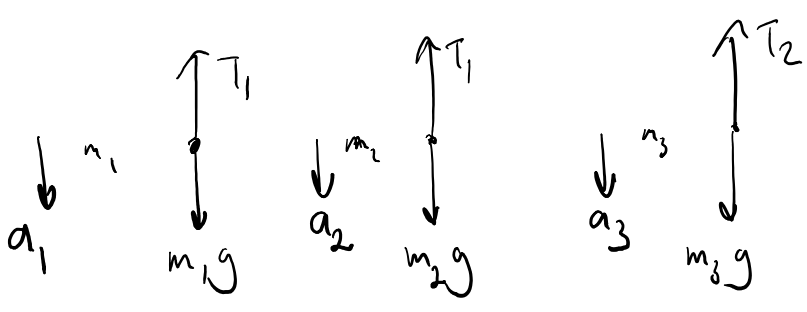
$m_1 = 1\,kg$

$m_2 = 2\,kg$

$m_3 = 3\,kg$

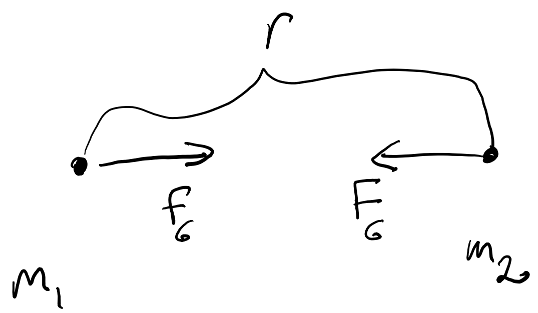
Find the acceleration of m_1

- direction
- magnitude



$m_1 a_1 = m_1 g - T_1$ $\qquad m_2 a_2 = m_2 g - T_1$ $\qquad m_3 a_3 = m_3 g - 2T$

Gravitational Force



$$F_G = G\frac{m_1 m_2}{r^2}$$

$$\boxed{F = mg}$$

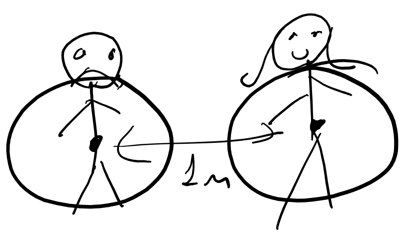
Ex/ Calculate the force of attraction between Ayşe ($m = 50$ kg) and Ali ($m = 70$ kg)

$r = 1$ m

$G = 6.67\ 10^{-11}\ \frac{Nm^2}{kg^2}$

$\frac{kg\ m/s^2\ m^2}{kg^2}$

$\frac{m^3}{kg\ s^2}$



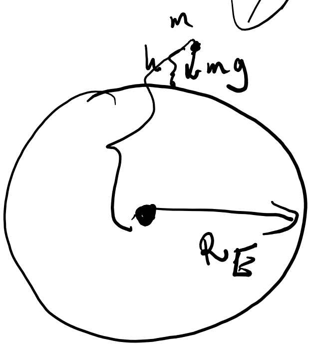
$$F_G = G\frac{m_1 m_2}{r^2} = 6.67\ 10^{-11}\ \frac{50\ 70}{1^2}$$

$$\simeq 35.7\ 10^{-9}\ N$$

$$\simeq 3.6\ 10^{-10}\ N$$

$$W_A \cong m_2 g \cong 500 \text{ N}$$

$$F = G\,\frac{m_1 m_2}{r^2}$$



Motion on earth's surface

$$h \sim 10, 100, 1000 \text{ m}$$

$$F_G = G\,\frac{m_E\, m}{(R_E + h)^2}$$

$$R_E \cong 6000 \text{ km}$$

$$\frac{h}{R_E} \sim \frac{1\cancel{000}\text{ m}}{6\,000\,\cancel{000}\text{ m}} \sim 10^{-4}$$

$x = 0.01$

For small values of $x << 1$

neglect

$$(1+x)^n = 1 + n(1)^{n-1}x + \cancel{\binom{n}{2}(1)^{n-2}x^2} \ldots$$

0.01 $\qquad$ 0.0001

$$(1+x)^n \cong 1 + nx$$

$$x = 0.0003$$

$$(1+x)^2 = (1.0003)^2 \simeq 1.0006$$

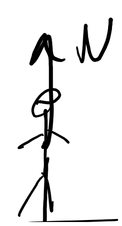
$$F_G = G\frac{m_E\, m}{(R_E+h)^2} = G\frac{m_E\, m}{R_E^2\left(1+\frac{h}{R_E}\right)^2}$$

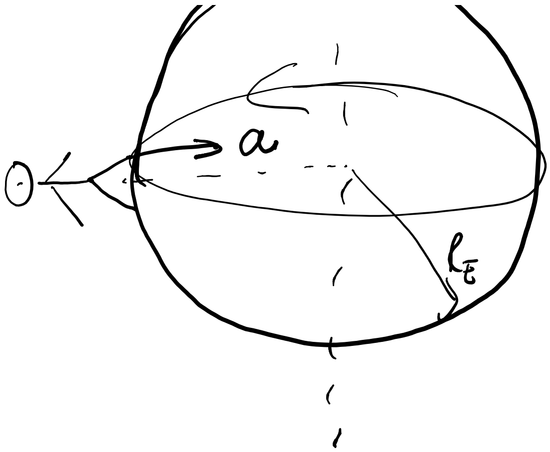
$$= G\frac{m_E\, m}{R_E^2}\left(1-2\frac{h}{R_E}\right)$$

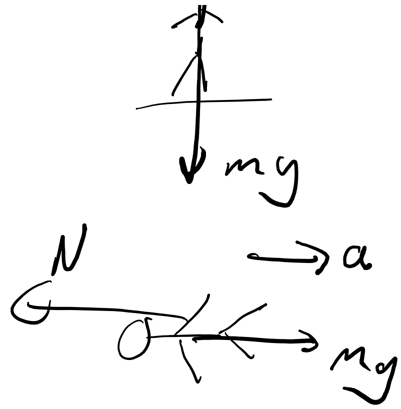
$$F_G = \underbrace{\left(G\frac{m_E}{R_E^2}\right)}_{g} m - \left(G\frac{m_E}{R_E^2}\right)2\frac{h}{R_E}\cdots$$

$$\boxed{g = G\frac{m_E}{R_E^2}}$$

Ex/ Find how fast the earth would need to rotate so that the mass on the equator feel weightless!







$$a = \frac{v^2}{R_E} = \omega^2 R_E$$

$$ma = mg$$

$$\omega^2 R_E = g$$

Weightlessness $N = 0$

$$\omega = \sqrt{\frac{g}{R_E}}$$

$$\omega \cong \sqrt{\frac{10\ m/s^2}{6\ 10^{6}\ m}} = \sqrt{\frac{1}{6}\ 10^{-5}}$$

$$= \sqrt{1.67\ 10^{-6}} = \sqrt{1.67}\ 10^{-3}$$

$$= 1.3\ 10^{-3}\ 1/s$$

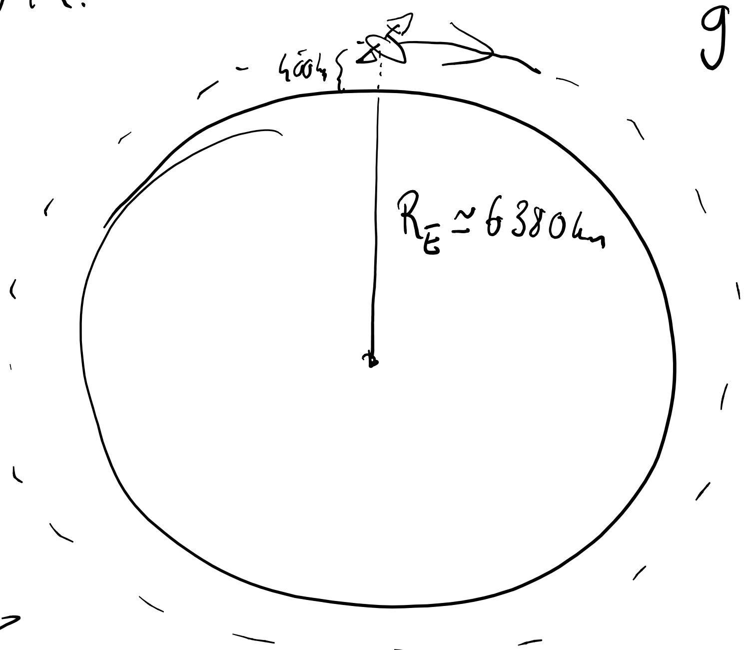
Current $\omega = ?$

$$T = \frac{2\pi}{\omega} = 24 \times 60 \times 60 \text{ sec}$$

$$\omega_{curr} = 7.27\ 10^{-5}$$

weightless astronauts!

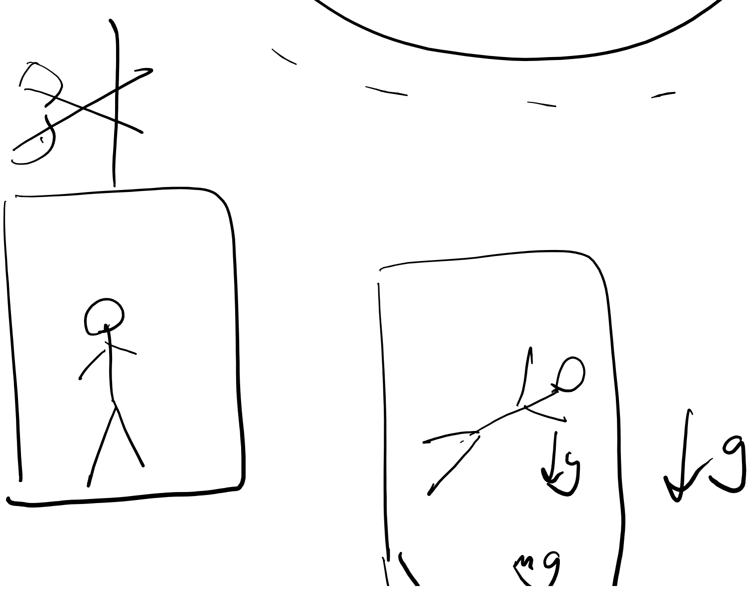
The ISS is about 400 km above earth.

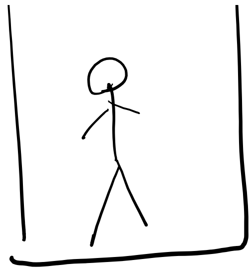


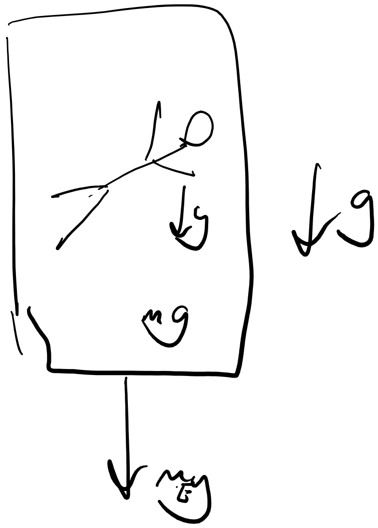
g @ ISS

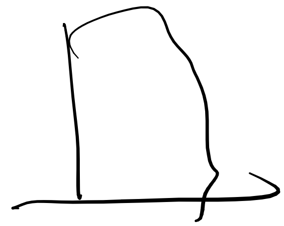
$g \neq 9.82$

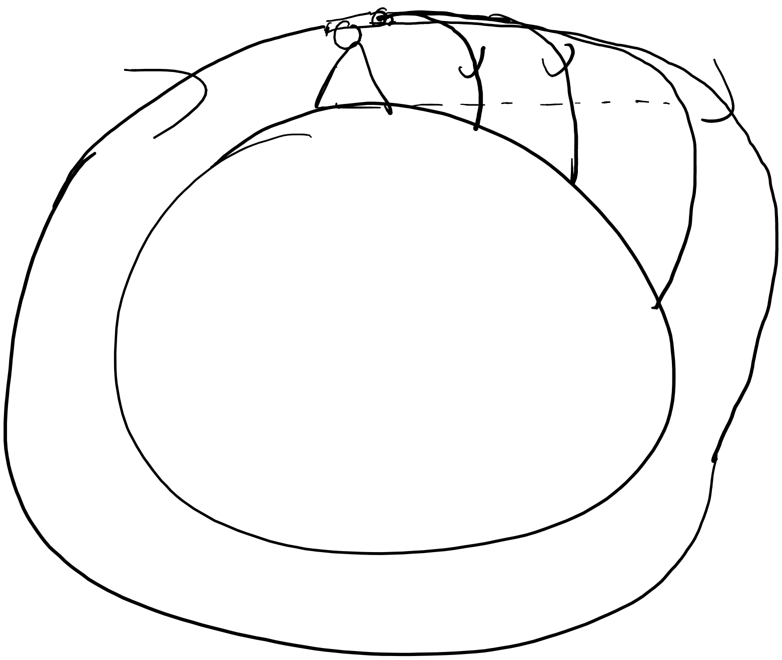
$\simeq 9.8\ \text{m/s}^2$

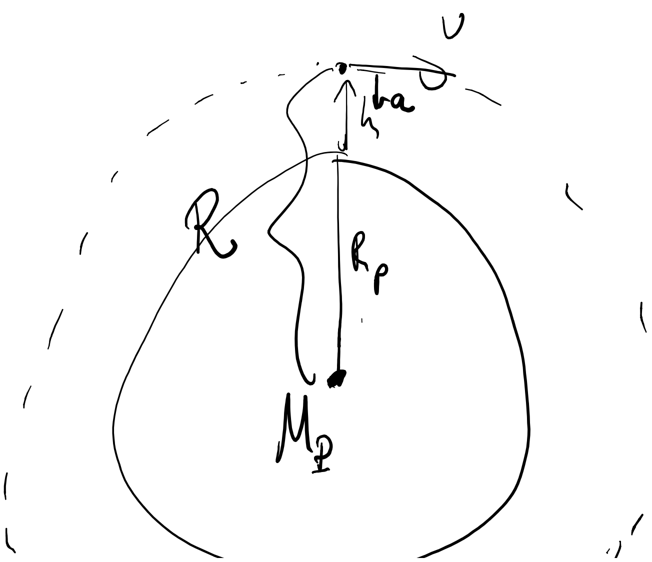










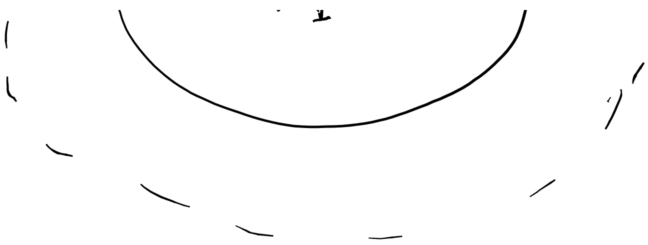


$a = \frac{v^2}{R}$ ↓

F_G

$ma = F_G$

$\cancel{m}\,\frac{v^2}{\cancel{R}} = G\,\frac{M_P\,\cancel{m}}{R^{\cancel{2}}}$



$$m\frac{v^2}{\cancel{R}} = G\frac{M_E m}{R^{\cancel{2}}}$$

$$\boxed{V = \sqrt{G\frac{M_E}{R}}}$$

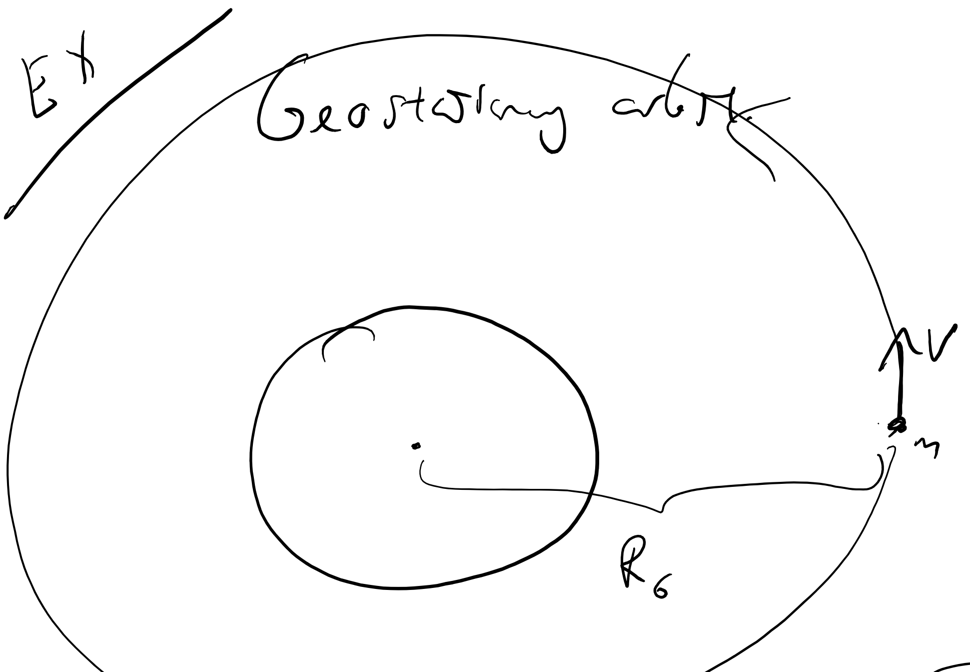
$$G\frac{M_E}{R_E^2} = g \qquad \simeq \sqrt{g R_E}$$

$$V \simeq \sqrt{10\ m/s^2\ 6\ 10^6\ m}$$

$$\simeq \sqrt{10^8\ m^2/s^2} = 10^4\ m/s$$

$$V_{GR} = 10\ km/s$$

Ex



$$\cancel{m}\frac{V^2}{R_G} = G\frac{M_E}{R_G^2}\cancel{m} \qquad V = \sqrt{G\frac{M_E}{R_G}}$$

$$T = 24\ h = \frac{2\pi R_G}{V}$$

V

$$24\times 60\times 60 = 2\pi \frac{R_G}{\sqrt{G\frac{M_E}{R_G}}} = 2\pi \frac{1}{\sqrt{G M_E}} R_G^{3/2}$$

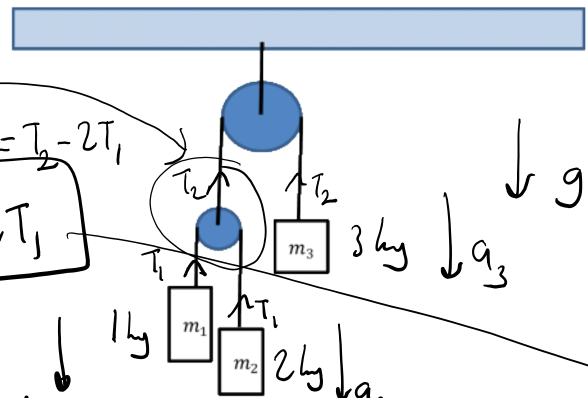
$$R_G = \dots$$

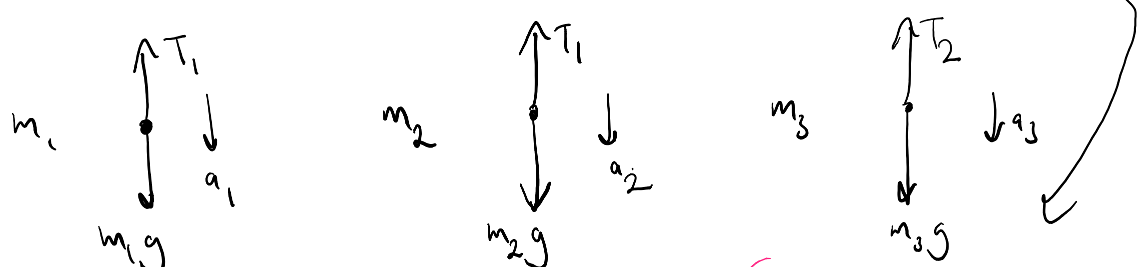
The three masses in the figure are $m_1 = 1.0kg$, $m_2 = 2.0kg$ and $m_3 = 3.0kg$. The acceleration due to gravity is given as $g = 10\,m/s^2$. There is no friction, the pulleys and the ropes are massless.

Find the direction and the magnitude of the acceleration of m_3.

$Ma = 0 = T_2 - 2T_1$

$\boxed{T_2 = 2T_1}$

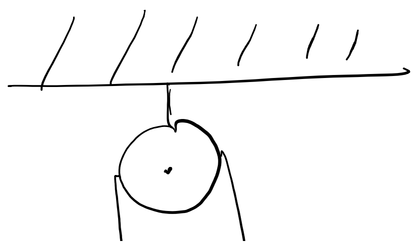


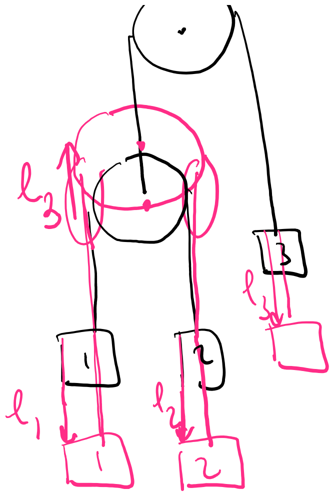


(1) $\boxed{m_1 a_1 = m_1 g - T_1}$

(2) $\boxed{m_2 a_2 = m_2 g - T_1}$

(3) $\boxed{m_3 a_3 = m_3 g - 2T_1}$





$$l_1 + l_2 + 2l_3 = 0$$

$$\frac{d^2}{dt^2}\left(\right.$$

$$\boxed{a_1 + a_2 + 2a_3 = 0} \quad (4)$$

$$(a_1 + a_3) = -(a_2 + a_3)$$

$$m_1 a_1 = m_1 g - T \quad (1)$$

$$m_2 a_2 = m_2 g - T \quad (2)$$

$$m_3 a_3 = m_3 g - 2T \quad (3)$$

$$a_1 = g - T$$

$+$

$$2a_2 = 2g - T \Rightarrow a_2 = g - \frac{T}{2}$$

$+$

$$3a_3 = 3g - 2T \Rightarrow a_3 = g - \frac{2}{3}T \Rightarrow 2a_3 = 2g - \frac{4}{3}T$$

$$\underbrace{a_1 + a_2 + 2a_3}_{\substack{0 \\ \text{by } (4)}} = g - T + g - \frac{T}{2} + 2g - \frac{4}{3}T$$

$$0 = 4g - T\left(1 + \frac{1}{2} + \frac{4}{3}\right)$$

$$4g = T\underbrace{\left(\frac{3}{2} + \frac{4}{3}\right)}_{\frac{17}{6}} \Rightarrow 4g = T\frac{17}{6}$$

$$T = \frac{24}{17}g$$

$$\boxed{T = \frac{240}{17}\,N}$$

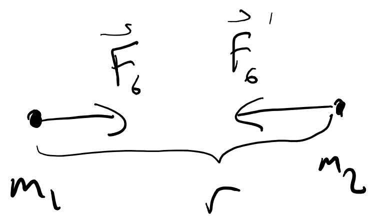
$$m_3 a_3 = m_3 g - 2T$$

$$a_3 = g - \frac{2}{3}T = g - \frac{2}{\cancel{3}}\frac{\cancel{24}^{8}}{17}g$$

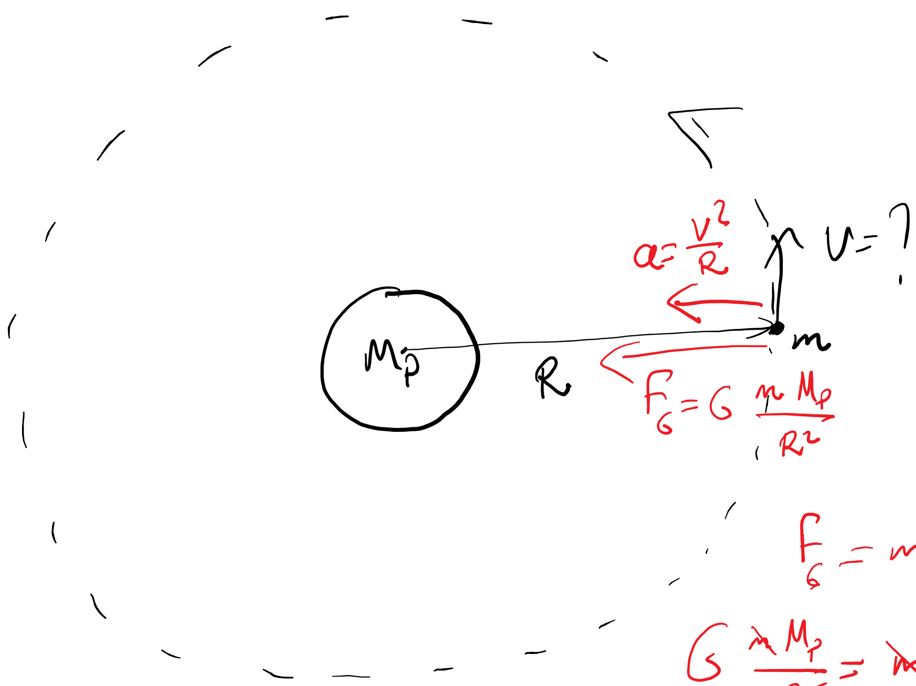
$$a_3 = g\left(1 - \frac{16}{17}\right) = \frac{1}{17} g = \boxed{\frac{10}{17}\ m/s^2}$$

going down!

Gravitation



$$\boxed{F_G = G\,\frac{m_1 m_2}{r^2}}$$



$$F_G = ma$$

$$G\,\frac{\cancel{m} M_p}{R^{\cancel{2}}} = \cancel{m}\,\frac{v^2}{\cancel{R}}$$

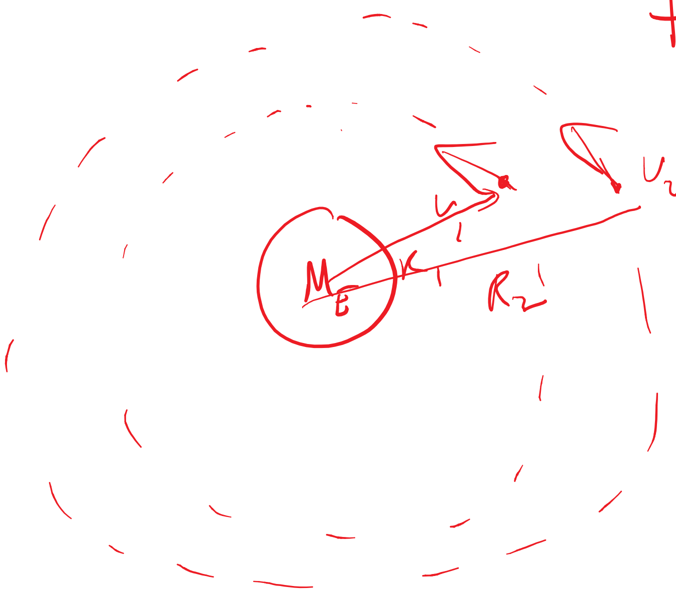
$$v = \sqrt{G\,\frac{M_p}{R}}$$

What is the period of orbital motion?

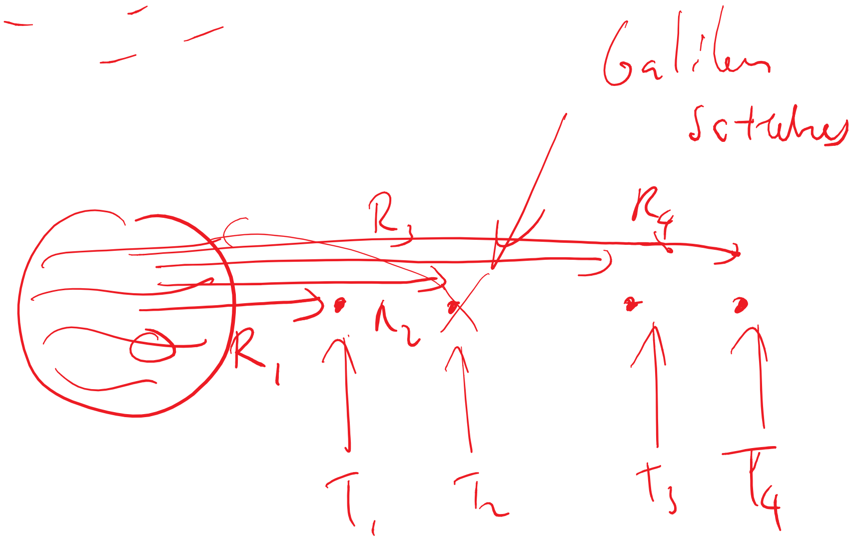
$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\quad} = 2\pi\,\frac{R^{3/2}}{\quad}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{G\frac{M_p}{R}}} = 2\pi\frac{R}{\sqrt{G M_p}}$$

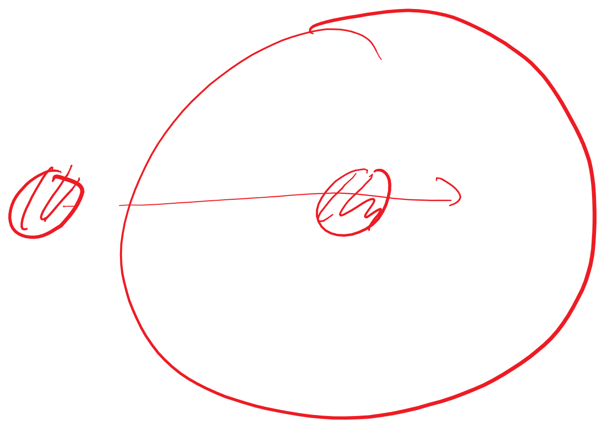
$$T^2 = 4\pi^2 \frac{R^3}{G M_p} \Rightarrow \boxed{\frac{R^3}{T^2} = \frac{G}{4\pi^2} M_p}$$

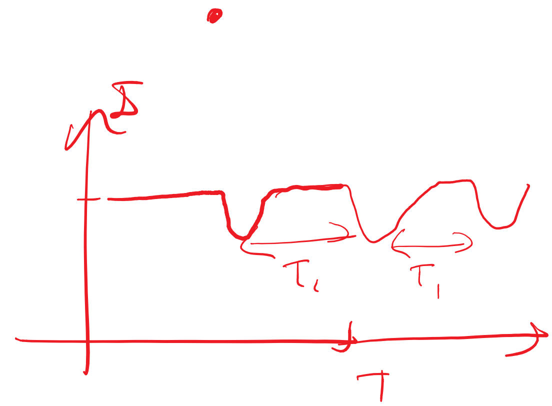


$$\frac{R_1^3}{T_1^2} = \frac{R_2^3}{T_2^2}$$



star

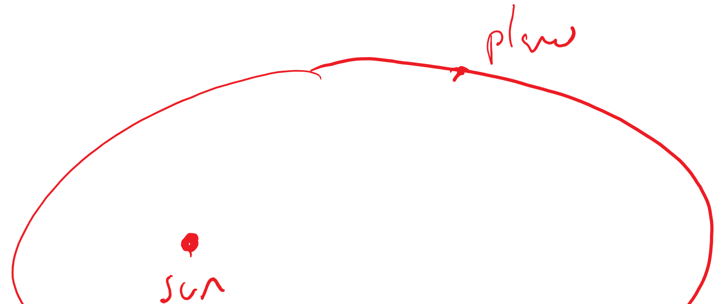




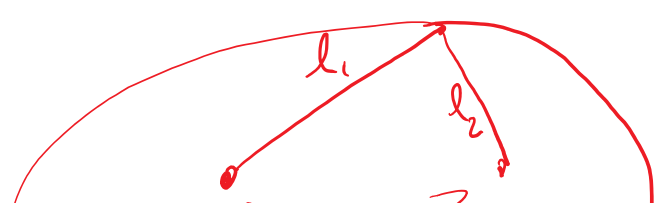
Kepler's Laws (Tycho Brahe)

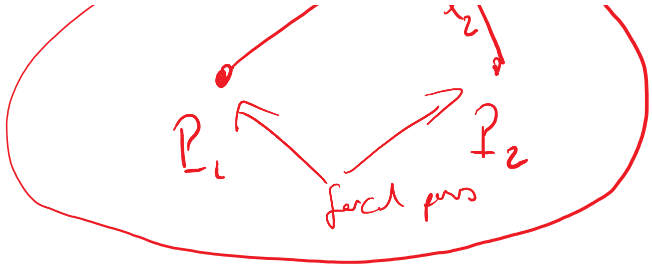
2nd Law $\dfrac{R^3}{T^2}$ is the same for all planets

1st Law Orbit of a satellite is an ellipse with sun at one of the foci.



$l_1 + l_2 = \text{constant}$





2nd Law

(Angular Momentum Conservation)