Week 02 (Oct 3-Oct 7)

Friday, September 23, 2016 2:55 PM

PHYSICS 101 - Instructor: M. Özgür OKTEL- 2016

QUIZ-2

A particle's position is given by

$x(t) = 4 - 12t + 3t^2$

where t is in seconds and x is in meters.

a) What is the particles velocity at t=1 seconds?
b) Is it moving in positive or negative x direction at t=1 seconds?
c) What is the particles speed at t=1 seconds?
d) Is the speed increasing or decreasing at t=1 seconds?
e) Plot the acceleration of the particle as a function of time. What is its maximum value?
f) Is there ever an instant where the velocity is zero? If so give the time t, otherwise answer no

Answer Box

a)	b)	c)	d)	e)	f)

f) $v(t) = 0$

$-12 + 6t = 0$ $\boxed{t = 2\ sec}$ v is zero!

$x(t) = 4 - 12t + 3t^2$

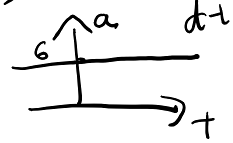
a) $v(t) = \frac{dx}{dt} = \boxed{-12 + 6t}$

$v(t=1) = -12 + 6 = \boxed{-6\ m/s}$

b) Particle is moving towards $-\infty$.

c) $s = |v| = 6\ m/s$

d) $a(t) = \frac{dv}{dt} = 6\ m/s^2$ $a > 0$ $v < 0 \Rightarrow$ speed is decreasing

e) 

$a_{max} = 6\ m/s^2$

$v = 6\ m/s \longleftarrow \bullet \longrightarrow a$ $\frac{dv}{dt} = a$

$v(t+dt) = v(t) + dt\, a$

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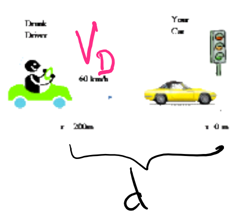
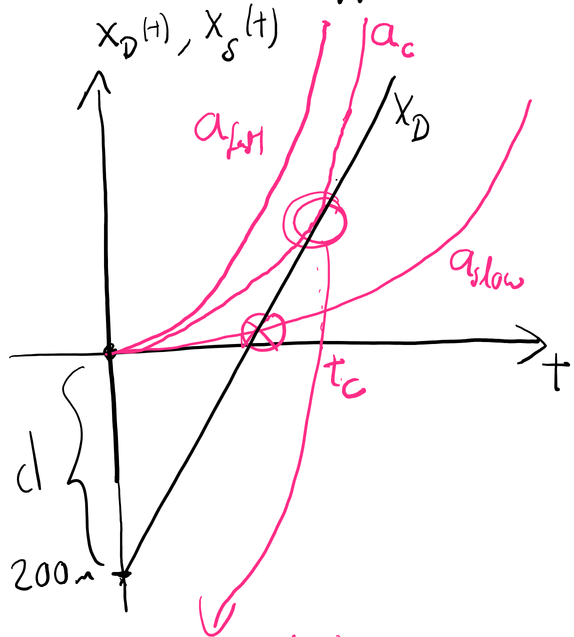
QUIZ-3





You are waiting at a red light in your brand new car. As soon as the light turns green you notice a drunk driver is approaching your car with constant velocity 60 km/h, and is 200 m behind you. You start accelerating immediately.

a) What is the minimum (constant) acceleration you need so that the drunk driver does not catch your car. (Give your result in SI units).
b) Sadly, although your car looks very good, it can only supply half of the acceleration you found in (a). Where does the drunk driver hit your car?

$x_D(t) = -d + V_D t$

$V_D(t) = V_D$

$x_s(t) = \frac{1}{2} a t^2$

$V_s(t) = at$

$x_D(t_c) = x_s(t_c)$

$V_D(t_c) = V_s(t_c)$

$-d + V_D t_c = \frac{1}{2} a t_c^2$ (1)

$V = a t_c$ (2)

$V_S(t) = a t$

$V_D = a t_c$ ②

$$(2) \to t_c = \frac{V_D}{a} \Rightarrow -d + V_D \frac{V_D}{a} = \frac{1}{2} a \frac{V_D^2}{a^2}$$

$$\frac{V_D^2}{a} = \frac{1}{2}\frac{V_D^2}{a} + d$$

$$\frac{1}{2}\frac{V_D^2}{a} = d \Rightarrow \boxed{a = \frac{1}{2}\frac{V_D^2}{d}}$$

1°) Goal ✓ 2°) Units ✓ $[a] = \frac{m}{s^2} = \frac{(m/s)^2}{m} = m/s^2$ ✓

3°) Limits ✓ $V_D \to 0 \quad a \to 0$ ✓

$d \to 0 \quad a \to \infty$ ✓

convert this to m/s 1 hr = 60x60 sec

$$V_D = 60 \text{ km/h} \Rightarrow = 60\,10^3 \text{ m/hour}$$

$$= \frac{6\,10^4}{60\;60} \text{ m/sec}$$

6) $a = \frac{1}{2}\frac{1}{2}\frac{V_D^2}{d} = \frac{1}{4}\frac{V_D^2}{d}$

$$x_D(t) = x_S(t)$$

$$-d + V_D t = \frac{1}{2}\left(\frac{1}{4}\frac{V_D^2}{d}\right)t^2$$

$\Rightarrow t = ?$

$x_D(t) = ?$

d

$$
\begin{tikzcd}
x(t) \arrow[r, bend left, "\frac{d}{dt}"] & v(t) \arrow[l, bend left, "\int dt"] \arrow[r, bend left, "\frac{d}{dt}"] & a(t) \arrow[l, bend left, "\int dt"]
\end{tikzcd}
$$

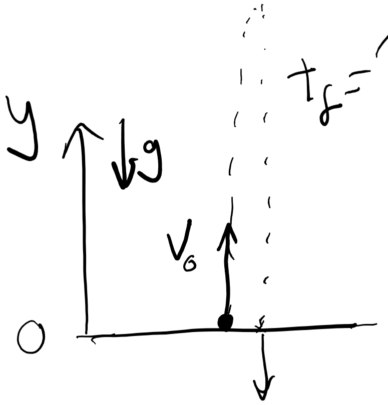
Motion with constant acceleration.

$$a(t) = a$$
$$v(t) = at + v_0$$
$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

Motion on earth's surface has constant gravitational acceleration

$$g \cong 9.82 \ m/s^2$$

1°) Throw an object up and measure the time for it to fall back.



$$a_y = -g$$
$$V(t) = -gt + V_0$$
$$y(t) = -\frac{1}{2}gt^2 + V_0 t + \cancel{y_0} \rightarrow 0$$

$$y(t_f) = 0$$

$y_0=0$

$$y(t_f)=0$$

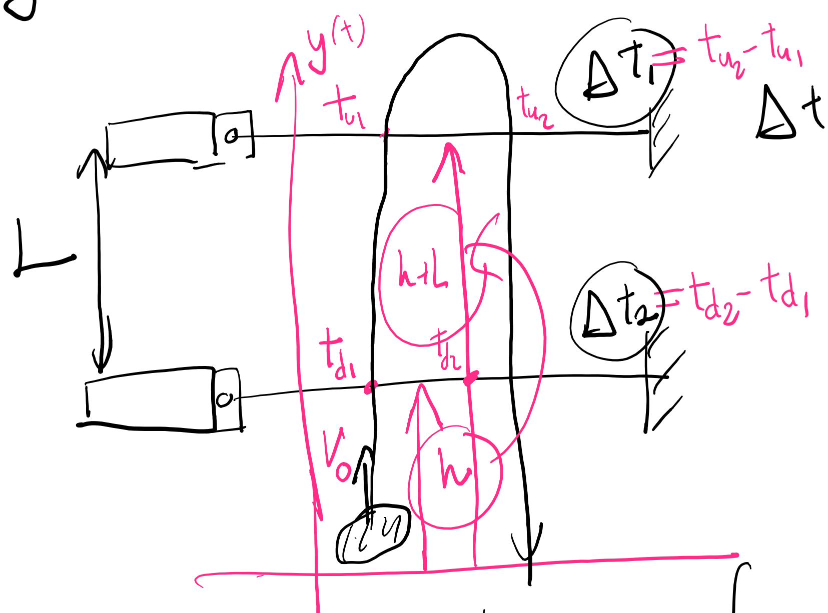
$$-\frac{1}{2}g t_f^2 + v_0 t_f = 0 \nearrow t_f = 0$$

$$v_0 \cancel{t_f} = \frac{1}{2} g t_f^{\cancel{2}}$$

$$\boxed{t_f = \frac{2v_0}{g}}$$

$$\frac{m/\cancel{s}}{m/s^{\cancel{2}}} = s \checkmark$$

$g = 9.822659...$



Δt can be measured very accurately!

Can we calculate g from just $\Delta t_1, \Delta t_2, L$

$$g = f(\Delta t_1, \Delta t_2, L)$$

$$y(t) = v_0 t - \frac{1}{2} g t^2$$

$$y(t_d) = h \Rightarrow v_0 t - \frac{1}{2} g t^2 = h$$

$$0 = \frac{g}{2} t^2 - v_0 t + h$$

$ax^2 + bx + c = 0$

$$0 = \frac{g}{2}t^2 - v_0 t + h$$

$$ax^2 + bx + c = 0$$

$$x = -\frac{b}{2a} \mp \frac{1}{2a}\sqrt{b^2 - 4ac} \qquad t_{d_{1,2}} = \frac{v_0}{2\frac{g}{2}} \mp \frac{1}{2\frac{g}{2}}\sqrt{v_0^2 - 4\frac{g}{2}h}$$

$$t_{d_{1,2}} = \frac{v_0}{g} \mp \frac{1}{g}\sqrt{v_0^2 - 2gh}$$

$$\Delta t_2 = t_{d_2} - t_{d_1}$$

$$= \cancel{\frac{v_0}{g}} + \frac{1}{g}\sqrt{v_0^2 - 2gh} - \left(\cancel{\frac{v_0}{g}} - \frac{1}{g}\sqrt{v_0^2 - 2gh}\right)$$

replace h by h+L

$$\Delta t_2 = \frac{2}{g}\sqrt{v_0^2 - 2gh} \Rightarrow \frac{g^2}{4}(\Delta t_2)^2 = \cancel{v_0^2} - 2g\cancel{h}$$

$$\Delta t_1 = \frac{2}{g}\sqrt{v_0^2 - 2g(h+L)} \Rightarrow -\ \frac{g^2}{4}(\Delta t_1)^2 = \cancel{v_0^2} - 2g(\cancel{h}+L)$$

$$\frac{g^{\cancel{2}}}{4}\left((\Delta t_2)^2 - (\Delta t_1)^2\right) = 2\cancel{g}L$$

$$\boxed{g = \frac{8L}{(\Delta t_2)^2 - (\Delta t_1)^2}}$$

1°) Goal ✓

2°) units

$[L] \to m$

$[\Delta t] \to s$

$\frac{m}{s^2}$ ✓

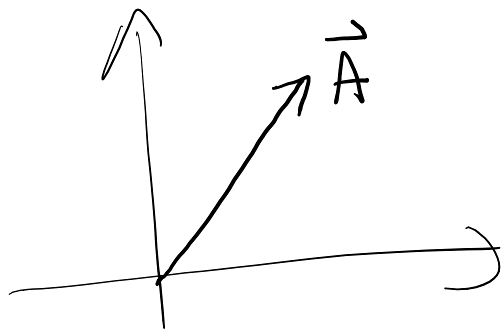
3°) limits ✓

$L \to 0 \quad g \to 0$ ✓

$$g = \underbrace{9.826152983}_{\text{10 significant figures.}}$$

$$\underbrace{\text{Motion in higher dimensions.}}_{\text{Represent all quantities}}$$

$$\boxed{\text{Vectors}}$$

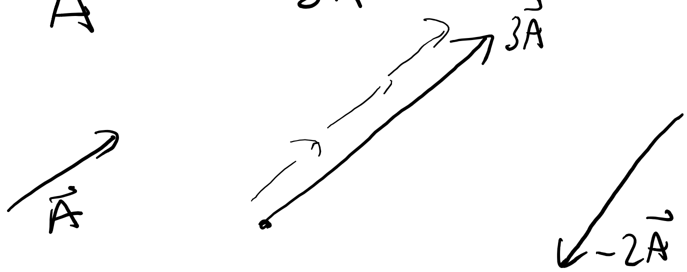


vector : (magnitude, direction)

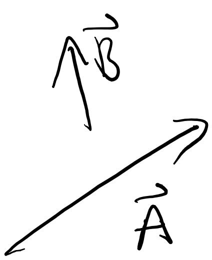
Scalars
~~numbers~~

Vectors can be multiplied by ~~numbers~~ scalars.

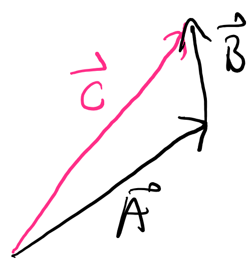
$\vec{A}$ $\qquad 3\vec{A} \qquad -2\vec{A}$



Vectors can be added

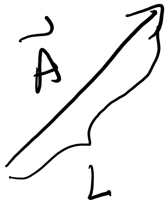


$\vec{C} = \vec{A} + \vec{B}$



The length of a vector is called norm
magnitude

unit vectors:

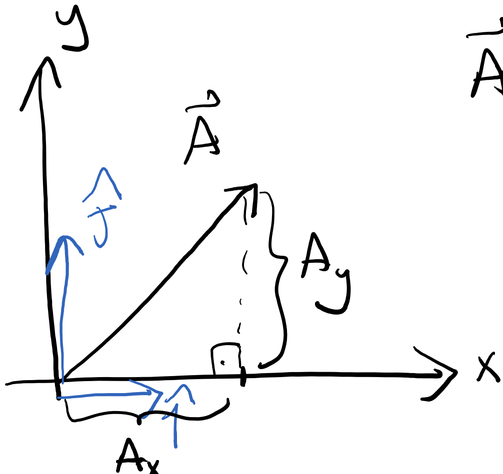


$L = |\vec{A}| \rightarrow$ number

vectors with magnitude 1

$\hat{a}, \hat{b}, \hat{c} \ldots$

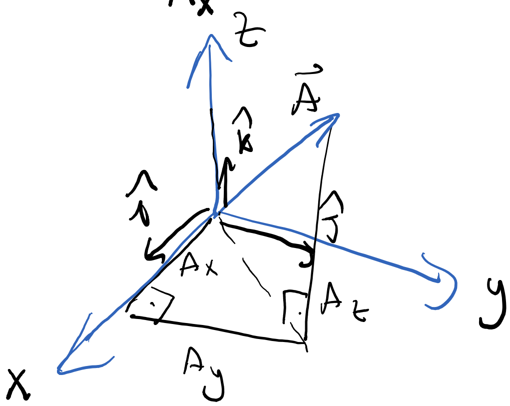
Components of a vector



$$\vec{A} = (A_x, A_y)$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$



$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{A} + \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) + (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

Vector multiplication.

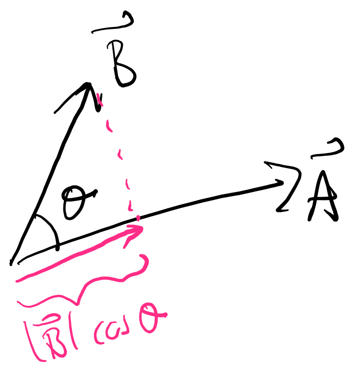
$$\vec{A} \cdot \vec{B} = C$$

number
scalar

Scalar product
dot product.

$\vec{A} \cdot \vec{B}$ → dot product.

Geometric definition

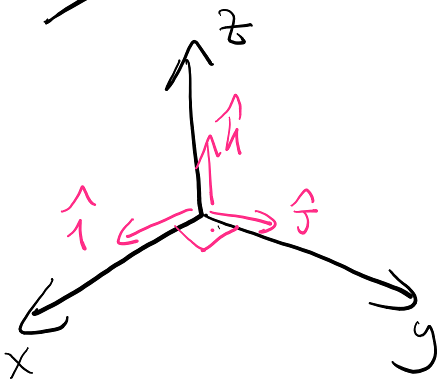


projection of $\vec{B}$ onto $\vec{A}$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

$$\vec{A} \cdot \vec{A} = |\vec{A}||\vec{A}| \underbrace{\cos(0)}_{1} = |\vec{A}|^2$$

$$|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}}$$



$$\hat{\imath} \cdot \hat{\imath} = |\hat{\imath}|^2 = 1$$

$$\hat{\jmath} \cdot \hat{\jmath} = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = \cancel{|\hat{\imath}||\hat{\jmath}| \cos\left(\tfrac{\pi}{2}\right)} \; 0$$

$$\hat{\imath} \cdot \hat{k} = 0$$

$$\hat{\jmath} \cdot \hat{k} = 0$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}) \cdot (B_x \hat{\imath} + B_y \hat{\jmath} + B_z \hat{k})$$

$$= A_x B_x \underbrace{\hat{\imath} \cdot \hat{\imath}}_{1} + \cancel{A_x B_y \hat{\imath} \cdot \hat{\jmath}}^{0} + \ldots \; \cancel{\hat{\imath} \cdot \hat{k}}^{0}$$

$$+ A_y B_y \underbrace{\hat{\jmath} \cdot \hat{\jmath}}_{} + A_z B_z \underbrace{\hat{k} \cdot \hat{k}}_{1}$$

$$\boxed{(\vec{A}\cdot\vec{B}) = A_x B_x + A_y B_y + A_z B_z}$$

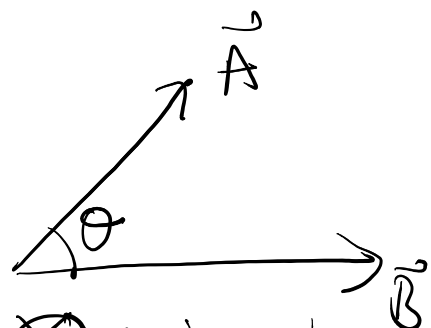
vectors

$$\vec{A} \times \vec{B} = \vec{C}$$

Vector product

cross product.

(This only works in 3D)



$\vec{C} \otimes$ ← into the screen

$\vec{C} = \vec{A} \times \vec{B}$

- Magnitude of $\vec{C}$

  $|\vec{C}| = |\vec{A}||\vec{B}| \sin\theta$

- $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$!

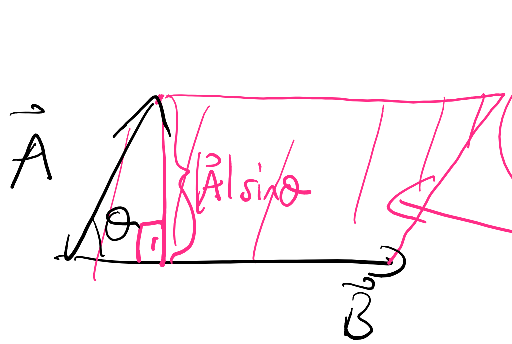
- $\vec{C}$ direction is determined by the right hand rule

$\vec{D} = \vec{B} \times \vec{A}$

$\odot$ coming out of the screen

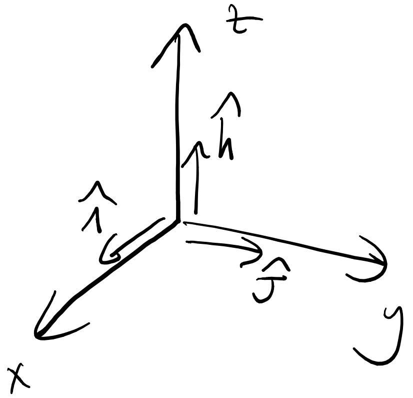
$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$



$$|(\vec{A}\times\vec{B})| = |\vec{A}||\vec{B}|.\sin\theta$$

area spanned by $\vec{A}, \vec{B}$



$$\hat{i}\times\hat{i} = 0 = \hat{j}\times\hat{j} = \hat{k}\times\hat{k}$$

$$\hat{i}\times\hat{j} = \hat{k}$$

$$\hat{j}\times\hat{k} = \hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j}$$

Determinant

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - B_yA_z) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - B_xA_y)$$

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QUIZ-4

Given the following two vectors

$\vec{a} = 4\hat{i} - 3\hat{j} + 12\hat{k}$

$\vec{b} = \hat{i} + \hat{j} + \hat{k}$ $\quad \vec{b}\cdot\hat{i} = 1$

$\alpha = \cos^{-1}\frac{1}{\sqrt{3}}$

a) Calculate $|\vec{a}|$.
b) Find $\vec{a}+\vec{b}$.
c) Find $\vec{c}$ so that $\vec{a}-\vec{b}+\vec{c}=0$
d) Find the angle between $\vec{a}$ and $\vec{b}$.
e) If $\vec{d}=\vec{a}\times\vec{b}$, calculate $|\vec{d}|$.
f) Find the angle between $\vec{b}$ and the x axis.

α $\hat{i}$

$$|\vec{a}|^2 = \vec{a}\cdot\vec{a}$$

$$|\vec{a}| = \sqrt{(4\hat{i}-3\hat{j}+12\hat{k})\cdot(4\hat{i}-3\hat{j}+12\hat{k})}$$

$$= \sqrt{\underbrace{\underbrace{4^2+3^2}_{5^2}+12^2}_{13^2}} = \boxed{13}$$

$$\underbrace{\vec{b}\cdot\hat{i}}_{1} = \underbrace{|\vec{b}|}_{\sqrt{3}}\cos\theta$$

f) Find the angle between b and the x axis.

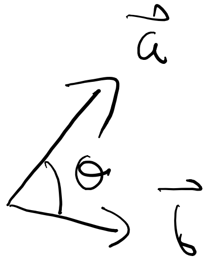
b) $\vec{a}+\vec{b} = 4\hat{i}-3\hat{j}+12\hat{k}+\hat{i}+\hat{j}+\hat{k} = \boxed{5\hat{i}-2\hat{j}+13\hat{k}}$

c) $\vec{a}-\vec{b}+\vec{c}=0$

$$\vec{c}=\vec{b}-\vec{a} = \hat{i}+\hat{j}+\hat{k}-4\hat{i}+3\hat{j}-12\hat{k}$$

$$= \boxed{-3\hat{i}+4\hat{j}-11\hat{k}}$$

d)



$$\vec{a}\cdot\vec{b} = \overset{13}{|\vec{a}|}\,\overset{\sqrt{3}}{|\vec{b}|}\cos\theta \qquad |\vec{b}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

$$(4\hat{i}-3\hat{j}+12\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}) = 13\sqrt{3}\cos\theta$$

$$4-3+12 = 13\sqrt{3}\cos\theta$$

$$\cancel{13} = \cancel{13}\sqrt{3}\cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{3}} \Longrightarrow \boxed{\theta = \cos^{-1}\frac{1}{\sqrt{3}}}$$

e) $\vec{d} = \vec{a}\times\vec{b}$

$$\cos^2\theta+\sin^2\theta = 1$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$\sin\theta = \sqrt{1-\frac{1}{3}}$$

$$= \sqrt{\frac{2}{3}}$$

$$|\vec{d}| = |\vec{a}|\,|\vec{b}|\sin\theta$$

$$= 13\ \cancel{\sqrt{3}}\ \sqrt{\frac{2}{\cancel{3}}}$$

$$\boxed{|\vec{d}| = 13\sqrt{2}}$$

$$\boxed{|\vec{d}| = 13\sqrt{2}}$$

Motion in 3 dimensions

1D

$x(t)$

$\frac{d}{dt}$

$v(t)$

$\frac{d}{dt}$

$a(t)$

$\Longrightarrow$

$\vec{r}(t)$ position vector

$\vec{v}(t)$ velocity vector

speed $= |\vec{v}|$

$\vec{a}(t)$ acceleration vector

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

$$\vec{v} = \frac{d}{dt}\left(\vec{r}(t)\right)$$

$$\vec{v} = \frac{d}{dt}\left(x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}\right)$$

$$= \left(\frac{dx(t)}{dt}\right)\hat{i} + \left(\frac{dy(t)}{dt}\right)\hat{j} + \left(\frac{dz(t)}{dt}\right)\hat{k}$$

$$= v_x(t)\,\hat{i} + v_y(t)\,\hat{j} + v_z(t)\,\hat{k}$$

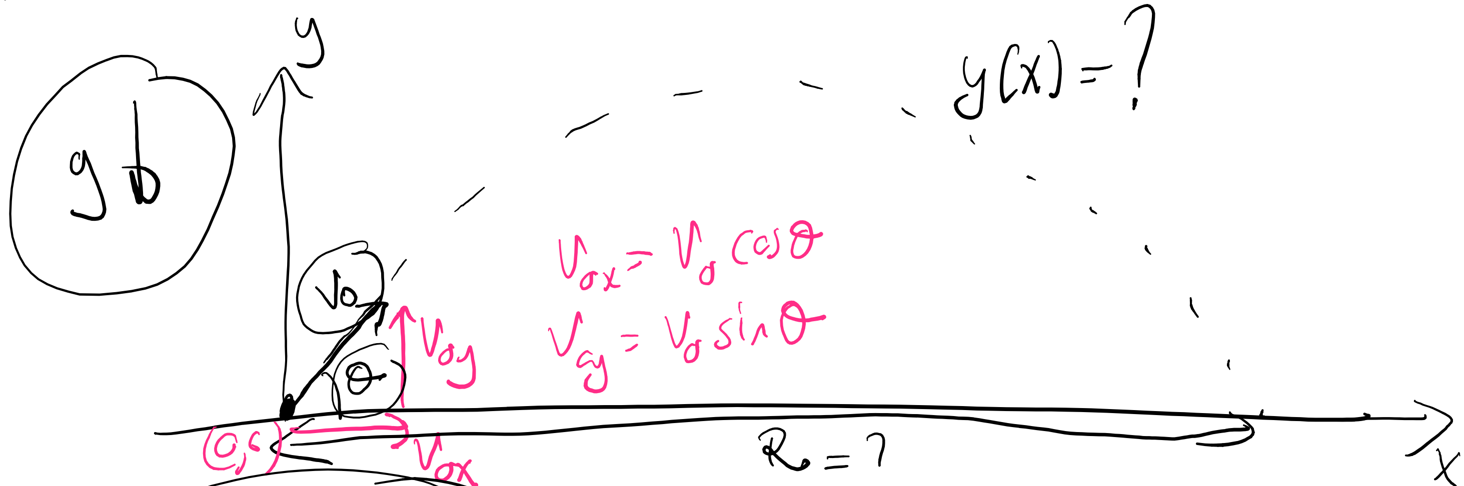
$$\vec{a} = \frac{d}{dt}(\vec{v}) = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$a_x = \frac{d}{dt}v_x(t) = \frac{d^2}{dt^2}x(t)$$

$$a_y = \frac{d}{dt}v_y(t) = \frac{d^2}{dt^2}y(t)$$

$$a_z = \frac{d}{dt}v_z(t) = \frac{d^2}{dt^2}z(t)$$

Projectile motion



x(t) = ?

y(t) = ?

$$\vec{a} = \cancel{0\hat{\imath}} - g\hat{\jmath}$$

$$\vec{V}(t) = V_{0x}\,\hat{\imath} + (V_{0y} - gt)\,\hat{\jmath}$$

$$\vec{r}(t) = \underbrace{(X_0 + V_{0x}t)}_{X(t)}\,\hat{\imath} + \underbrace{\left(y_0 + V_{0y}t - \frac{1}{2}gt^2\right)}_{y(t)}\,\hat{\jmath}$$

motion in $\hat{\imath}$ direction is with constant velocity

motion in $\hat{\jmath}$ direction is motion with constant acceleration

Range ?

$$\boxed{\begin{aligned} x(t) &= V_0(\cos\theta)t \\ y(t) &= V_0\sin\theta\, t - \frac{1}{2}gt^2 \end{aligned}}$$

time of flight t_f $\quad y(t_f) = 0$

$$V_0\sin\theta\, \cancel{t_f} - \frac{1}{2}g t_f^{\cancel{2}} = 0$$

$$\frac{1}{2}g t_f = V_0\sin\theta \Rightarrow \boxed{t_f = \frac{2V_0\sin\theta}{g}}$$

$$R = x(t_f) = V_0\cos\theta\, t_f$$

$$\boxed{R = \frac{2V_0^2\sin\theta\cos\theta}{g}}$$

$$t = \frac{x}{V_0\cos\theta}$$

$$x(t) = V_0\cos\theta\, t$$

$y(x) = ?$

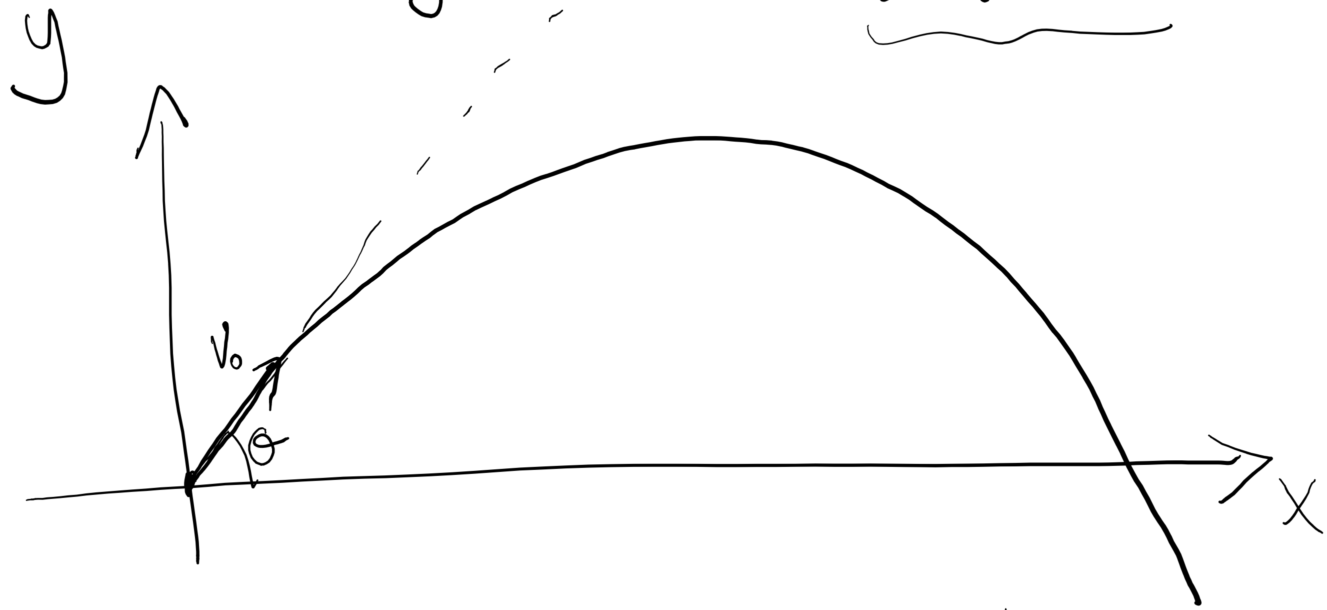
$$y(x) = ?$$

$$x(t) = V_0 \cos\theta \, t$$

$$y(t) = V_0 \sin\theta \, t - \frac{1}{2} g t^2$$

$$y(x) = \cancel{V_0} \sin\theta \frac{x}{\cancel{V_0}\cos\theta} - \frac{1}{2} g \frac{x^2}{V_0^2 \cos^2\theta}$$

$$y(x) = x \tan\theta - \underbrace{\frac{1}{2}\left(\frac{g}{V_0^2}\right)\frac{x^2}{\cos^2\theta}}$$



For a given speed V_0, which angle gives the <u>maximum</u> range

$$R(\theta) = \frac{2 V_0^2 \sin\theta \cos\theta}{g} = \frac{V_0^2}{g} \sin 2\theta \qquad \frac{dR}{d\theta} = 0$$

$$\frac{dR}{d\theta} = \frac{2 V_0^2}{g} \frac{d}{d\theta}(\sin\theta \cos\theta) = \frac{2 V_0^2}{g}(\cos^2\theta - \sin^2\theta) = 0$$

$$\frac{\quad}{d\theta} = \frac{2v_0}{g}\frac{d}{d\theta}(\sin\theta\cos\theta) = \frac{\quad}{g}(\cos\theta - \sin\theta\ldots = 0$$

$$\underbrace{\cos\theta\underbrace{\frac{d}{d\theta}\sin\theta}_{\cos\theta} + \sin\theta\underbrace{\frac{d}{d\theta}\cos\theta}_{-\sin\theta}}$$

$$\cos^2\theta = \sin^2\theta$$

$$\boxed{\theta = \frac{\pi}{4}}$$