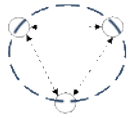
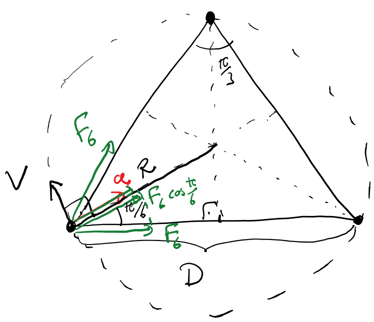
Week 06 (Oct 31 - Nov 4)

Friday, September 23, 2016 2:57 PM

Phys 101- Instructor: M. Özgür OKTEL – 2016 QUIZ-14

Assume that three planets each with the same mass M/3 form an equilateral triangle of side D. What should be their rotation **period** around their common center of mass (which is the center of the dashed circle in the figure) so that the system is in equilibrium. (This configuration is in general known as Klempler's rosette)





$$\underbrace{R\cos\frac{\pi}{6}}_{\frac{\sqrt{3}}{2}} = \frac{D}{2}$$

$$\boxed{R = \frac{1}{\sqrt{3}} D}$$

$$|\vec{a}| = \frac{v^2}{R}$$

$$F_{net} = 2F_G \cos\frac{\pi}{6} = \cancel{2}\, G\frac{(M/3)(M/3)}{D^2}\frac{\sqrt{3}}{\cancel{2}}$$

$$F_{net} = \left(\frac{M}{3}\right) a$$

$$G\frac{M^{\cancel{2}}/9}{D^{\cancel{2}}}\cancel{\sqrt{3}} = \frac{\cancel{M}}{3}\frac{v^2}{\frac{\cancel{D}}{\cancel{\sqrt{3}}}}$$

$$v^2 = \cancel{3}\, G\frac{M}{\underset{3}{\cancel{9}}D} \Rightarrow v = \sqrt{G\frac{M}{3D}}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi\, D/\cancel{\sqrt{3}}}{\sqrt{G\frac{M}{\cancel{3}D}}} = \frac{2\pi D^{3/2}}{\sqrt{GM}}$$

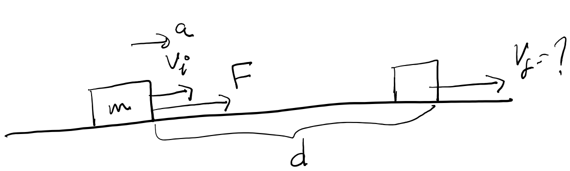
$$[T] = sec = \frac{m^{3/2}}{\sqrt{\frac{m^3}{\cancel{kg}\, s^2}\cancel{kg}}} = s\frac{\cancel{m^{3/2}}}{\cancel{m^{3/2}}} = s$$

1°) Calc ✓

2°) Units ✓

3°) Limit ✓

Work and Kinetic Energy



$$a = \frac{F}{m} \qquad v_f = v_i + at$$

$$t = \frac{v_f - v_i}{}$$

$$\overline{\left(\frac{F}{m}\right)}$$

$$d = v_i t + \frac{1}{2} a t^2$$

$$= v_i \frac{(v_f - v_i)}{F/m} + \frac{1}{2}\cancel{\left(\frac{F}{m}\right)} \frac{(v_f - v_i)^2}{\left(\frac{F}{m}\right)^{\cancel{2}}}$$

$$\frac{F}{m} d = v_i (v_f - v_i) + \frac{1}{2}(v_f - v_i)^2$$

$$F d = m (v_f - v_i)\left[v_i + \frac{1}{2} v_f - \frac{1}{2} v_i\right]$$

$$F d = m (v_f - v_i)\frac{1}{2}\left[v_i + v_f\right] = \underbrace{\frac{1}{2} m v_f^2}_{K} - \frac{1}{2} m v_i^2$$

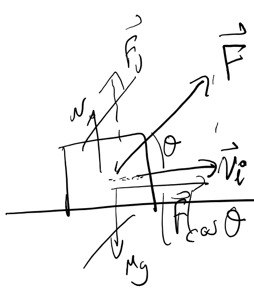
$$\boxed{K = \frac{1}{2} m v^2} \qquad \boxed{W = Fd}$$

$$\Uparrow$$

$$\Delta K = K_f - K_i = F d$$

$$\boxed{\Delta K = W}$$

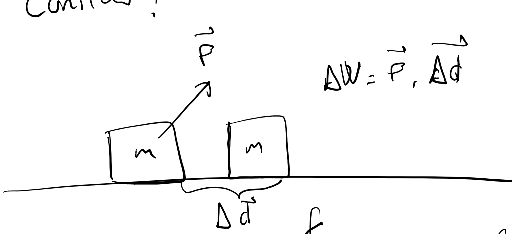
Work-Energy theorem



$$\boxed{W = \vec{F} \cdot \vec{d}}$$

$$\boxed{\Delta K = W}$$

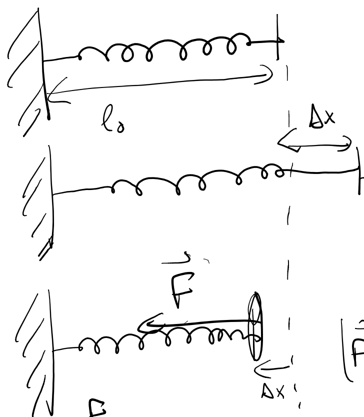
What if the force is not a constant?



$$\Delta W = \vec{F} \cdot \Delta\vec{d}$$

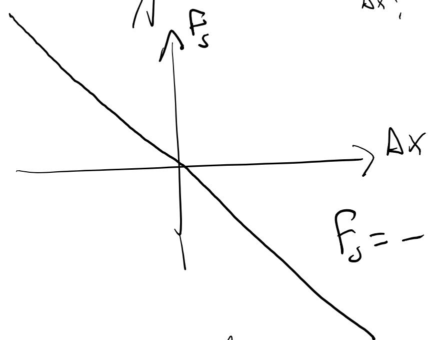
$$W = \int_i^f \vec{F} \cdot \vec{dl} \quad \left(\text{line integral}\right)$$

Ideal Spring

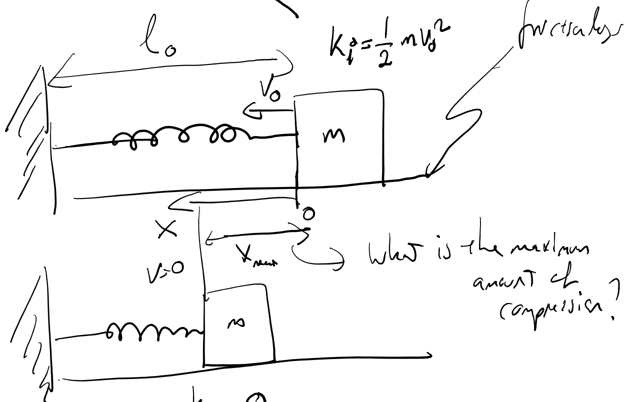


$|F| = k\,|\Delta x|$

$|\vec{F}| = k\,|\Delta x|$



$F_s = -k\,\Delta x$



what is the maximum amount of compression?

$K_f = 0$

$$\Delta K = K_f - K_i = -\frac{1}{2} m v_0^2 = W$$

$$= \int \vec{F}_s \cdot d\vec{l}$$

$$= -\int_0^{x_{max}} k x \, dx$$

$$-\frac{1}{2} m v_0^2 = -k \underbrace{\int_0^{x_{max}} x \, dx}$$

$F = -kx$

$\frac{N}{m} = [k]$

$$\cancel{-}\frac{1}{\cancel{2}} m v_0^2 = \cancel{-}k \frac{x_{max}^2}{\cancel{2}}$$

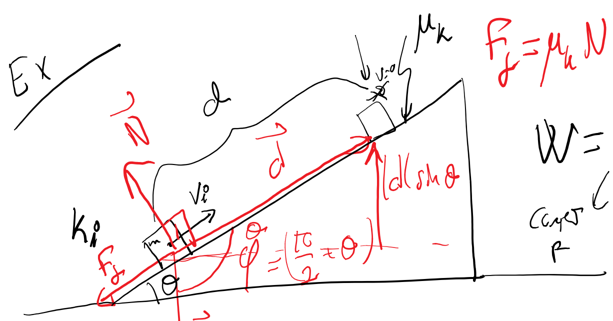
$$\boxed{x_{max} = \sqrt{\frac{m}{k}}\, v_0}$$

$[x_{max}] = m$

$$\sqrt{\frac{kg}{\frac{kg\, m/s^2}{m}}}\ m/s$$

μ_k $F_f = \mu_k N = \mu_k mg \cos\theta$

Ex

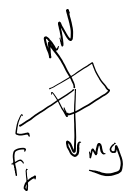


$F_f = \mu_k N = \mu_k m g \cos\theta$

$W = \int \vec{F} \cdot d\vec{l}$

const. F $= \vec{F} \cdot \vec{d}$

$K_i = \frac{1}{2} m v_i^2$ $\quad K_f = 0 \quad \Delta K = -\frac{1}{2} m v_i^2$



$W = W_g + W_f + W_N$

$W_g = m\vec{g} \cdot \vec{d} = mg\,d \cos\left(\frac{\pi}{2}+\theta\right)$, where $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$W_f = \vec{f}_f \cdot \vec{d} = -f_f d$

$W_N = \vec{N} \cdot \vec{d} = 0$ as $\vec{N} \perp \vec{d}$

$$W = -mgd\sin\theta - F_f d$$

$$\Delta K = -\frac{1}{2} m v_i^2 = (-mg\sin\theta - F_f)\, d$$

$$\cancel{-}\frac{1}{2}\cancel{m} v_i^2 = \cancel{-}(\cancel{m}g\sin\theta + \mu_k \cancel{m}g\cos\theta)\, d$$

$$\frac{1}{2} v_i^2 = g\,d(\sin\theta + \mu_k\cos\theta)$$

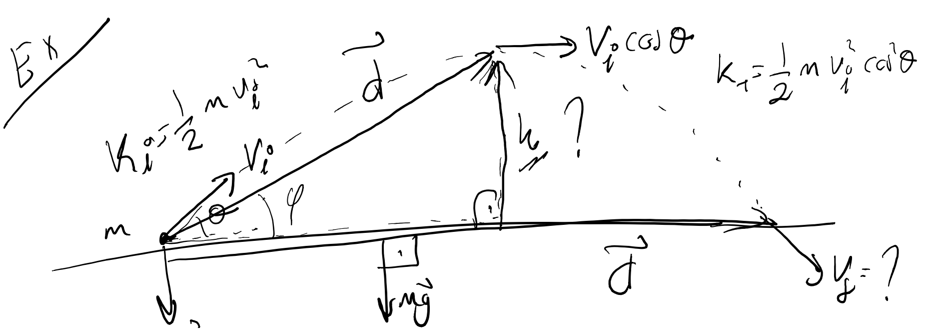
$$\boxed{d = \frac{v_i^2}{2g(\sin\theta + \mu_k\cos\theta)}}$$

1°) Goal

2°) $\frac{m^2/s^2}{m/s^2} = m$ ✓

3°) $v_i \to 0 \quad d \to 0$ ✓

Ex



$K_i = \frac{1}{2} m v_i^2$ $\quad K_T = \frac{1}{2} m v_i^2 \cos^2\theta$

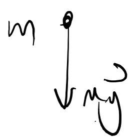
$K_T - K_i = \vec{F} \cdot \vec{d}$

$= -mgh$

$\vec d$ $\quad v_f = ?$

$= mg$ ...

$\frac{1}{2}\cancel{m} v_i^2(\cos^2\theta - 1) = -\cancel{m} g h$



$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = W$$

$$= \int \vec F \cdot d\vec l$$

$$= \vec F \cdot \vec d$$

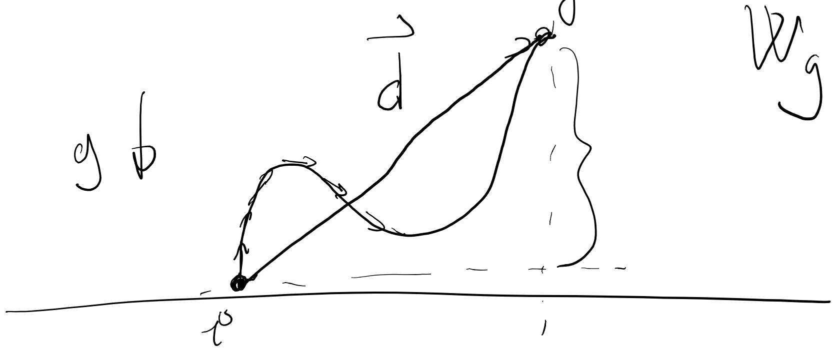
$$= \underbrace{m\vec g \cdot \vec d}$$

$$h = \frac{v_i^2}{2g}(1-\cos^2\theta)$$

$$\boxed{h = \frac{(v_i \sin\theta)^2}{2g}}$$

$$v_i^2 = v_f^2 \Rightarrow \boxed{v_f = v_i}$$

$K \rightarrow$ scalar!



$$W_g = \int_i^f \vec F \cdot d\vec l \qquad \vec F \rightarrow (m\vec g)$$

$$= m\vec g \cdot \underbrace{\int_i^f d\vec l}_{\vec d}$$

$$W_g = m\vec g \cdot \vec d$$

$$= -mgh$$

$$\Delta K = W$$

$$\Delta K = \underbrace{W_g} + W_F$$

$-mgh$

$$\Delta K + mgh = W_f$$

$$\left(\frac{1}{2} m v_f^2 + \underbrace{mg\, h_f}\right) - \left(\frac{1}{2} m v_i^2 + mg h_i\right) = W_f$$

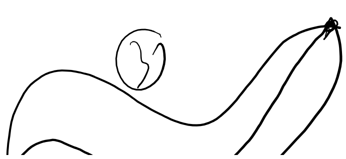
$U_g = mgh \Rightarrow$ Gravitational Potential energy!

$$E = K + U_g$$

$$\boxed{\Delta E = W_f}$$

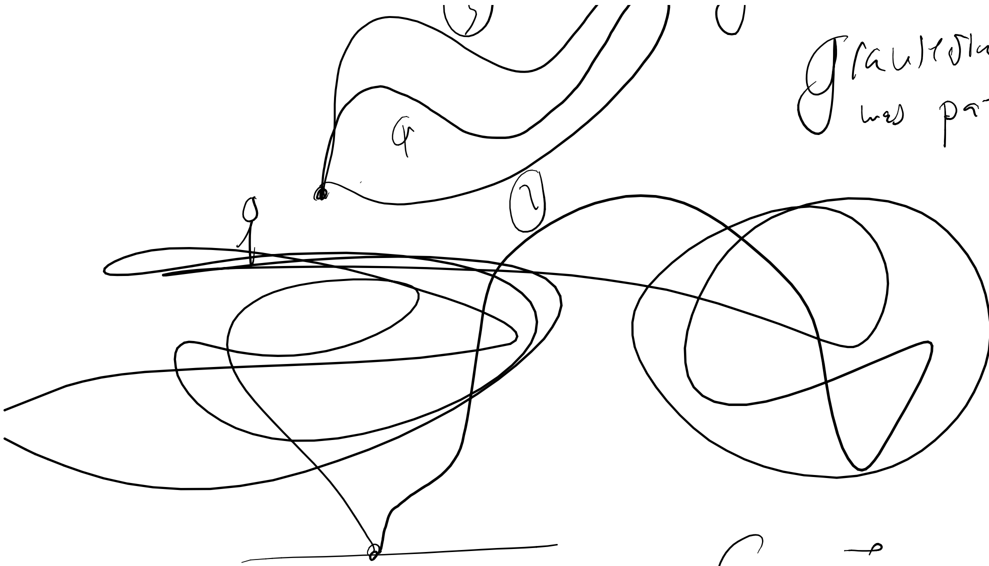
Can we define a potential energy for the other forces as well?

$$W = \int \vec{F} \cdot \vec{dl} = -mg(h_f - h_i)$$
$$= -(U_{g(\text{final})} - U_{g(\text{initial})})$$



$\int$

Gravitational work ... the independent

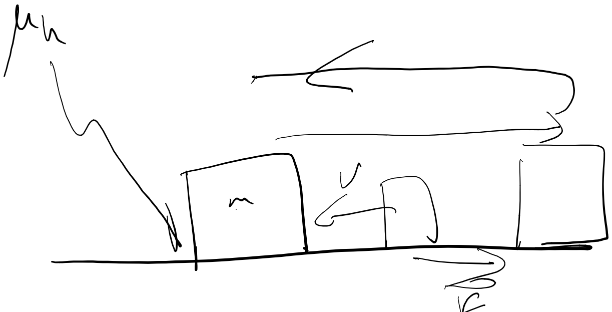


Gravitational work was path independent

$$\oint \vec{F}_g \cdot d\vec{l} = 0$$

For any force if $\oint \vec{F} \cdot d\vec{l} = 0$

a potential energy can be defined!



$$\vec{v} \cdot \vec{F} < 0$$

$$\oint \vec{F}_f \cdot d\vec{l} < 0$$

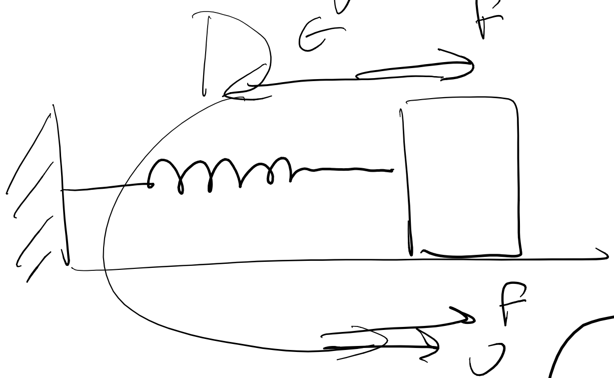
$$\neq 0$$

Friction is path dependent

$\Longrightarrow$ No potential possible

Nonconservative force (Dissipative force)

Nonconservative force (Dissipative force)



$\vec{v}, \vec{F}$ changes sign

$$\oint \vec{F} \cdot d\vec{l} = 0$$

$\boxed{\text{Conservative force}}$

$$\boxed{U_s = \frac{1}{2} k x^2}$$

compression or stretching!

$$E = K + U_g + U_s + \ldots\ldots$$

mechanical energy

$$\boxed{\Delta E = W_{n.c}}$$

Ex

$\Delta = ?$

Ex

$\Delta = ?$

$$E_i = mg(h + l_0) \qquad E_f = \frac{1}{2}k\Delta^2 + mg(l_0 - \Delta)$$

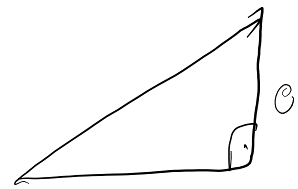
$$mgh + \cancel{mgl_0} = \frac{1}{2}k\Delta^2 + \cancel{mgl_0} - mg\Delta$$

$$mgh = \frac{1}{2}k\Delta^2 - mg\Delta$$

$$0 = \frac{k}{2}\Delta^2 - mg\Delta - mgh$$

$$\Delta = \frac{mg}{k} \mp \frac{1}{k}\sqrt{m^2g^2 + 4\frac{k}{2}mgh}$$

$$= \frac{mg}{k} \mp \sqrt{\left(\frac{mg}{k}\right)^2 + 2\frac{mg}{k}h}$$

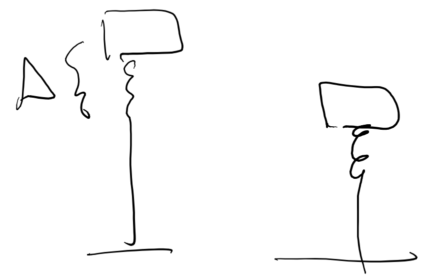


$\frac{mg}{k}$ 2

$h=0$

$\Delta = ?$

$$\boxed{\Delta = \frac{mg}{k} + \sqrt{\left(\frac{mg}{k}\right)^2 + 2\frac{mg}{k}h}}$$



$$mg\Delta = \frac{1}{2}k\Delta^2$$

$$\boxed{\frac{2mg}{k} = \Delta}$$

Nov 14-18

No Phys 101 classes

We'll have make-up lectures

NAME:	
STUDENT NO:	
SIGNATURE:	
DEPARTMENT	
SECTION	

PHYSICS 101- Instructor: M. Özgür OKTEL- 2016

QUIZ-12

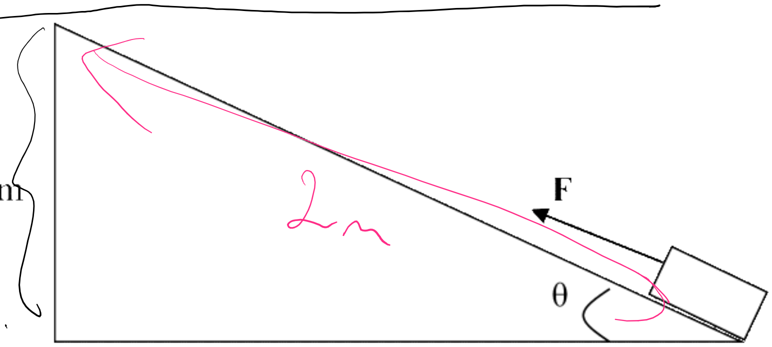
A mass m=1kg is at the bottom of an inclined plane as shown in the figure. What is the minimum work that is needed to be done by the force $\vec{F}$, (directed as shown in the figure) to get the mass to the top of the inclined plane if

a) There is no friction between the mass and the plane.

b) The friction coefficient between the mass and the plane is $\mu = 1/\sqrt{3}$.

(Assume g= 10 m/s^2, the inclination of the plane is θ=30 degrees, and the height of the inclined plane is 1m)

ref line



$$\Delta E = E_f - E_i = K_f + U_f - (K_i + U_i)$$

$= 0$ for minimum wo

$$a) \quad E_i = \overset{U_i}{-mgh} + \overset{K_i}{0} = -1kg\ 10\ m/s^2\ 1m = -10\ kg \ldots$$

J

$$E_f = \overset{K_f}{0} + \overset{U_f}{0} = 0$$

$$\Delta E = E_f - E_i$$

W

W

$) = \underline{\underline{W}}$

h

$\frac{m^3}{s^2}$

calc

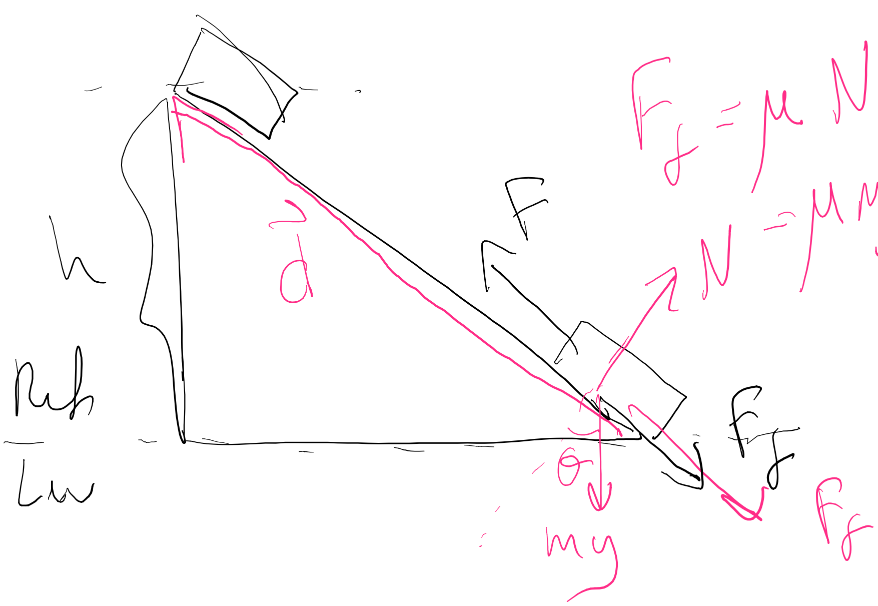
$= (10Ts)$

$= 0 - (-10)$

$$\boxed{W = 10\,J}$$

$\Delta E = W$

6



$F_f = \mu N$

$N = \mu mg\cos\theta$

$E_i = U_i^0 + K_i^0$

$= 0 + 0 =$

$E_f = mgh + K_f^0$

$\Delta E = E_f - E_i =$

$$\Delta E = W$$

work done

$$= W_F + W_\mu$$

$$W_\mu = \vec{F_f} \cdot \vec{d} = -\mu mg\cos\theta\, d$$

$= \boxed{10 \text{ Joules}}$

0 J

$= 10 \text{ J}$

10 J

by friction

$$= -\frac{1}{\cancel{3}} 2\, 10\, \frac{\cancel{3}}{\cancel{2}}\, \cancel{2} =$$

$$\Delta E = W_f + W_\mu$$

$$10 = W_f - 10 \Rightarrow \boxed{W_f = 2\ldots}$$

$$[\text{Energy}] = \text{Joules} = kg\, m^2/s^2$$

Calories

KWh

Define <u>Power</u>

$$P = \frac{dE}{dt} \Rightarrow [P] = \frac{[E]}{s} = \frac{kg\ldots}{s\ldots}$$

$-10J$

$\boxed{20J}$

$\frac{\text{kg}\ \text{m}^2/\text{s}^2}{\text{s}}$

$$P = \frac{\quad}{dt} \Rightarrow$$

$= kg$

$= Watt$

Horsepower $\cong$ 750 watt.

Power

Energy

$$K = \frac{1}{2} m v^2$$

$$P = \frac{d}{dt}\left(\frac{1}{2} m v^2\right) = \frac{m}{2} \frac{d}{dt}(v^2) = \frac{m}{\cancel{2}} \cancel{2}$$

$$= V \underbrace{m}_{F}$$

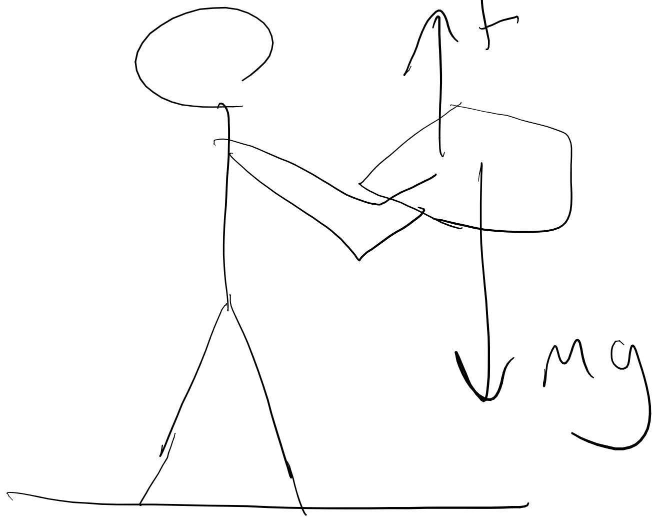
$$\boxed{P = \vec{F} \cdot \vec{v}}$$

m^2/s^3

t

$V \frac{dV}{dt}$

$a = FV$



$$W = \vec{F} \cdot \vec{d}$$

Summary so far

$$\Delta E = W$$

$$E = K + U$$

$$K = \frac{1}{2} m v^2$$

$$W = \int_i^f \vec{F} \cdot d\vec{r}$$

$$= \vec{F} \cdot \vec{d}$$

constant force

$= 0$

$\overrightarrow{d\theta}$

$\vec{d}$

Constant force

Can have different sources

All conservative forces can be expressed in terms of a potential.

$$U_s = \frac{1}{2} k x^2 \quad \text{(spring}$$

$$U_g = m g h \quad \text{(} h \text{ is th... above a ...}$$

We derived this for $\underbrace{\vec{F}_g = m...}_{\text{only an appro...}}$

$$\vec{F}_{\cdot} = G \frac{M_E m}{r^2}$$

e height

ref. line

$\vec{G}$

ximum

$R \rightarrow$ distance

$$F_G = G \frac{\quad}{R^2}$$

Potential energy for gravitation.

Ex

m drop a

$W_r = ?$ $\quad F_G = G \frac{M_E}{x^2} m$

all forces

$$\Delta K = W$$

$$= \int_i^f \vec{F} \cdot d\vec{l} = \int_r^{R_E} G \frac{M_E}{x^2} m$$

$$G M_E m \left[\frac{-1}{\quad} \right]^{R_E}$$

$R \to$ distance from the center of the earth.

near(?)
from distance r

$$\int \frac{1}{x^2}\,dx = \frac{-1}{x}$$

$(\,dx\,)$

$$= -G M_E m \left[-\frac{1}{x} \right]_r$$

$$K_f - K_i^{0} = -G M_E m \left[\frac{1}{r} - \frac{1}{R_E} \right.$$

$$\frac{1}{2} m v_f^2 = -G M_E m \left[\frac{1}{r} - \frac{1}{R_E} \right.$$

$$\underbrace{\frac{1}{2} m v_f^2}_{K_f} \underbrace{- G \frac{M_E m}{R_E}}_{U_G} = -G \frac{M_E m}{r}$$

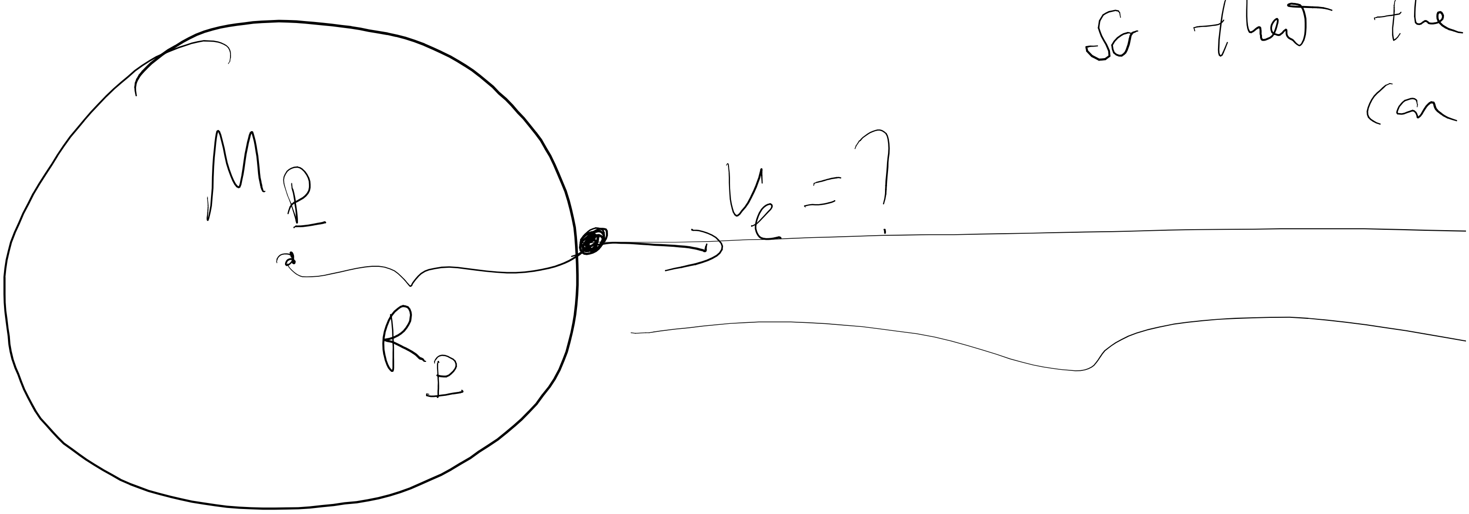
m_1 m_2

$$\boxed{U_G = -G\frac{m_1 m_2}{r}}$$

what should v

so that the

can

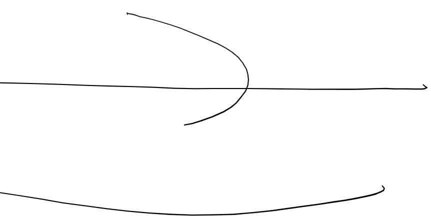


$$E_1 = \frac{1}{2} m v_e^2 - G\frac{M_P m}{R_P}$$

i? be

spaceship

escape?



$$E_f = \cancel{K_f}^{\,0} - \cancel{G\frac{Mm}{\infty}}^{\,0}$$

$$E_f = 0$$

$E_i = E_f$

$$\frac{1}{2}\cancel{m}V_e^2 - G\frac{M_E\cancel{m}}{R_E} = 0$$

$$V_e = \sqrt{2G\frac{M_E}{R_E}}$$