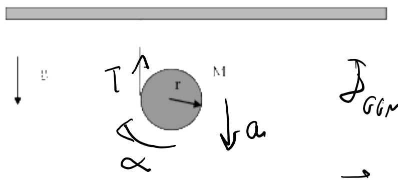
Week 11 (Dec 5 - Dec 9)

Friday, September 23, 2016 3:00 PM

Phys 101- Instructor: M. Özgür OKTEL – 2016 QUIZ-21

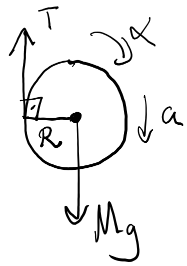
A disk of radius r and mass M is suspended from the ceiling by a string that is wound around the disk. When the disk is left free it starts falling down and rotating, unwinding the string. Find, the acceleration of the disk, the angular acceleration of the disk and the tension in the string. (Assume that the string always stays vertical)



$$I_{COM} = \frac{MR^2}{2}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

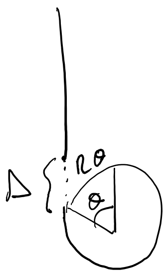
Free body diagram



(1) $$\boxed{Ma = Mg - T}$$

$$\tau_{com} = I_{com}\,\alpha$$

(2) $$\boxed{TR = \frac{MR^2}{2}\,\alpha}$$



$$\Delta = R\theta$$

$$\Downarrow$$

(3) $$\boxed{a = R\alpha}$$

$$T = \frac{M}{2}\underbrace{R\alpha}_{a} = \frac{Ma}{2}$$

$$Ma = Mg - \frac{Ma}{2}$$

$$Ma(1+\tfrac{1}{2}) = Mg \Rightarrow \boxed{a = \frac{2}{3}g}$$

$$\boxed{\alpha = \frac{a}{R} = \frac{2}{3}\frac{g}{R}}$$

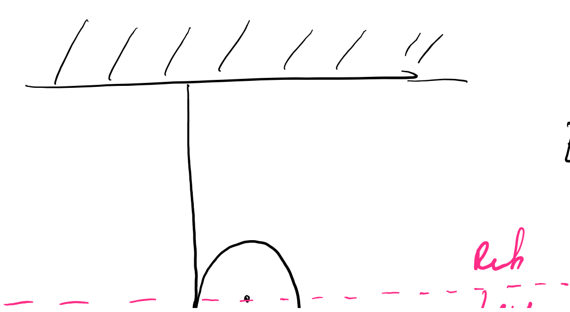
$$T = \frac{Ma}{2} = \frac{\cancel{M}}{\cancel{2}}\frac{\cancel{2}}{3}g = \boxed{\frac{Mg}{3}}$$

1°) Gad ✓

2°) Units $[\alpha] = \frac{1}{s^2}$ $\left[\frac{g}{R}\right] = \frac{\cancel{m}/s^2}{\cancel{m}} = \frac{1}{s^2}$ ✓

3°) Limit $g = 0 \Rightarrow \alpha = 0$

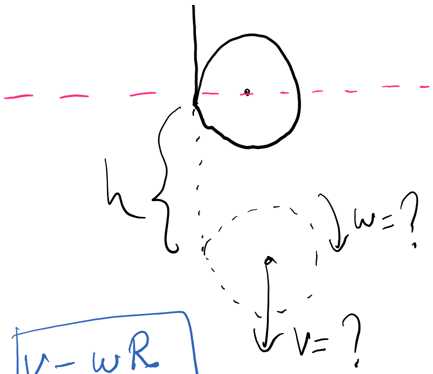
2nd Solution



Energy is conserved

Ref. $E_i = 0$

I_{com}



Ref Lev $\quad E_i = 0$

$$\boxed{V = \omega R}$$

$$E_f = -Mgh + \frac{1}{2}Mv^2 + \frac{1}{2}\underbrace{\frac{MR^2}{2}}_{I_{com}}\underbrace{\omega^2}_{\left(\frac{V}{R}\right)^2}$$

$$E_i = E_f$$

$$0 = -\cancel{M}gh + \frac{1}{2}\cancel{M}v^2 + \frac{1}{2}\frac{\cancel{M}\cancel{R^2}}{2}\frac{v^2}{\cancel{R^2}}$$

$$gh = \underbrace{\left(\frac{1}{2} + \frac{1}{4}\right)}_{\frac{3}{4}}v^2 \Rightarrow \boxed{V = \sqrt{\frac{4}{3}gh}}$$

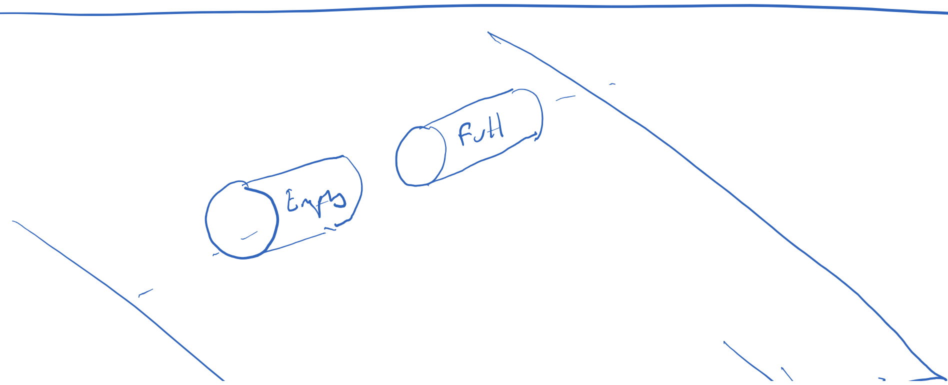
The disc falls with constant acceleration a

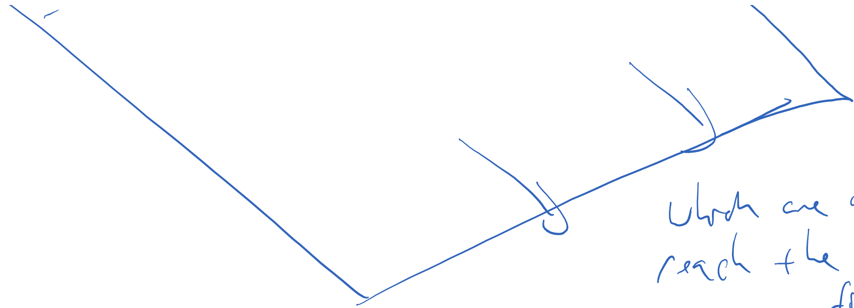
$$\frac{1}{2}at^2 = h \Rightarrow t = \sqrt{\frac{2h}{a}}$$

$$V = at = \sqrt{2ah}$$

$$\sqrt{2a\cancel{h}} = \sqrt{\frac{4}{3}g\cancel{h}}$$

$$\boxed{a = \frac{2}{3}g}$$

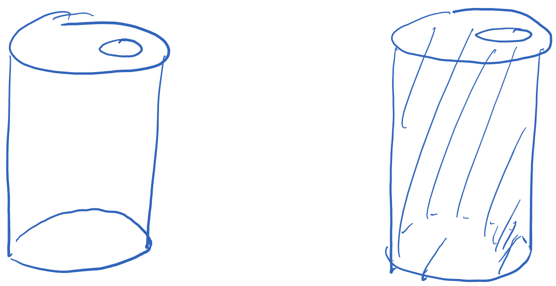




Which one will reach the bottom first.

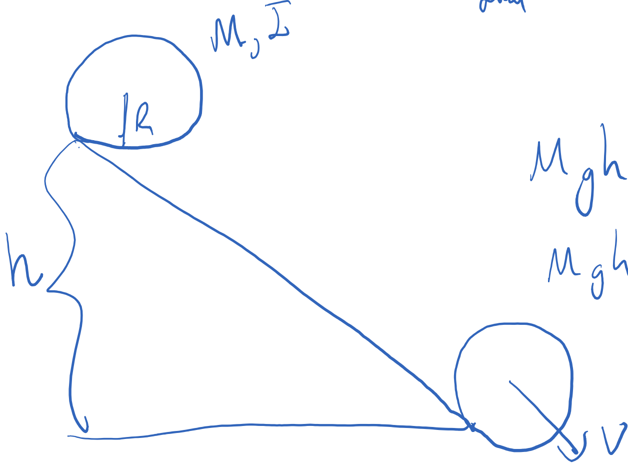
Filled	Empty	Equal	Depends on the order
13	4	4	1

M_0 $\quad\quad$ $M_0 + M_{fluid}$



I_0

$$I_{filled} = I_0 + \frac{M_{fluid} R^3}{2}$$



$$\omega = \frac{v}{R}$$

$$Mgh = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

$$Mgh = \left(\frac{1}{2} M + \frac{1}{2} \frac{I}{R^2}\right) v^2$$

$$V = \sqrt{\frac{2gh}{\left(1 + \frac{I}{MR^2}\right)}}$$

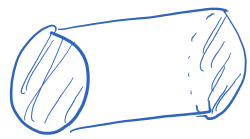
$$V_{empty} = \sqrt{\frac{2gh}{\left(1 + \frac{I_0}{M_0 R^2}\right)}}$$

$$V_{filled} = \sqrt{\frac{2gh}{\left(1 + \frac{I_0 + M_{fluid} R^3/2}{(M_0 + M_{fluid}) R^2}\right)}}$$

$M_0 R$ $(M_0 + M_{fluid}) R$

Empty can $I_0 \simeq MR^2 \Rightarrow$ most of the mass is distributed near $r=R$

$$\frac{I_0}{MR^2} \simeq 1$$



$$V_{empty} \simeq \sqrt{gh}$$

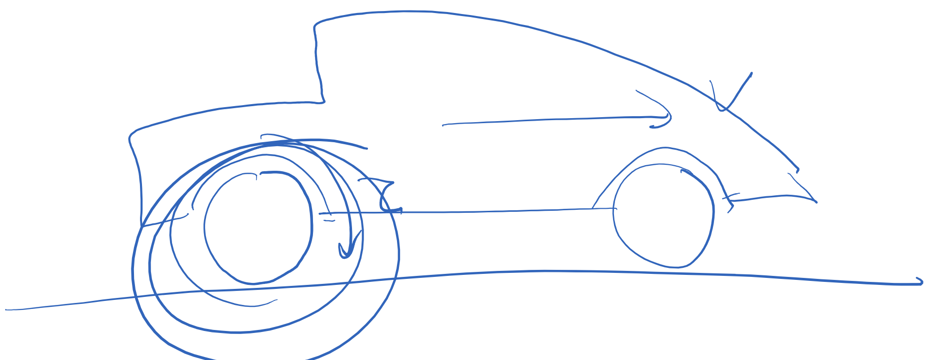
$$V_{filled} = \sqrt{\frac{2gh}{\left(1 + \frac{M_0 R^2 + M_f R^2/2}{(M_0 + M_{fluid}) R^2}\right)}}$$

$$V_{filled} = \sqrt{gh}\sqrt{\frac{2}{1 + \underbrace{\frac{M_0 + M_f/2}{(M_0 + M_f)}}_{<1}}}$$

>1

$$\boxed{V_{filled} \gtrsim V_{empty}}$$

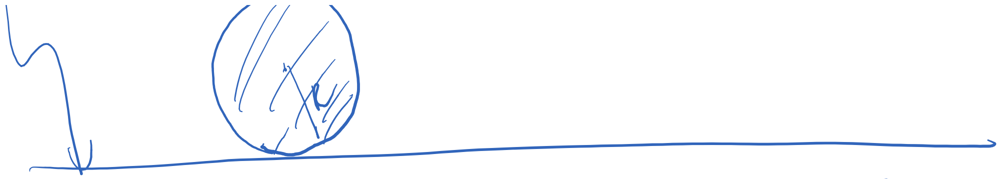
Rolling without slipping



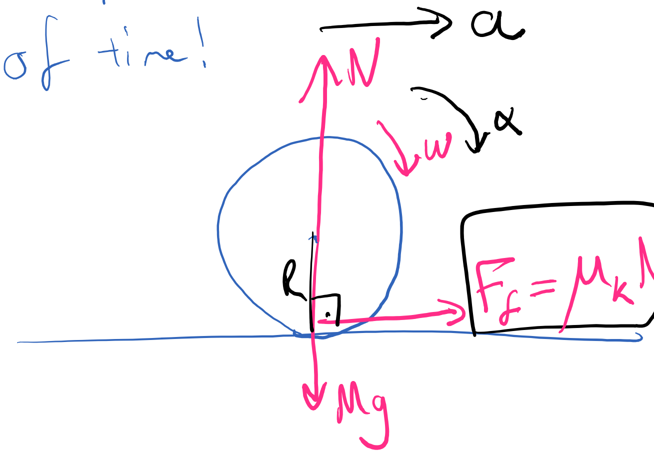
$\mu_s = \mu_k$

ω_0 $M,\ I = \frac{MR^2}{2}$

initial velocity is zero!



Find $\omega(t)$ and $v(t)$ as a function of time!



$$\boxed{F_f = \mu_k N} = \mu_k M g$$

$$Ma = F_f = \mu_k M g$$

$$\boxed{a = \mu_k g}$$

$$I\alpha = -F_f R$$

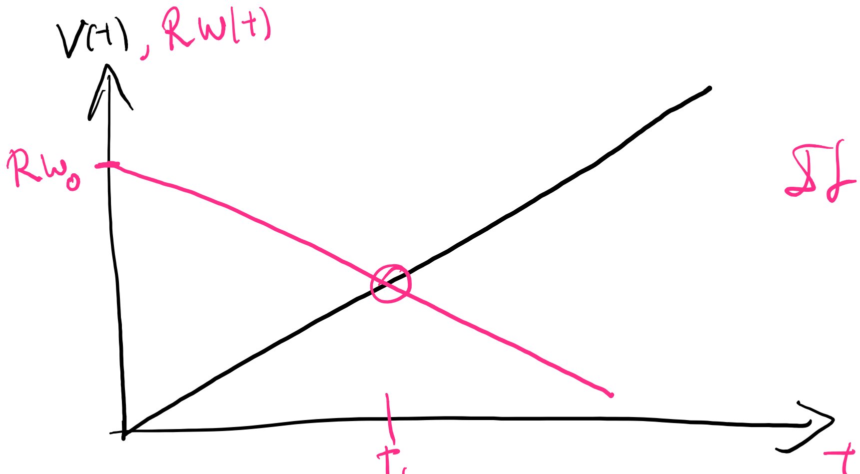
$$\alpha = -\frac{\mu_k \cancel{M} g \cancel{R}}{\frac{\cancel{M} R^{\cancel{2}}}{2}}$$

$$\boxed{\alpha = -\mu_k \frac{2g}{R}}$$

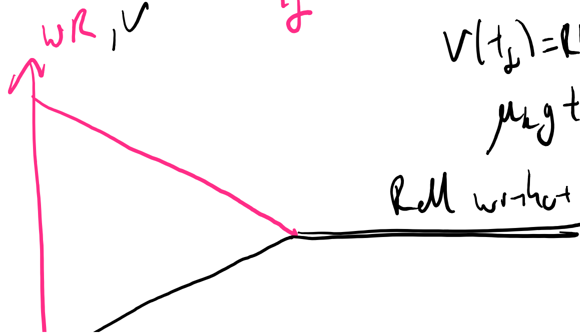
$$V(t) = at = \mu_k g t$$

$$\omega(t) = \omega_0 + \alpha t$$

$$= \omega_0 - \mu_k g \frac{2}{R} t$$



If $\omega R = V$
⇓
F_f is not kinetic anymore

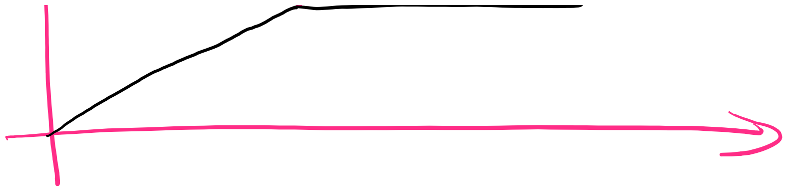


$$V(t_f) = R\omega(t_f)$$

$$\mu_k g t_f = \omega_0 R - 2\mu_k g t_f$$

$$3\mu_k g t_f = \omega_0 R$$

$$\boxed{t_f = \frac{\omega_0 R}{\ }}$$



$$\boxed{t_f = \frac{\omega_0 R}{3\mu_k g}}$$

$$\boxed{v_f = a t_f = \cancel{\mu_k g}\,\frac{\omega_0 R}{3\cancel{\mu_k g}} = \frac{1}{3}\omega_0 R}$$

$$\boxed{\omega_f = \frac{1}{3}\omega_0}$$

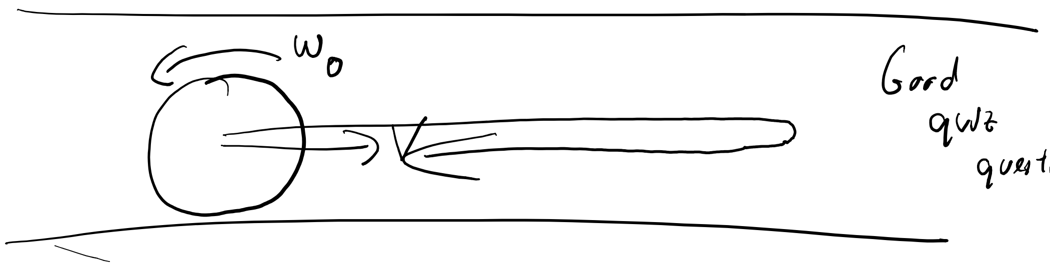
$$E_i = \frac{1}{2}\frac{MR^2}{2}\omega_0^2$$

$$= \frac{1}{4} MR^2\omega_0^2$$

$$E_f = \frac{1}{2} M v_f^2 + \frac{1}{2}\frac{MR^2}{2}\omega_f^2$$

$$= \frac{1}{2} M \frac{1}{9}\omega_0^2 R^2 + \frac{1}{4} MR^2 \frac{1}{9}\omega_0^2$$

$$= \frac{1}{4}\underbrace{\left(\frac{2}{9} + \frac{1}{9}\right)}_{\frac{1}{3}} M\omega_0^2 R^2 = \boxed{\frac{1}{3}E_i}$$



Good quiz question!

x		θ
v	$\Longrightarrow$	ω
a		α
M	$\Longrightarrow$	I
$\vec{F}$	$\Longrightarrow$	$\vec{\tau}$

$$K = \frac{1}{2} m v^2 \Longrightarrow K = \frac{1}{2} I \omega^2$$

$$\vec{p} = m\vec{v} \Longrightarrow ?\ \vec{L} = I\vec{\omega}$$

angular momentum.

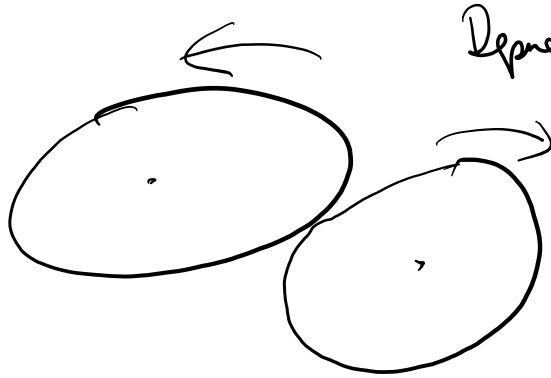
$$\frac{d\vec{p}}{dt} = \vec{F}$$

$\Downarrow$

closed system $\vec{P}_{tot}$ is conserved

$$\frac{d}{dt}(L) = I \underbrace{\frac{d\omega}{dt}}_{\alpha} = \tau$$

Is there a Newton's third law for torques?



Depends on which torques we are talking about!

As long as I am concerned about one axis of rotation torques come in equal and opposite pairs! $\Longrightarrow$ Angular momentum for a $\boxed{\text{closed}}$ system is conserved!

$\Downarrow$

$\boxed{\text{A system for which } \underline{\text{external torque}} \text{ is zero.}}$

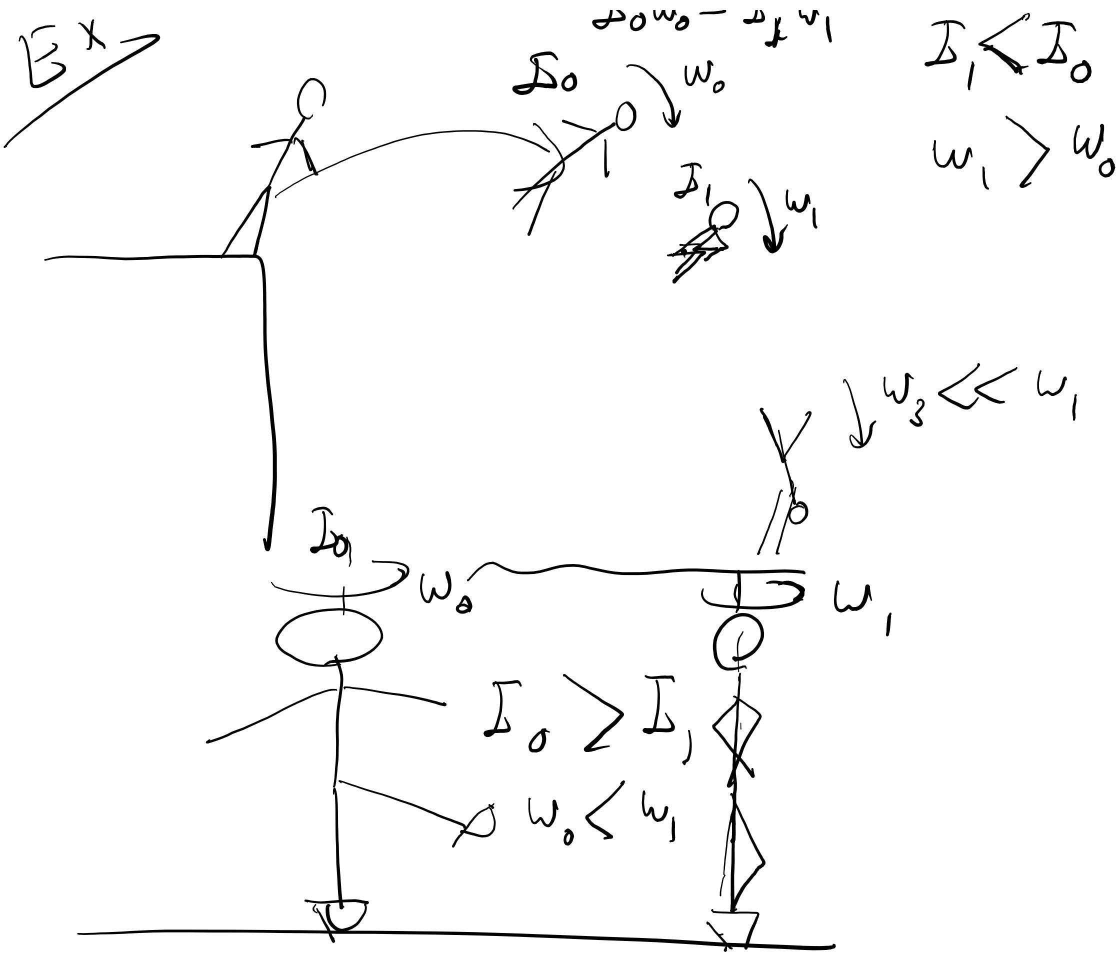
Ex

$$I_0 \omega_0 = I_f \omega_1$$

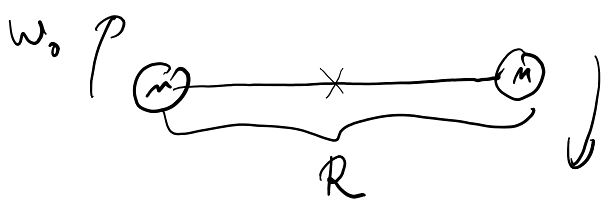
I_0 ω_0

$$I_1 < I_0$$

Ex



Ex



1°) Find the tension in the string connecting the two masses.

2°) If R is slowly reduced to $\frac{R}{2}$, what is the final angular velocity?

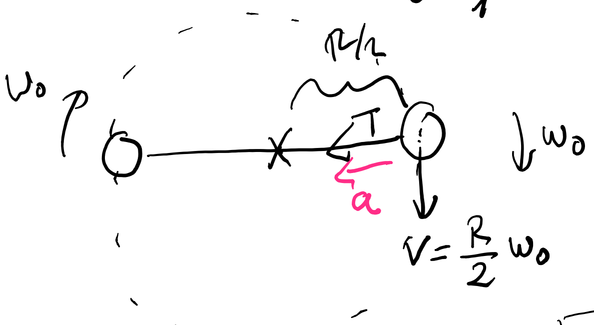
3°) How much work needs to be done to reduce $R \to R/2$?

$$I_0 = \sum_i r_i^2 m_i = m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2 = \boxed{\frac{1}{2} m R^2}$$

1°)

$$\boxed{I_0 = \sum_i r_i^2 m_i = m\left(\tfrac{R}{2}\right)^2 + m\left(\tfrac{R}{2}\right)^2 = \tfrac{1}{2} m R^2}$$

1°)



$$a = \frac{v^2}{(R/2)} = \frac{R}{2}\omega_0^2$$

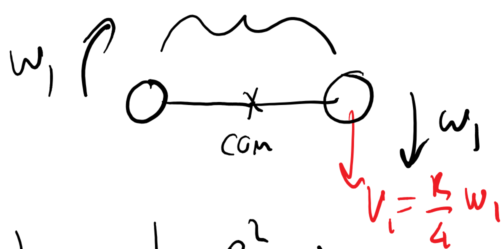
$$T = ma$$

$$\boxed{T = m\frac{R}{2}\omega_0^2}$$

$$L_i = I_0 \omega_0 = \frac{1}{2} m R^2 \omega_0$$

$$E_i = 2 \cdot \frac{1}{2} m \left(\frac{R}{2}\omega_0\right)^2 = \frac{1}{4} m R^2 \omega_0^2$$

2°)



$$I_f = 2m\left(\frac{R}{4}\right)^2 = \frac{1}{8} m R^2$$

$$L_f = \frac{1}{8} m R^2 \omega_1$$

$$L_i = L_f$$

$$\frac{1}{2} \cancel{mR^2} \omega_0 = \frac{1}{8} \cancel{mR^2} \omega_1 \Rightarrow \boxed{\omega_1 = 4\omega_0}$$

$$E_f = 2 \cdot \frac{1}{2} m \left(\frac{R}{4} 4\omega_0\right)^2$$

$$\boxed{E_f = m R^2 \omega_0^2}$$

3°)

$$\Delta E = E_f - E_i$$

$$= m R^2 \omega_0^2 - \frac{1}{4} m R^2 \omega_0^2 = \frac{3}{4} m R^2 \omega_0^2$$

$$W = \Delta E = \frac{3}{4} m R^2 \omega_0^2$$

$$W = \left(\vec{F} \cdot \vec{d} = \left(T\right) \tfrac{R}{2} \right)$$

$$W = \int \vec{F} \cdot d\vec{l} = \int_{R} T(r)\,dr$$

no angular momentum?

Angular momentum conserved

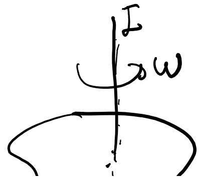
$L \neq 0$

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

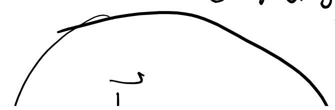
use it to derive

$\vec{L} = I\vec{\omega}$ for rigid bodies

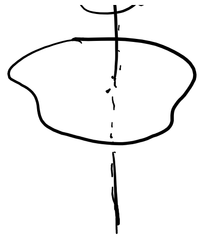
Angular Momentum



$L = I\omega$

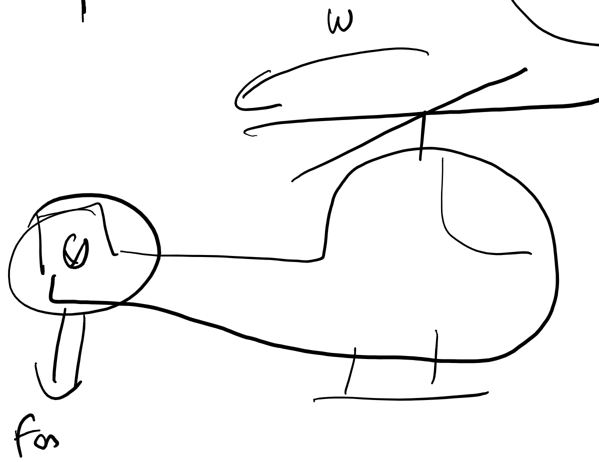


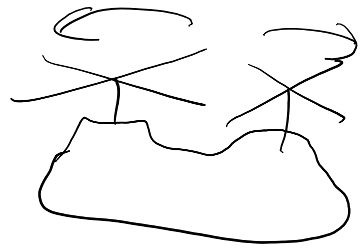
Closed system = no external net torque



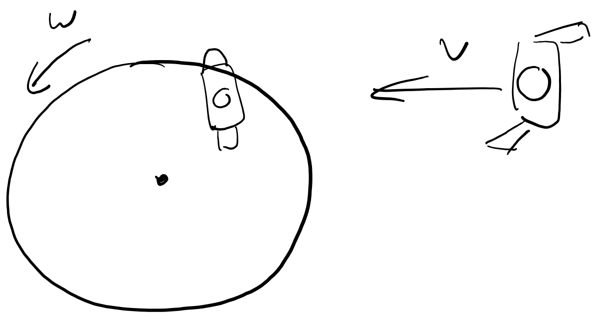
$L = I\omega$

$\vec{L}_{tot}$ is conserved. $\Leftarrow$ no external net torque



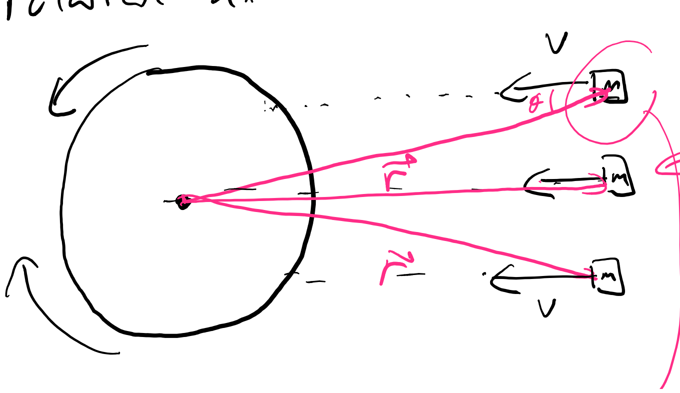


Simple example top



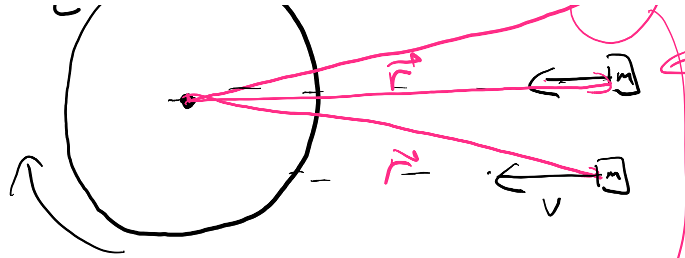
Even an object moving with constant velocity can carry angular momentum!

Angular momentum is defined w.r.t the rotation axis

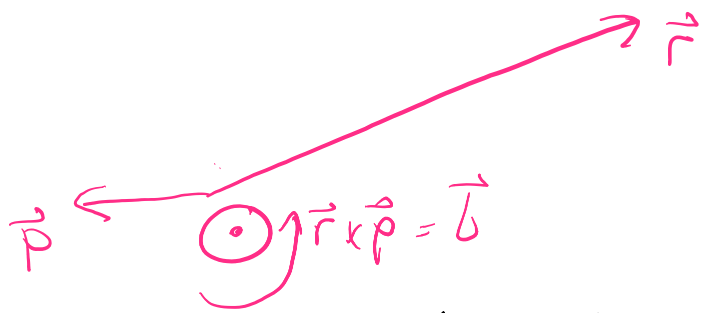


$\vec{L} = \vec{r} \times \vec{p}$

$|\vec{L}| = |\vec{r}||\vec{p}|\underbrace{\sin\theta}_{0}$



$$|\vec L| = |\vec r||\vec p|\sin\theta$$



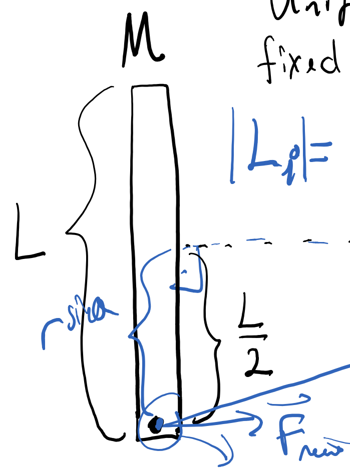
$$\vec r \times \vec p = \vec L$$

$$\vec L = \vec r \times \vec p \neq \vec p \times \vec r$$

$$\vec r \times \vec p = -\vec p \times \vec r$$

EX

Uniform rod on a frictionless table fixed at one end but free to rotate



$$|L_i| = mv|\vec r|\sin\theta = \boxed{mv\frac{L}{2}}$$

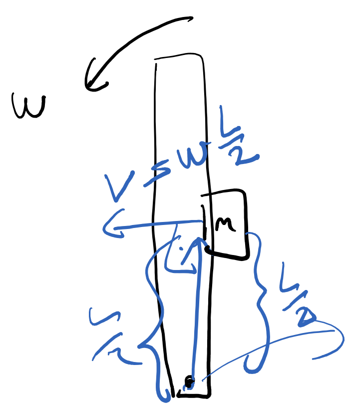
~~Energy~~

~~Momentum!~~

Angular Momentum

$$E_i = \frac{1}{2}mv^2$$

After the collision the mass sticks to the rod



(1°) $w = ?$

(2°) Energy lost during collision.

$$I_{edge} = \frac{ML^3}{3}$$

$$E_f = \frac{1}{2} I_{system} w^2$$

$$L_f = I_{rod}\, w + |\vec r \times \vec p|$$

$$= \frac{ML^3}{3} w + \frac{L}{2} m w \frac{L}{2} = \overbrace{\left(I_{rod} + m\left(\frac{L}{2}\right)^2\right)}^{I_{system}} w$$

$$= \frac{ML^3}{3}\omega + \frac{L}{2} m \omega \frac{L}{2} = \left(I_{rod} + m\left(\frac{L}{2}\right)^2\right)\omega$$

$$E_f = \frac{1}{2}\frac{ML^3}{3}\omega^2 + \frac{1}{2} m \left(\omega \frac{L}{2}\right)^2$$

$$L_i = L_f$$

$$m v \frac{\cancel{L}}{2} = \left(\frac{ML^{\cancel{3}}}{3} + m\frac{L^{\cancel{2}}}{4}\right)\omega$$

1^0) Gad

$$\boxed{\omega = \frac{m}{\left(\frac{2}{3}M + \frac{m}{2}\right)} \frac{V}{L}}$$

$$E_i = \frac{1}{2} m v^2$$

$$E_f = \frac{1}{2}\left(\frac{ML^3}{3} + m\frac{L^3}{4}\right)\omega^2$$

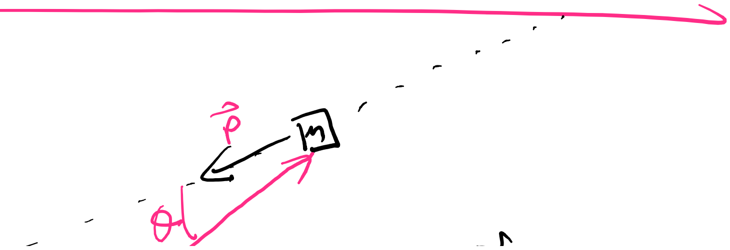
$$= \frac{1}{2}\left(\frac{M}{3}+\frac{m}{4}\right)\cancel{L^2}\frac{m^2}{\left(\frac{2M}{3}+\frac{m}{2}\right)^2}\frac{v^2}{\cancel{L^2}}$$

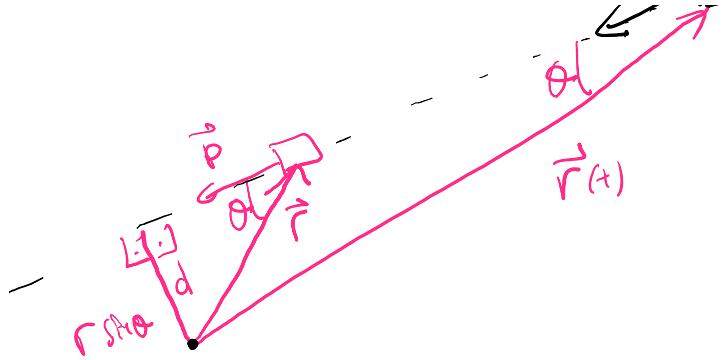
$$= \frac{1}{2} m^2 \frac{\cancel{\left(\frac{M}{3}+\frac{m}{4}\right)}}{4\left(\frac{M}{3}+\frac{m}{4}\right)^{\cancel{2}}} v^2$$

$$E_f = \frac{1}{2} m v^2 \underbrace{\left[\frac{m}{\left(m+\frac{4}{3}M\right)}\right]}_{<1}$$

$$\Delta E = E_i - E_f = \frac{1}{2} m v^2\left[1 - \frac{m}{m+\frac{4}{3}M}\right]$$

$$= \boxed{\frac{1}{2} m v^2 \frac{4M}{3m+4M}}$$





$$\vec{L} = \vec{r} \times \vec{p}$$

$$|\vec{L}| = |\vec{r}|\,|\vec{p}| \sin\theta$$

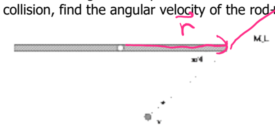
$$= \underbrace{|\vec{r}| \sin\theta}_{\text{constant}}\, |\vec{p}|$$

$|\vec{L}| =$ (distance of closest approach) (momentum)

Let's take our quiz!

Phys 101- Instructor: M. Özgür OKTEL – 2016 QUIZ-22

A thin rod of length L is fixed to a horizontal table from its midpoint but is free to rotate. A small particle of mass m is moving with velocity v as shown in the figure. The particle hits the rod at its end point, making a 45 degree angle with the rod. If the particle sticks to the rod after the collision, find the angular velocity of the rod+particle system after the collision.



$$I = \frac{ML^2}{12}$$

$$L_i = \vec{r} \times \vec{p} = |\vec{r}|\,|\vec{p}| \sin\frac{\pi}{4}$$

$$= \frac{L}{2}\, m v \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} m v L$$

$$L_f = \left(I + m\frac{L^2}{4}\right)\omega$$