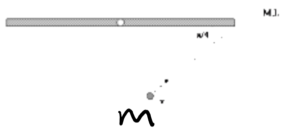
Week 12 (Dec 12 - Dec 16)

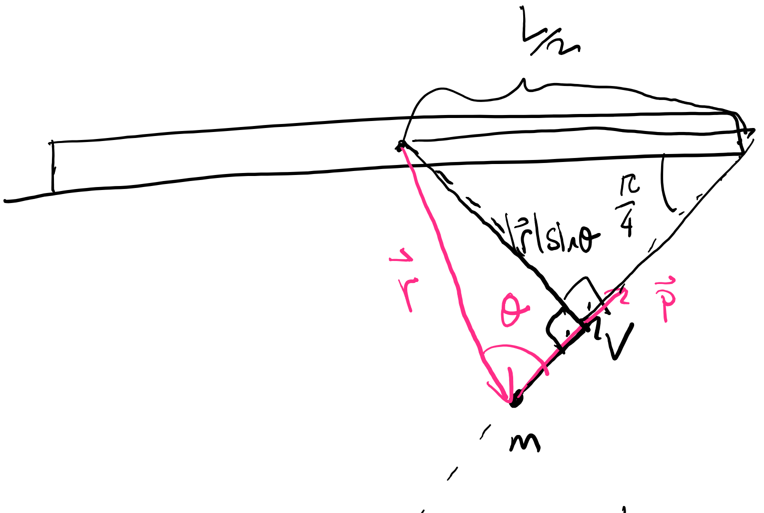
Friday, September 23, 2016 3:00 PM

Phys 101- Instructor: M. Özgür OKTEL – 2016 QUIZ-22

A thin rod of length L is fixed to a horizontal table from its midpoint but is free to rotate. A small particle of mass m is moving with velocity v as shown in the figure. The particle hits the rod at its end point, making a 45 degree angle with the rod. If the particle sticks to the rod after the collision, find the angular velocity of the rod+particle system after the collision.



Angular momentum



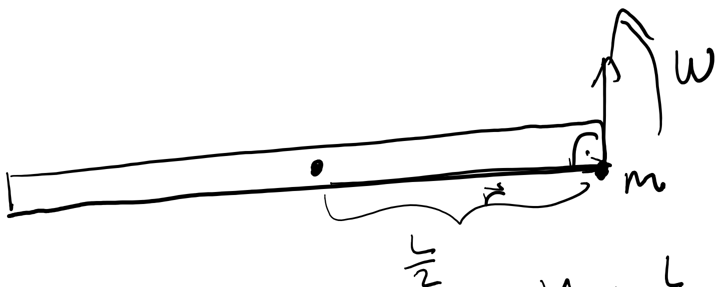
$$L_i^o = |\vec{r} \times \vec{p}|$$

$$= |\vec{r}|\,|\vec{p}|\sin\theta$$

$$= |\vec{r}|\sin\theta\,|\vec{p}|$$

$$= mv\frac{L\sqrt{2}}{4}$$

From geometry $|\vec{r}|\sin\theta = \frac{L}{2}\sin\frac{\pi}{4} = \frac{L\sqrt{2}}{4}$



$$V = w\frac{L}{2}$$

$$L_f = I_{rod}\,w + mw\left(\frac{L}{2}\right)^2$$

$$= \left(I_{rod} + m\left(\frac{L}{2}\right)^2\right) w$$

$$\frac{ML^3}{12}$$

$$L_i^o = L_f$$

$$mvL\frac{\sqrt{2}}{\cancel{4}} = \left(\frac{ML^3}{\cancel{12}_3} + m\frac{L^3}{\cancel{4}}\right) w$$

$$\boxed{\omega = \frac{m\sqrt{2}}{\left(\frac{M}{3}+m\right)}\,\frac{V}{L}}$$

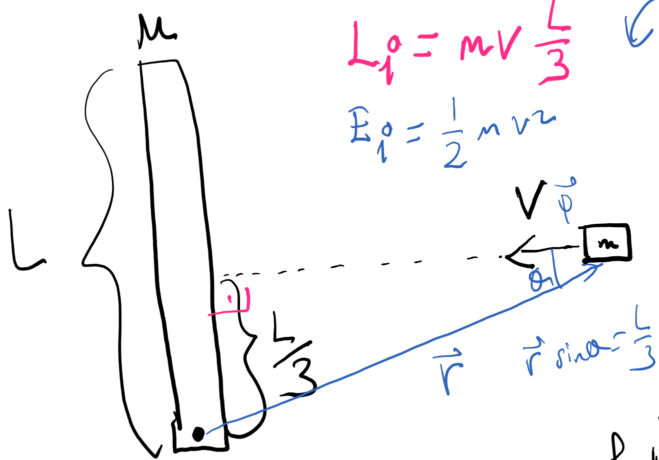
Ex

Again on a frictionless table, uniform rod fixed from end point.

Small mass m makes an elastic collision with the rod.

Energy is conserved!

Find the angular velocity of the rod and the velocity of m after the collision.

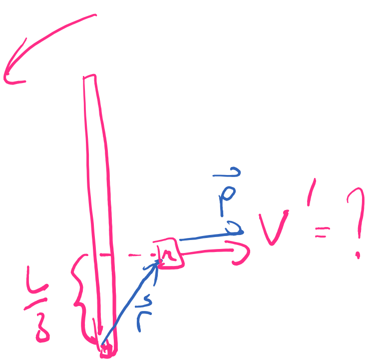


$$L_i = mV\frac{L}{3}$$

$$E_i = \frac{1}{2} m v^2$$

$$\vec{r}\sin\theta = \frac{L}{3}$$

$\omega = ?$



* Angular Momentum conserved ✓

* Energy is conserved!

$$I_{rod} = \frac{ML^2}{3}$$

$$L_f = I_{rod}\,\omega - mv'\frac{L}{3}$$

$$E_f = \frac{1}{2} m v'^2 + \frac{1}{2} I_{rod}\,\omega^2$$

$$L_i = L_f$$

$$mv\frac{\cancel{L}}{\cancel{3}} = \frac{ML^{\cancel{2}}}{\cancel{3}}\,\omega - mv'\frac{\cancel{L}}{\cancel{3}}$$

$$\boxed{v' = \frac{MLw}{m} - v}$$

$$E_i = E_f$$

$$\frac{1}{\cancel{2}} m v^2 = \frac{1}{\cancel{2}} m v'^2 + \frac{1}{\cancel{2}} \frac{ML^3}{3} w^2$$

$$m v^2 = m \left(\frac{M}{m} L w - v \right)^2 + \frac{ML^3}{3} w^2$$

$$m v^2 = m \left(\frac{M^2}{m^2} L^2 w^2 - 2 \frac{M}{m} L w v + v^2 \right) + \frac{ML^3}{3} w^2$$

$$\cancel{m v^2} = \cancel{m v^2} - 2 M L w v + \frac{M^2}{m} L^2 w^2 + \frac{1}{3} M L^2 w^2$$

$$0 = -2MLwv + ML^2 w^2 \left(\frac{1}{3} + \frac{M}{m} \right)$$

$$0 = w \cancel{ML} \left[L w \left(\frac{1}{3} + \frac{M}{m} \right) - 2v \right]$$

$$\cancel{w = 0}$$

initial state

$$w = \frac{2v}{\left(\frac{1}{3} + \frac{M}{m} \right) L}$$

$$\boxed{w = \frac{6m}{(m+3M)} \frac{v}{L}}$$

$$v' = \frac{M}{m} L w - v$$

$$= \frac{6M}{(m+3M)} v - v = \boxed{\left(\frac{6M}{3M+m} - 1 \right) v}$$

$$= \frac{6m}{(m+3M)} V - V = \left(\frac{\quad}{3M+m} - 1\right) V$$

$$\boxed{V' = \frac{3M-m}{3M+m} V}$$

$$\frac{M}{\cancel{m}} \cancel{L} \frac{6\cancel{m}}{(m+3M)} \frac{V}{\cancel{L}}$$

$$\boxed{\omega = \frac{6m}{(m+3M)} \frac{V}{L} \qquad V' = \frac{3M-m}{3M+m} V}$$

1°) Goal ?

2°) Units ✓

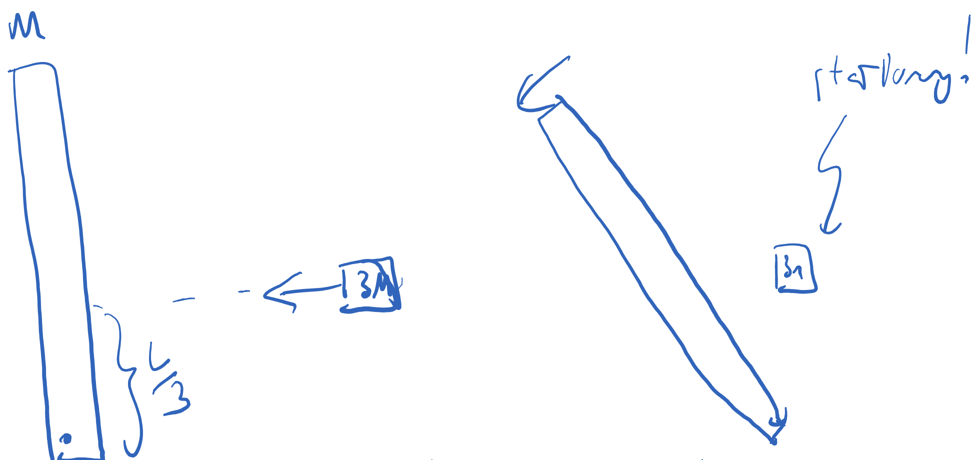
3°) Limits

$m \to 0$, $\omega \to 0$ ✓

$M \to \infty$, $V' = V$ ✓

$$[\omega] = \frac{kg}{\cancel{kg}} \frac{\cancel{m}/s}{\cancel{m}} \checkmark \qquad \frac{m}{s} = \frac{kg}{kg} \frac{m}{s} \checkmark$$

$$V' = 0 \text{ if } m = 3M$$

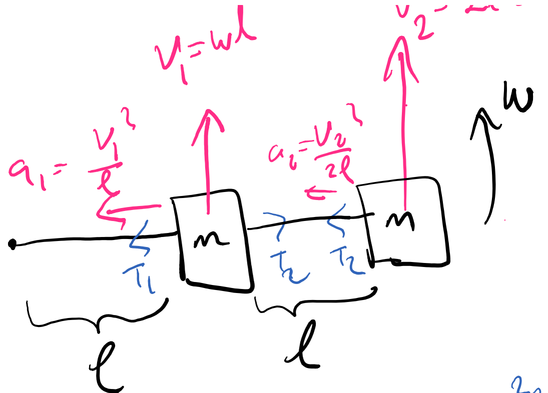


"sweet spot"

EX Two identical masses are tied to a pivot point as shown. They are rotating with ω on a frictionless table.

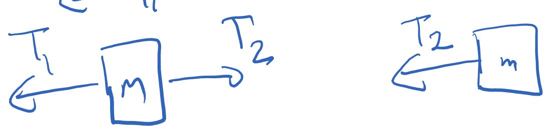
$v_1 = \omega l$ $\quad v_2 = 2l\omega$

a) Find the tension in



a) Find the tension in both strings

b) If the first string breaks find the V_{com} and ω' of the masses



$m\omega^2 l = T_1 - T_2$

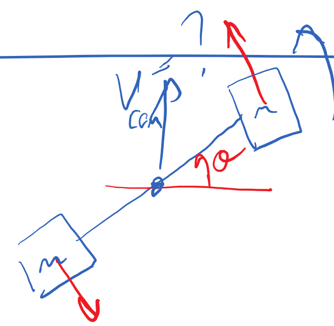
$m\omega^2 2l = T_2$

$T = m\omega^2 l + T_2$

$T_2 = 2m\omega^2 l$

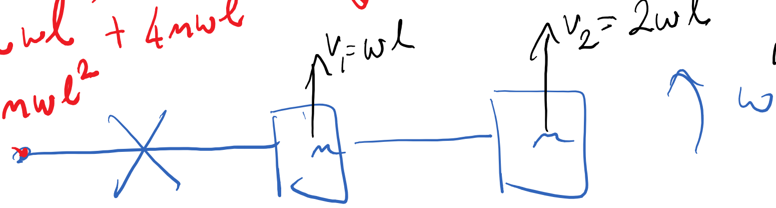
$T_1 = 3m\omega^2 l$

$$\underbrace{P_f = 2m}_{\text{total mass}} V_{COM}$$



• P conserved!

$L_i^o = m\omega l^2 + 4m\omega l^2$
$= 5m\omega l^2$



$$\boxed{P_i^o = 3m\omega l}$$

$E_i^o = \frac{1}{2}m\omega^2 l^2 + \frac{1}{2}m 4\omega^2 l^2 = \frac{5}{2}m\omega^2 l^2$

$P_i^o = P_f$

$3\cancel{m}\omega l = 2\cancel{m} V_{com} \implies V_{com} = \frac{3}{2}\omega l$

• Energy should also be conserved!

$L_f = I_{com}\omega' + \vec{r}\times\vec{p}_{com}$

$E_f = \frac{1}{2}(2m)V_{com}^2 + \frac{1}{2}I_{com}\omega'^2$

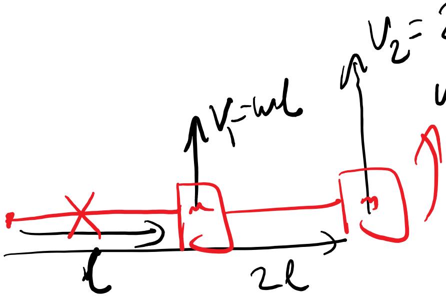
$I_{com} = 2m\frac{l^2}{4} = \frac{ml^2}{2}$ $\quad E_i = E_f$

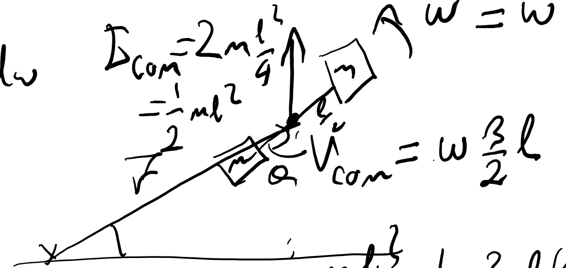
$$\frac{5}{2} m\omega^2 l^2 = \frac{1}{2}\, 2m\left(\frac{3}{2}\omega l\right)^2 + \frac{1}{2}\,\frac{ml^2}{2}\,\omega'^2$$

$$\frac{5}{2}\cancel{m}\omega^2\cancel{l^2} = \frac{9}{4}\cancel{m}\omega^2\cancel{l^2} + \frac{1}{4}\cancel{m}\cancel{l^2}\omega'^2$$

$$\frac{1}{4}\omega^2 = \frac{1}{4}\omega'^2 \Longrightarrow \boxed{\omega' = \omega}$$

$$\boxed{L = I_{com}\,\omega + \vec{r}\times\vec{p}_{com}}$$





$$L_f = \frac{ml^2}{2}\omega' + \frac{3}{2}l(2m)\frac{3}{2}\omega l$$

$$= \frac{1}{2}m\omega l^2 + \frac{9}{2}m\omega l^2$$

$$= \frac{10}{2}m\omega l^2 = L_i$$

$$L_i = l\,m\omega l + 2l\,m\omega 2l$$

$$= 5m\omega l^2$$

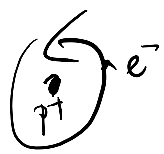
$$\vec{L} = \vec{r}\times\vec{p}$$

$$[L] = m\, kg \frac{m}{s} = kg \frac{m^2}{s} = kg \frac{m^2}{s^2} s = \underline{\underline{Js}}$$

Angular momentum is quantized!!

Fundamental constant Planck's constant $\hbar$

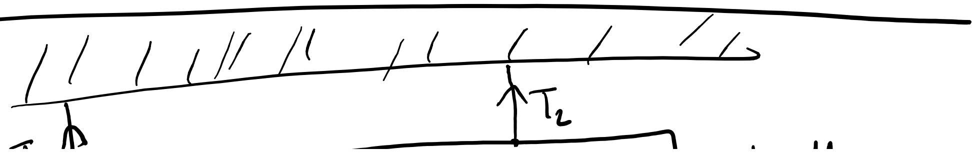
$$\hbar \simeq 1.05\ 10^{-34}\ Js$$



L	orbital	
$0\hbar$	$\Rightarrow s$	(sharp)
$1\hbar$	$\Rightarrow p$	(principal)
$2\hbar$	$\Rightarrow d$	(diffuse)
$3\hbar$	$\Rightarrow f$	⋮
⋮	g	
	h	

$$\underbrace{\Delta x}_{m}\ \underbrace{\Delta p}_{kg\, m/s} \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \gtrsim \hbar$$

Uncertainty relations



Quiz 23

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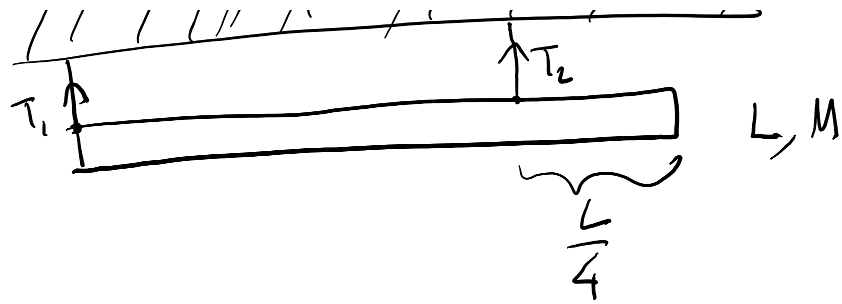
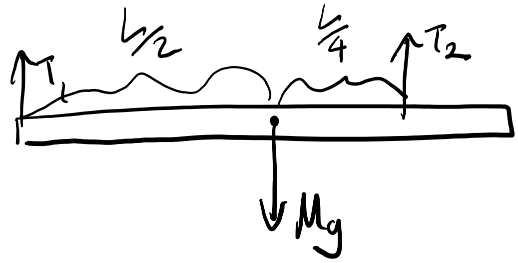


a) $T_1, T_2 = ?$



$\vec{F} = m\vec{a} = 0$

$\boxed{Mg = T_1 + T_2}$

$\vec{\tau} = I\vec{\alpha} = 0$ $\Rightarrow$ use C.O.M (you could have used any point!)

$\vec{\tau} = 0$

$\tau_1 = \frac{L}{2} T_1$

$\tau_2 = \frac{L}{4} T_2$

$\vec{\tau}_1 + \vec{\tau}_2 = 0$

$\frac{\cancel{L}}{\cancel{2}} T_1 = \frac{\cancel{L}}{\cancel{4}_2} T_2$

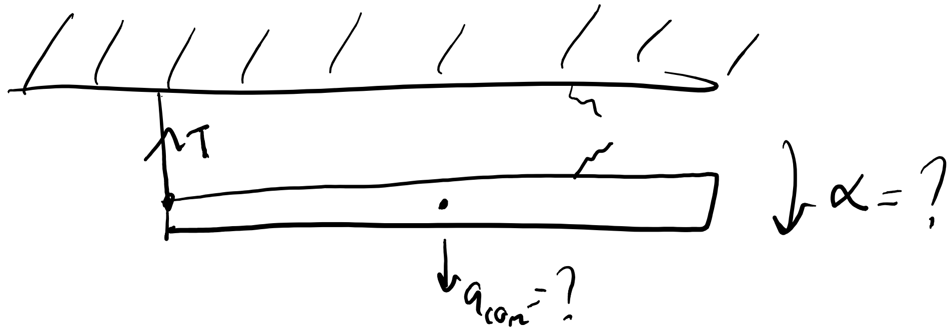
$\boxed{T_1 = \frac{T_2}{2}}$

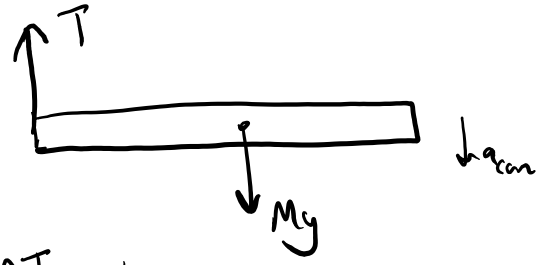
$Mg = \frac{T_2}{2} + T_2$

$\Rightarrow T_2 = \frac{2}{3} Mg$

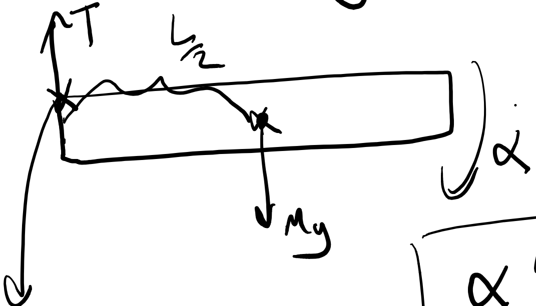
$\Rightarrow T_1 = \frac{1}{3} Mg$

b)





$$\boxed{M a_{com} = Mg - T} \quad ①$$



that point is momentarily stationary!

$$\boxed{I_{com}\,\alpha = T\frac{L}{2}} \quad ②$$

$$\boxed{\alpha\frac{L}{2} = a_{com}} \quad ③$$

$$I_{com} = \frac{ML^2}{12}$$

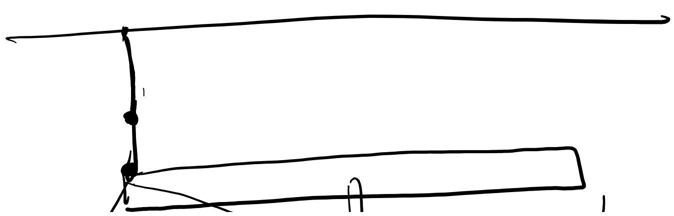
$$T\frac{\cancel{L}}{\cancel{2}} = \frac{M L^{\cancel{2}}}{\cancel{12}_{6}}\,\frac{2a_{com}}{\cancel{L}} \Rightarrow T = \frac{M a_{com}}{3}$$

$$\cancel{M} a_{com} = \cancel{M} g - \cancel{M}\frac{a_{com}}{3}$$

$$\frac{4}{3}a_{com} = g \qquad \boxed{a_{com} = \frac{3}{4}g}$$

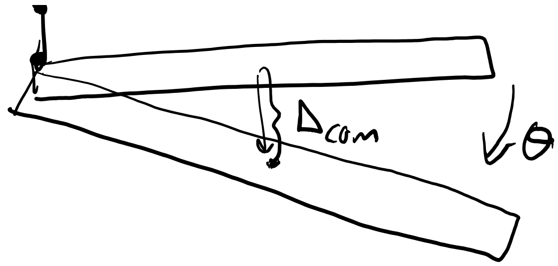
$$\boxed{T = \frac{Mg}{4}}$$

$$\alpha = \frac{2}{L}a_{com} = \boxed{\frac{3}{2}\frac{g}{L}}$$



θ is small

$$\Delta_{com} - \frac{L}{2}\sin\theta = 0$$



$$\Delta_{com} - \frac{L}{2}\sin\theta = 0$$

$$\Delta_{com} - \frac{L}{2}\theta = 0$$

$$\boxed{a_{com} - \frac{L}{2}\alpha = 0}$$

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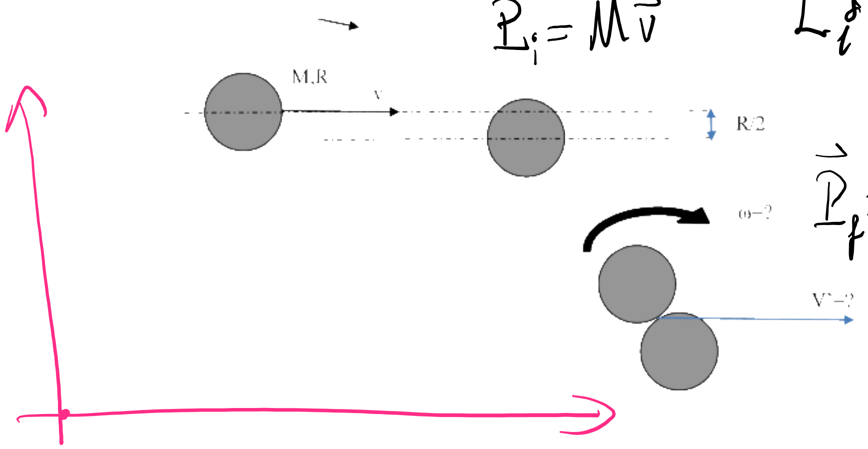
$\vec{P}_i = M\vec{v}$

$L_i = \vec{r} \times \vec{p}_{com} + I_{com}\omega$



$\vec{P}_f = 2M\vec{v}'$ Momentum ✓

Energy X

Angular mom. ✓

A uniform disc of radius R and mass M is sliding with velocity v towards an identical disc at rest. The centers of the discs are misaligned by R/2 in the direction of the velocity. There is no friction.

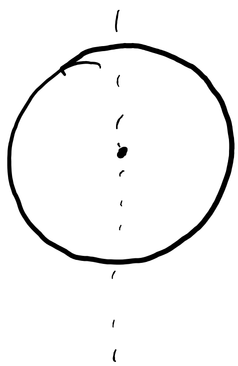
When the two discs collide they stick to each other without significant deformation. Find ✓

a) The center of mass velocity of the combined two disc system.

The angular frequency of rotation about the center of mass

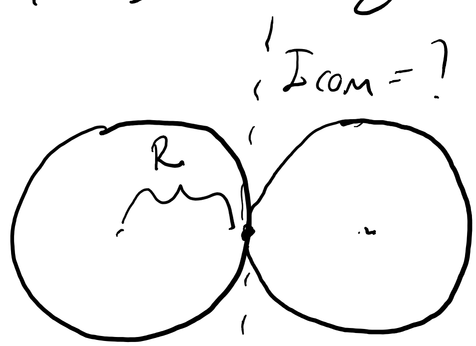
a) $\vec{P}_i = \vec{P}_f \Rightarrow M|\vec{v}| = 2M|\vec{v}'| \Rightarrow |\vec{v}'| = \frac{|\vec{v}|}{2}$ ✓

b)



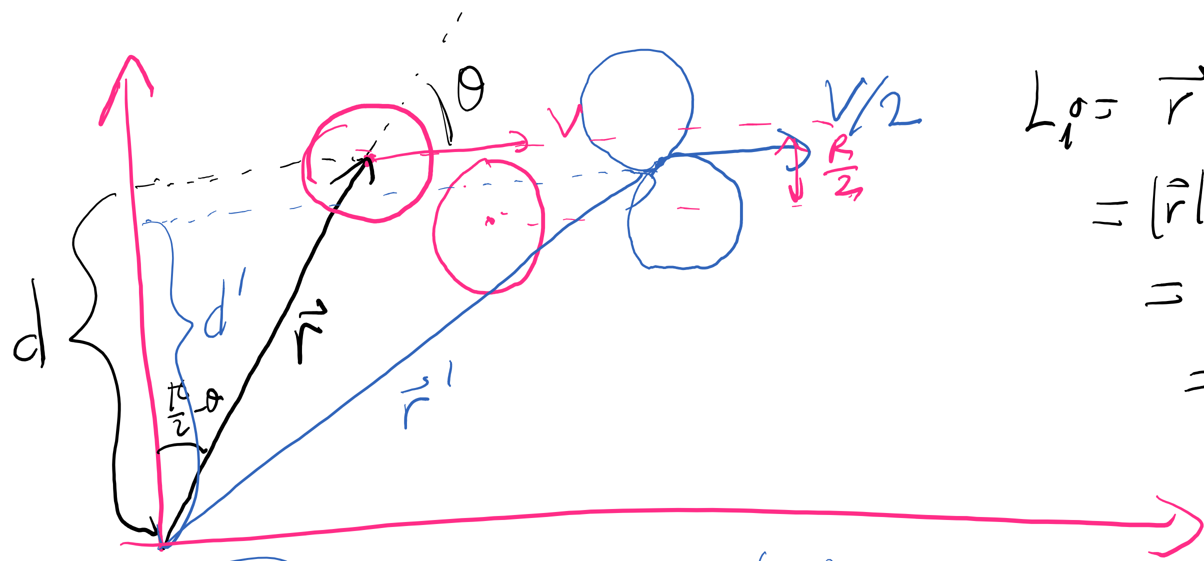
$I_{DISC} = \frac{MR^2}{2}$

$I_{com} = ?$



$$I_{com} = \left(\frac{MR^2}{2} + MR^2\right)2$$

$$= 3MR^2$$



$$L_i^o = \vec{r} \times (M\vec{v})$$
$$= |\vec{r}|M|\vec{v}|\sin\theta$$
$$= Mv|\vec{r}|\cos\left(\frac{\pi}{2}-\theta\right)$$
$$= Mvd$$

$$\boxed{d-d' = \frac{R}{4}}$$

by geometry

$$L_f^{(com)} = \vec{r}\,' \times 2M\frac{\vec{v}}{2} = (\vec{r}\,' \times M\vec{v})$$
$$= Mvd'$$

$$L_f = Mvd' + I_{com}\,\omega$$

$$L_i^o = L_f$$

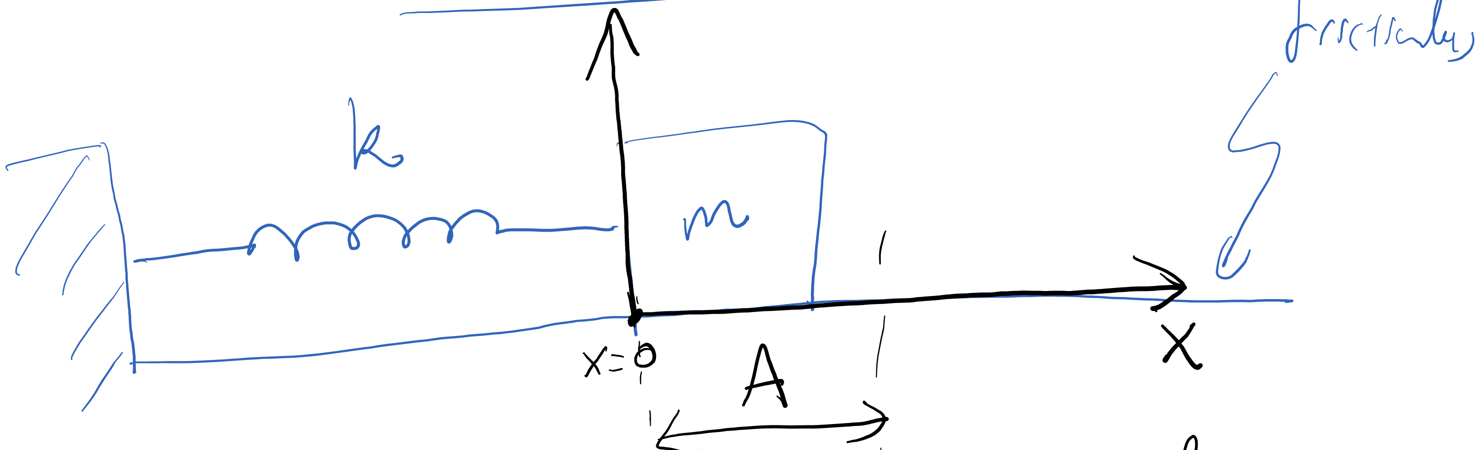
$$\cancel{M}vd = Mvd' + (3\cancel{M}R^2)\omega$$

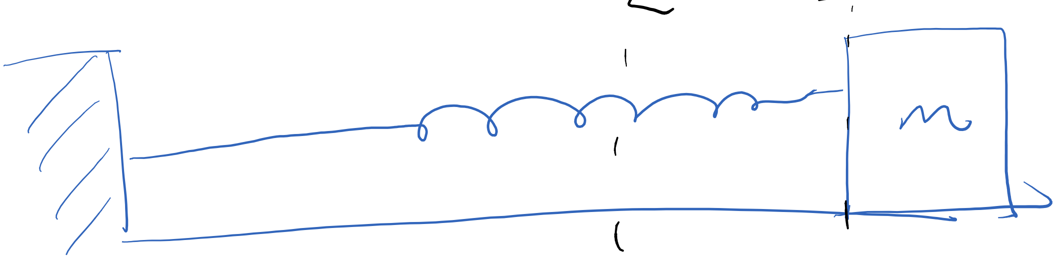
$$v\underbrace{(d-d')} = 3R^2\omega$$

$$v\frac{\cancel{R}}{4} = 3R^{\cancel{2}}\omega$$

$$\boxed{\omega = \frac{v}{12R}}$$

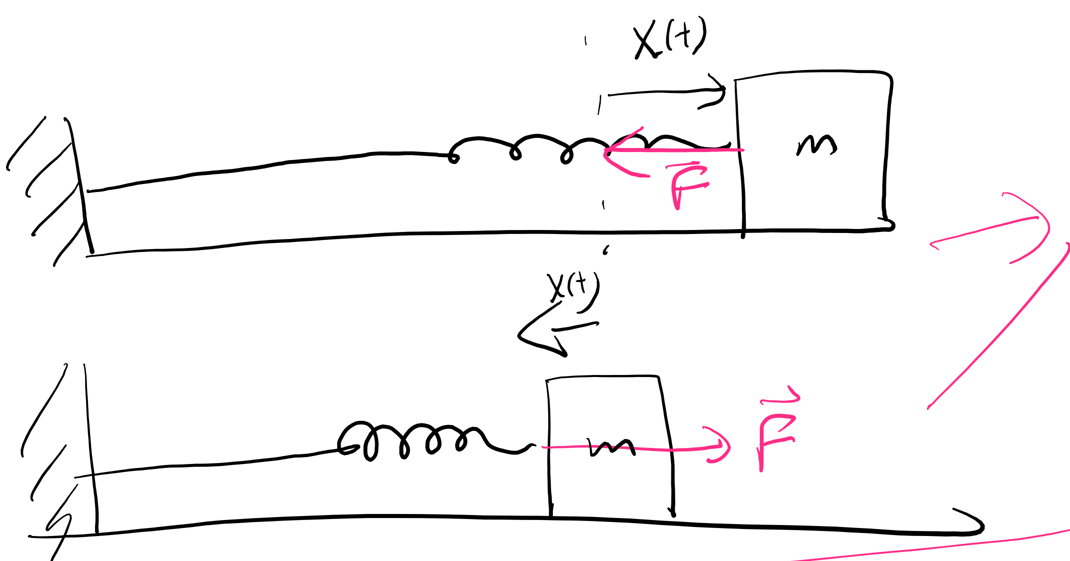
Oscillations





let go from this configuration

$x(t) = ?$



$$\vec{F} = -k\vec{x}$$

$\Downarrow$ 1D

$$F = -kx$$

$$F = ma$$

$$a(t) = \frac{d^2x(t)}{dt^2}$$

$$-k\,x(t) = m\,a(t)$$

$$\boxed{\frac{d^2x(t)}{dt^2} = -\frac{k}{m}\,x(t)}$$

$$\frac{d^2}{du^2}\sin u = -\sin u \qquad \frac{d^2}{du^2}(\cos u) = -\cos u$$

Guess the solution

$$x(t) = C_1\sin(\omega t) + C_2\cos(\omega t)$$

$$\frac{dx}{dt}(t) = \omega C_1\cos(\omega t) - \omega C_2\sin(\omega t)$$

$$\frac{d^2x}{dt^2}(t) = -\omega^2 C_1\sin(\omega t) - \omega^2 C_2\cos(\omega t)$$

Does this satisfy our diff. eqn?

$$\frac{d^2x}{dt^2} = -\frac{k}{m}\,x$$

$$-\omega^2\big(C_1\sin(\omega t) + C_2\cos(\omega t)\big) = -\frac{k}{m}\big(C_1\sin(\omega t) + C_2\cos(\omega t)\big)$$

If $\omega^2 = \frac{k}{m}$ eqn. is satisfied!

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$X(t) = C_1 \sin\left(\sqrt{\frac{k}{m}}\, t\right) + C_2 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

we need initial conditions

$$X(t=0) = A$$

$$\frac{dx}{dt}(t=0) = 0$$

$$C_1 \underbrace{\sin\left(\sqrt{\frac{k}{m}}\, 0\right)}_{0} + C_2 \underbrace{\cos\left(\sqrt{\frac{k}{m}}\, 0\right)}_{1} = A$$

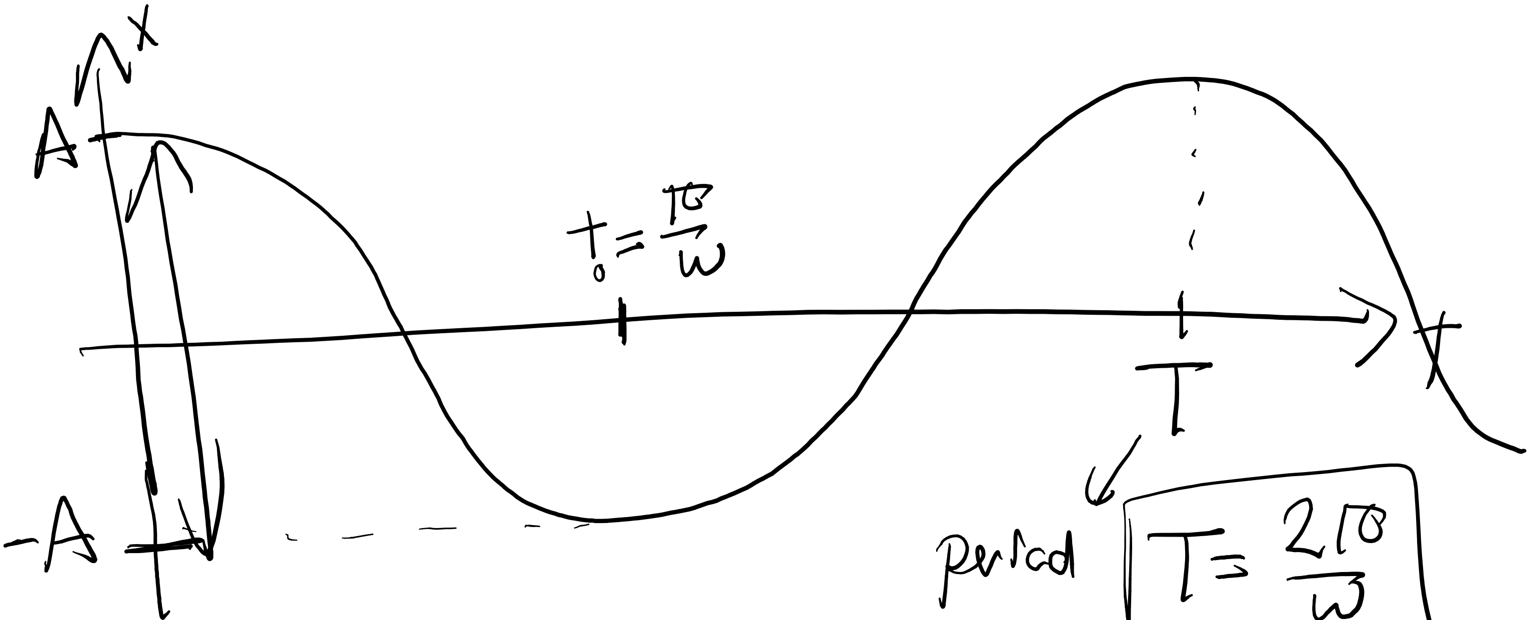
$$\boxed{C_2 = A}$$

$$C_1 \sqrt{\frac{k}{m}} \underbrace{\cos\left(\sqrt{\frac{k}{m}}\, 0\right)}_{1} - \sqrt{\frac{k}{m}}\, C_2 \underbrace{\sin\left(\sqrt{\frac{k}{m}}\, 0\right)}_{0} = 0$$

$$\boxed{C_1 = 0}$$

$$\boxed{X(t) = A \cos\left(\sqrt{\frac{k}{m}}\, t\right)}$$

$$\boxed{x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t\right)}$$



period $\boxed{T = \frac{2\pi}{\omega}}$

$A \rightarrow$ amplitude

$\omega \rightarrow$ (angular) frequency $\qquad \nu = \frac{1}{T} = \frac{\omega}{2\pi}$ frequency

$T = \frac{2\pi}{\omega} \rightarrow$ period