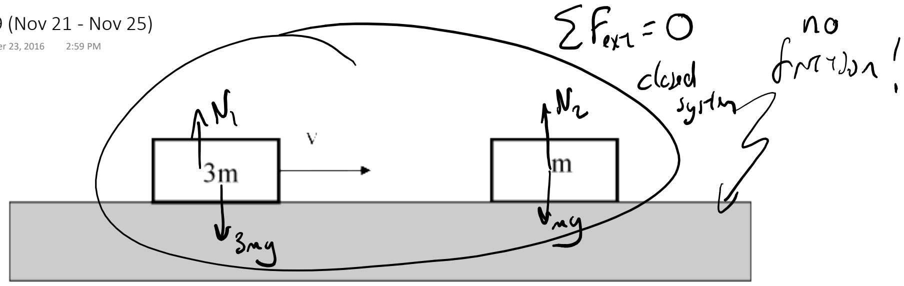
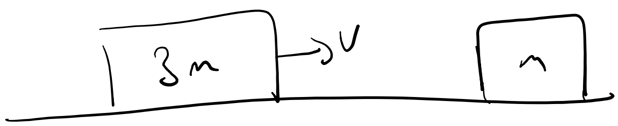
Week 09 (Nov 21 - Nov 25)

Friday, September 23, 2016 2:59 PM



Initial velocities of the two masses are as shown above, find the direction and magnitude of the velocity of both masses after they undergo a completely inelastic collision (i.e. stick to each other). What is the amount of energy lost during collision?

Momentum is conserved



$P_i = 3mv$

$P_f = 4m v_f$

$$P_i = P_f$$
$$3\cancel{m}v = 4\cancel{m}v_f \Rightarrow \boxed{v_f = \frac{3}{4} v}$$

$$E_i = ? \qquad \frac{1}{2} 3m v^2 + \cancel{\frac{1}{2} m\, 0^2} = K_i = \frac{3}{2} m v^2$$

$$E_f = \qquad \frac{1}{2}(4m) v_f^2 = \frac{1}{2} 4m\left(\frac{3}{4}v\right)^2 = 2m \frac{9}{16} v^2 = \frac{9}{8} m v^2$$

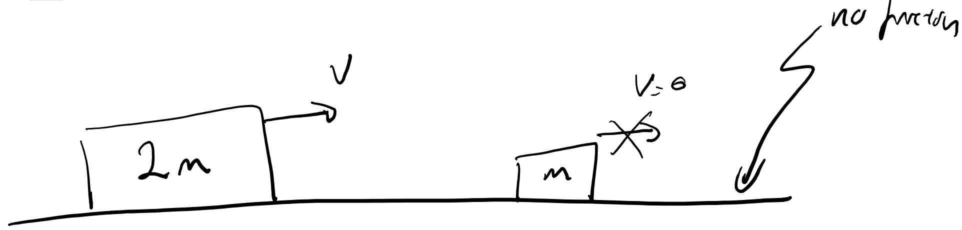
$$\Delta E = E_f - E_i = \frac{9}{8} m v^2 - \frac{3}{2} m v^2 = \left(\frac{9}{8} - \frac{12}{8}\right) m v^2$$
$$= \boxed{-\frac{3}{8} m v^2}$$

$\frac{3}{8} m v^2$ of energy was dissipated during the collision!

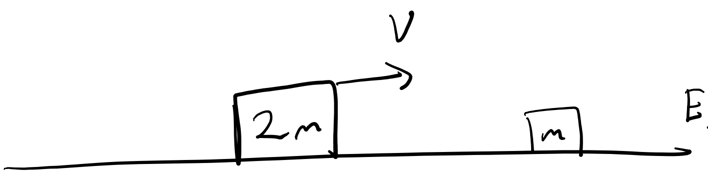
"Total Inelastic collision" ⇒ objects stick to each other

↑ between the two extremes inelastic collision.

* Elastic collision ⇒ No energy is lost during the collision!

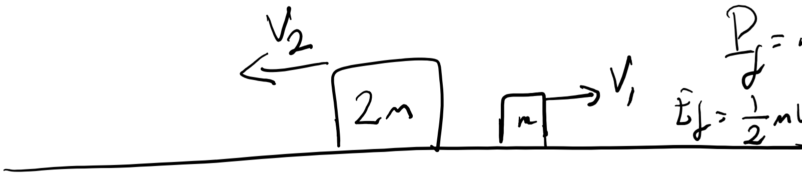


If these objects collide elastically find the final velocities of both objects.



$P_i = 2mV$

$E_i = \frac{1}{2}(2m)V^2$



$P_f = mV_1 - 2mV_2$

$E_f = \frac{1}{2}mV_1^2 + \frac{1}{2}2mV_2^2$

$\underline{P}$ is conserved

$$2\cancel{m}V = \cancel{m}V_1 - 2\cancel{m}V_2$$

$$2V = V_1 - 2V_2$$

$$2V_2 = V_1 - 2V$$

$$\boxed{V_2 = \frac{1}{2}V_1 - V}$$

E is conserved

$$\cancel{\tfrac{1}{2}}2\cancel{m}V^2 = \cancel{\tfrac{1}{2}}\cancel{m}V_1^2 + \cancel{\tfrac{1}{2}}2\cancel{m}V_2^2$$

$$2V^2 = V_1^2 + 2V_2^2$$

$$2v^2 = v_1^2 + 2\left(\frac{1}{2}v_1 - v\right)^2$$

$$\cancel{2v^2} = v_1^2 + \frac{1}{2}v_1^2 - 2v_1 v + \cancel{2v^2}$$

$$0 = \frac{3}{2}v_1^2 - 2v_1 v$$

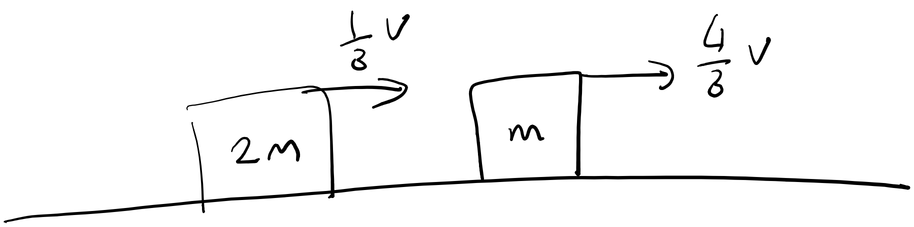
$$0 = v_1\left(\frac{3}{2}v_1 - 2v\right)$$

$v_1 = 0$ before the collision

$$\boxed{v_1 = +\frac{4}{3}v}$$

$$v_1 = \frac{4}{3}v \Rightarrow v_2 = \frac{1}{2}v_1 - v$$

$$v_2 = \frac{2}{3}v - v = -\frac{1}{3}v$$



check

$$P_f = \frac{1}{3}v\,2m + \frac{4}{3}mv = \frac{6}{3}mv = 2mv \checkmark$$

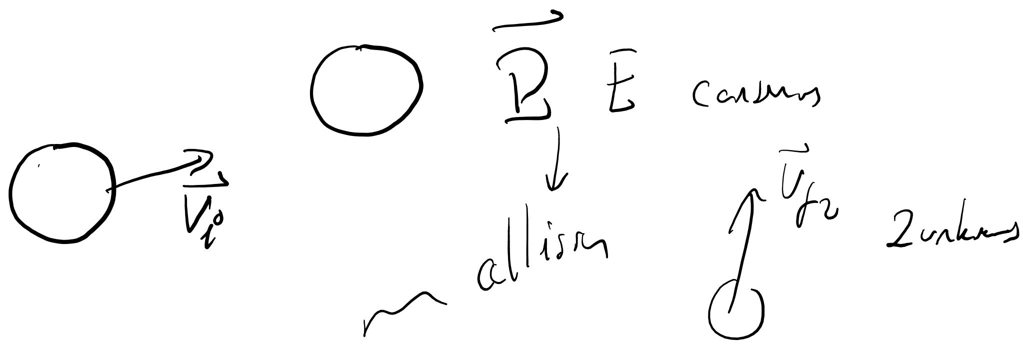
$$E_f = \frac{1}{2}2m\left(\frac{1}{3}v\right)^2 + \frac{1}{2}m\left(\frac{4}{3}\right)^2 v^2$$

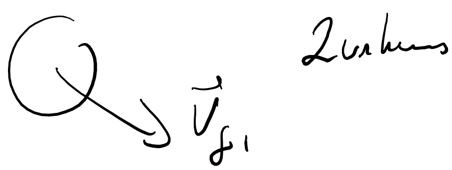
$$= m\frac{1}{9}v^2 + \frac{\cancel{16}\,8}{\cancel{2}\,9}mv^2 = \left(\frac{1}{9} + \frac{8}{9}\right)mv^2 = mv^2 \checkmark$$

Does elastic condition determine final velocities completely?

Collisions in 1D

$\boxed{\text{Only in 1D}}$

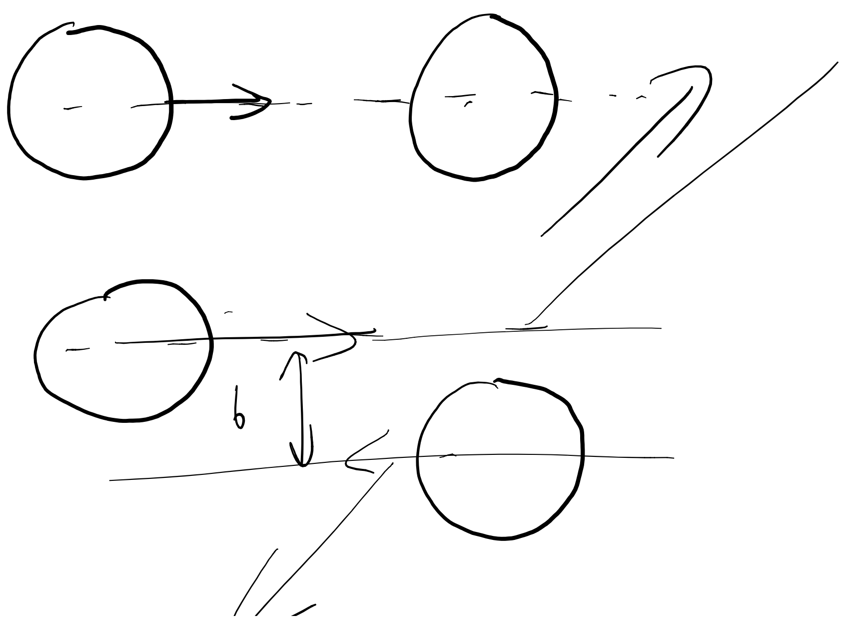


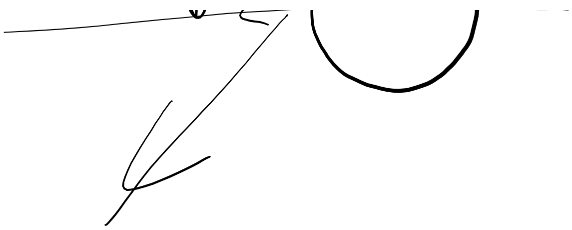


$\vec{P} \Rightarrow$ vector
2 equations

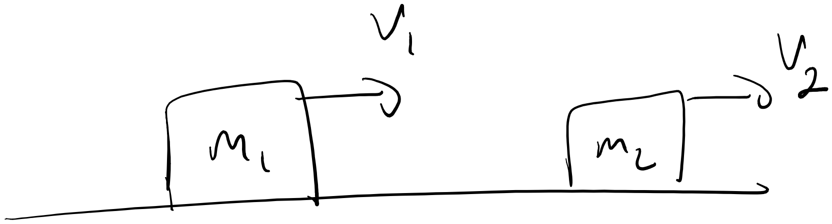
E $\Rightarrow$ scalar
1 equation

4 unknowns





in 2D and 3D the details of the collision are important to find the final velocities.



$$\underline{P} = m_1 V_1 + m_2 V_2 = M V_{CM}$$

$$= \underbrace{(m_1+m_2)}_{M} \underbrace{\frac{m_1 V_1 + m_2 V_2}{m_1+m_2}}$$

$$V_{CM} = \frac{d}{dt}\left(\underbrace{\frac{m_1 x_1 + m_2 x_2}{M}}\right)$$

center of mass $\longrightarrow x_{CM}$

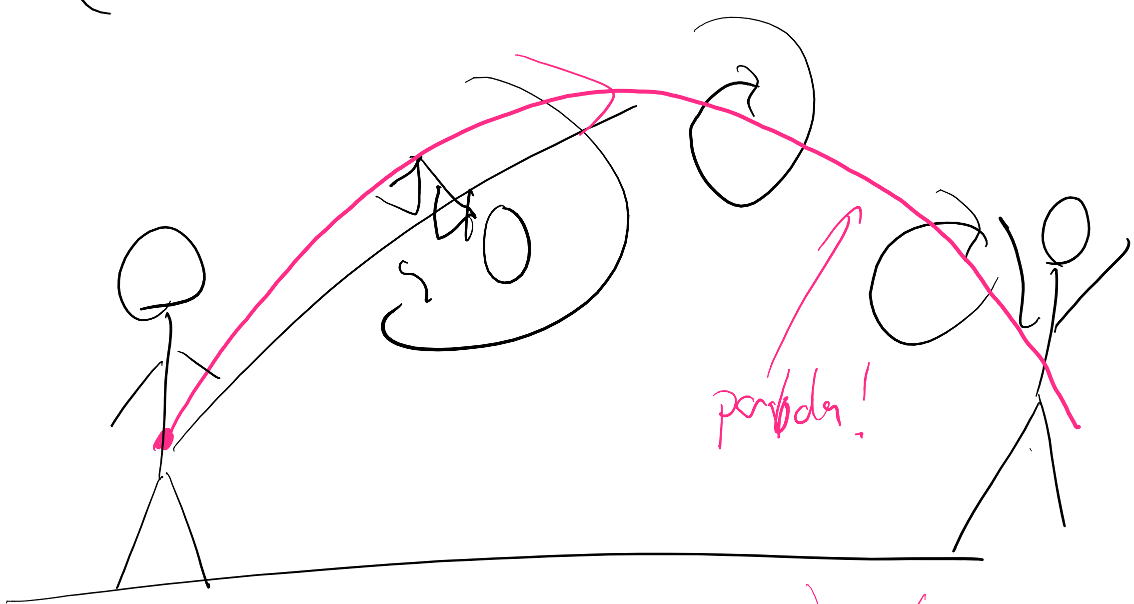
$$\boxed{\vec{P} = M \vec{V}_{CM}}$$

total momentum — total mass — velocity of the center of mass

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{(m_1 + m_2)}$$

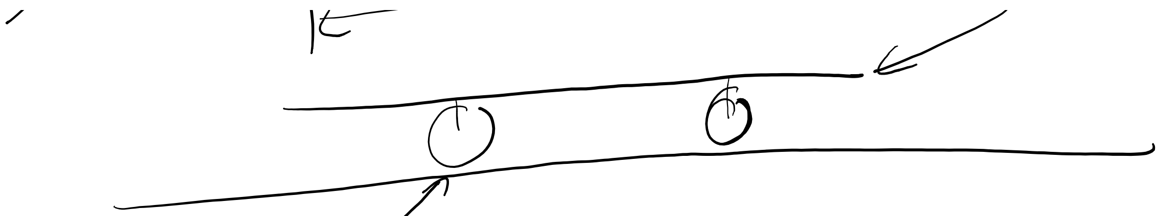
$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots\ldots}{(m_1 + m_2 + \ldots)} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i}$$

(Total momentum) = (total mass) (velocity of center of mass)



Motion $\Rightarrow$ (Motion of C.M.) + (Rotation around C.M.)

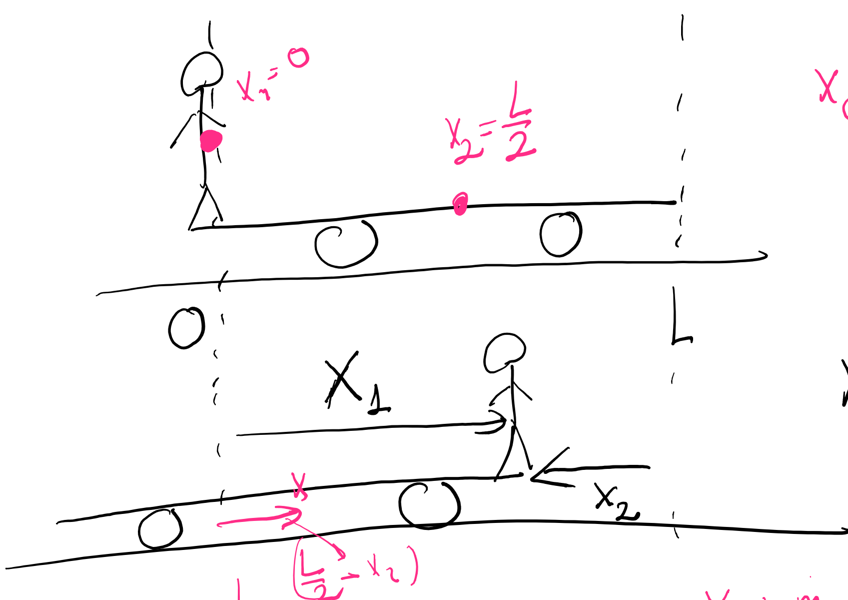
Ex



If I walk to the other edge of the car how far have I walked with respect to the ground?

$$\underline{P}_{total} = 0 = M_{total} v_{cm}$$

if $v_{cm} = 0 \Rightarrow$ position of x_{cm} does not change!



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_1 + x_2 = L$$

$$x_{cm} = \frac{m_2 \frac{L}{2}}{m_1 + m_2}$$

$$x_{cm} = \frac{m_1 x_1 + m_2 (\frac{L}{2} - x_2)}{m_1 + m_2}$$

$$x_2 = L - x_1$$

$$\frac{m_2 \frac{L}{2}}{\cancel{m_1 + m_2}} = \frac{m_1 x_1 + m_2 (\frac{L}{2} - x_2)}{\cancel{m_1 + m_2}}$$

$$\cancel{m_2 \frac{L}{2}} = m_1 x_1 + \cancel{m_2 \frac{L}{2}} - m_2 (L - x_1)$$

$$0 = m_1 x_1 - m_2 L + m_2 x_1$$

$$(m_1 + m_2) x_1 = m_2 L \Rightarrow \boxed{x_1 = \frac{L}{(m_1+m_2)} m_2}$$

$$x_1 = \frac{20 \cdot 10\cancel{0}}{18\cancel{0}} = \frac{100}{18} = \frac{50}{9} \simeq \boxed{5.6 \ m}$$

$$\boxed{x_1 = \frac{L \, m_2}{m_1 + m_2}}$$

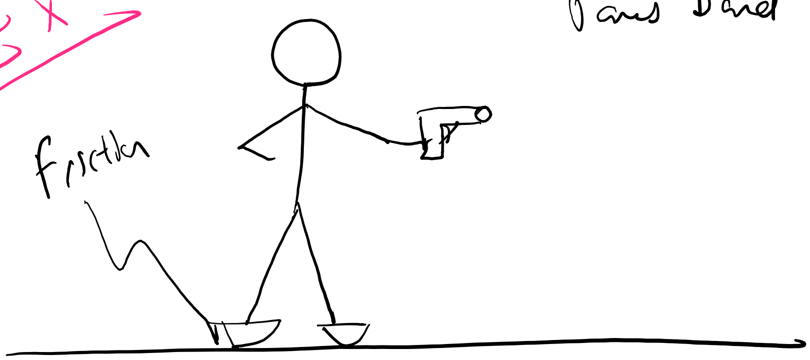
1°) Bad ✓

2°) Units ✓

3°) $m_2 \to \infty$ ✓

$$\lim_{m_2 \to \infty} \frac{L \, m_2}{m_1 + m_2} = L$$

Ex



James Bond on ice skates

M (Bond, clothes, gun, bullets)

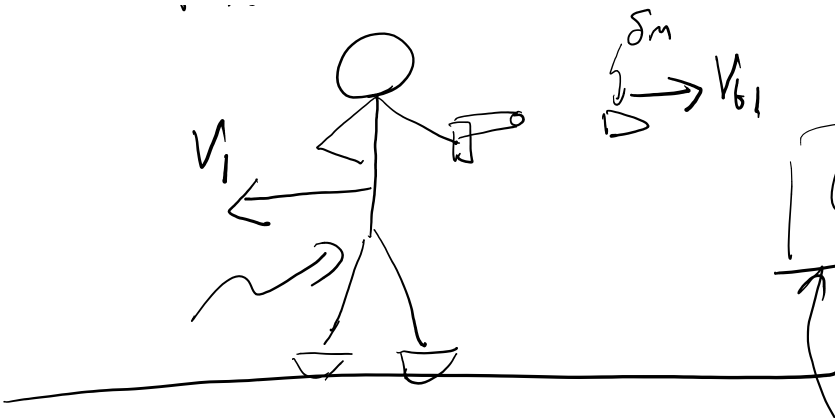
$\delta m \Rightarrow$ bullet weight.

$V_m \Rightarrow$ velocity of the bullet with respect to gun.

Initially 007 has zero velocity.

Find his velocity after he fires 2 bullets!

$\delta m \longrightarrow V_{t,1}$ $\qquad$ P conserved



P conserved

$$\boxed{(\delta m) V_{b1} = V_1 (M - \delta m)}$$

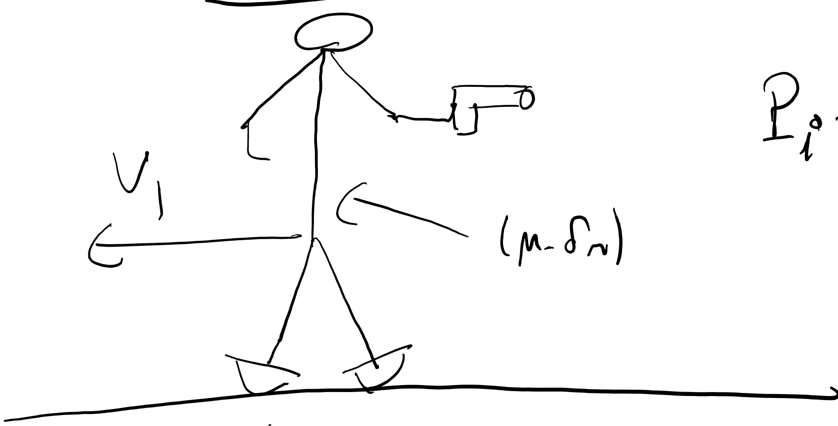
$$\boxed{V_{b1} + V_1 = V_M}$$

$$V_{b1} = V_M - V_1$$

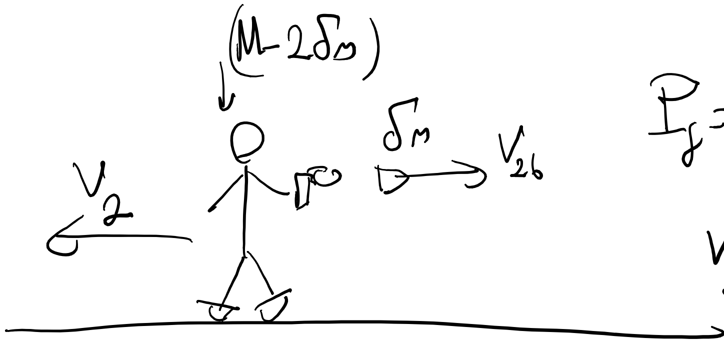
$$\delta m \, (V_m - V_1) = V_1 \; M - V_1 \delta m$$

$$(\delta m) V_m - \cancel{V_1 (\delta m)} = M V_1 - \cancel{V_1 \delta m}$$

$$\boxed{V_1 = \frac{\delta m}{M} V_M}$$



$$P_i = -(M - \delta m) V_1$$



$$P_f = (\delta m) V_{2b} - (M - 2\delta m) V_2$$

$$V_{2b} + V_2 = V_M$$

$$P_i = P_f \qquad -V_1 (M - \delta m) = (\delta m) V_{2b} - (M - 2\delta m) V_2$$

$$-V_1 M + V_1 \delta m = (\delta m)(V_m - V_2) - M V_2 + 2\delta m V_2$$

$$-V_1 M + V_1 \delta m = V_m(\delta m) - \cancel{V_2 \delta m} - M V_2 + \cancel{2}\delta m V_2$$

$$-V_m \delta m - V_1 M + V_1 \delta m = (-M + \delta m) V_2$$

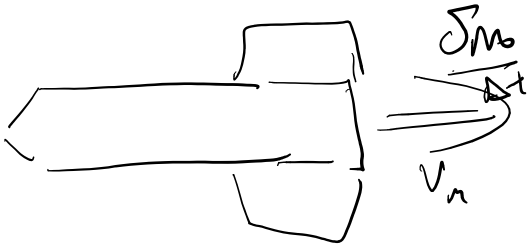
$$\frac{\delta m}{M} V_m \qquad \frac{\delta m}{M} V_m$$

$$-V_m \delta m - V_m \delta m + \frac{(\delta m)^3}{M} V_m = (-M + \delta m) V_2$$

$$\boxed{V_2 = \frac{V_m}{(M - \delta m)}\left(2\delta m - \frac{(\delta m)^3}{M}\right)}$$

If you take $\delta m \to 0$ limit

$\Delta t \to 0$

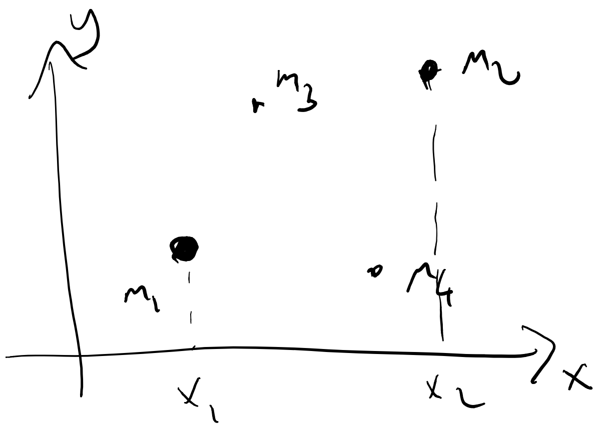


$\frac{\delta m}{\Delta t} \Rightarrow \mu$ kg/s

V_m

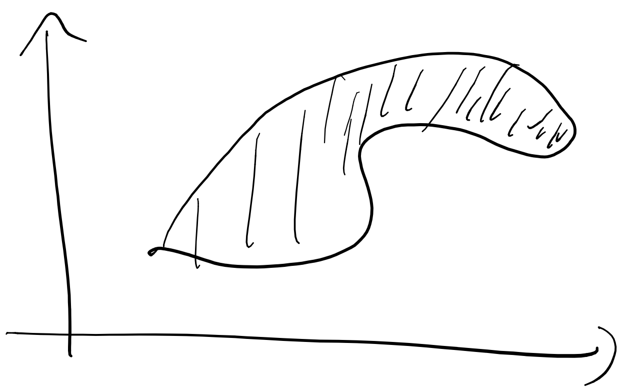
Study rocket eqn. in your book.

Center of mass for more complicated objects



$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + x_4 m_4}{(m_1+m_2+m_3+m_4)}$$

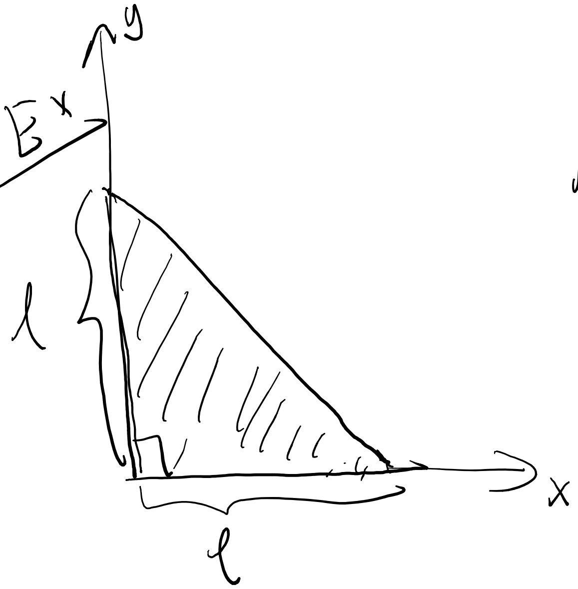
what happens if I have a continuous object?



$$x_{cm} = \frac{\sum_i x_i m_i}{\underbrace{\sum_i m_i}_{M}} \Rightarrow x_{cm} = \frac{1}{M}\int x\,dm$$

$$\vec{r}_{cm} = \frac{\int \vec{r}\,dm}{\int dm}$$

Ex

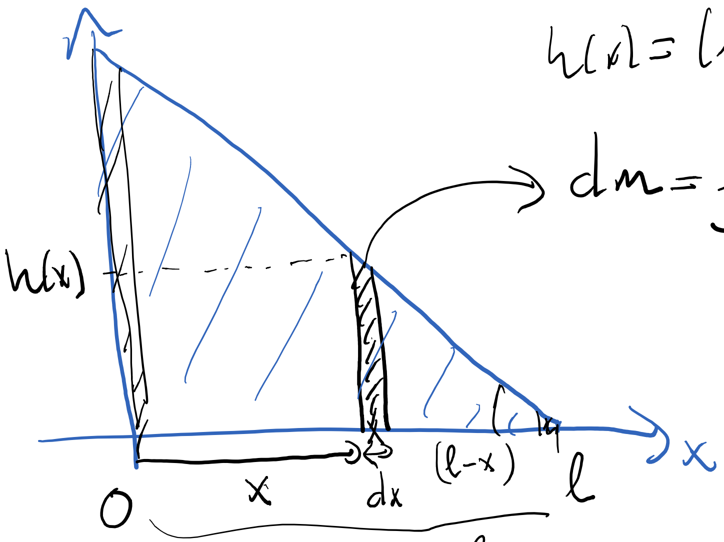


Made up of uniform density material.

$x_{cm} = ?$

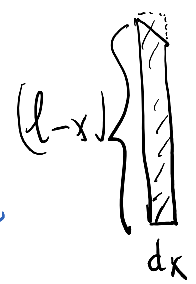
$y_{cm} = ?$

Find x_{cm} first



$h(x) = (l - x)$

$dm = \rho\, dA = \rho\,(l-x)\,dx$



neglect

$dA = dx\,(l-x) - \cancel{\frac{1}{2}(dx)^2}$

$A = \frac{l^2}{2} \qquad M = \rho\,\frac{l^2}{2}$

$dm = \rho\,(l-x)\,dx$

$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^l x\,\rho\,(l-x)\,dx$$

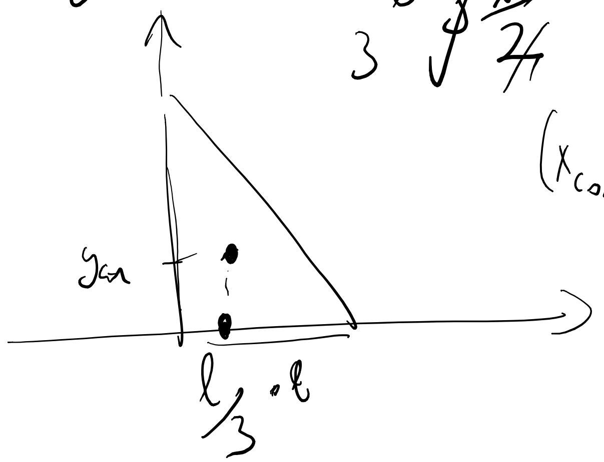
$$= \frac{1}{M}\,\rho\int_0^l (xl - x^2)\,dx$$

$$= \frac{1}{M}\,\rho\left\{ l\underbrace{\int_0^l x\,dx}_{\left.\frac{x^3}{2}\right|_0^l} - \underbrace{\int_0^l x^2\,dx}_{\left.\frac{x^3}{3}\right|_0^l} \right\}$$

$$x_{cm} = \frac{1}{M}\,\rho\left[\, l\,\frac{l^2}{2} - \frac{l^3}{3} \right] = \frac{\rho\, l^3}{\ldots}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$X_{cm} = \frac{1}{M}\rho\left[\ell\,\frac{\ell^3}{2} - \frac{\ell^3}{3}\right] = \frac{\rho\,\ell^3}{M}\underbrace{\left[\frac{1}{2}-\frac{1}{3}\right]}_{\frac{1}{6}}$$

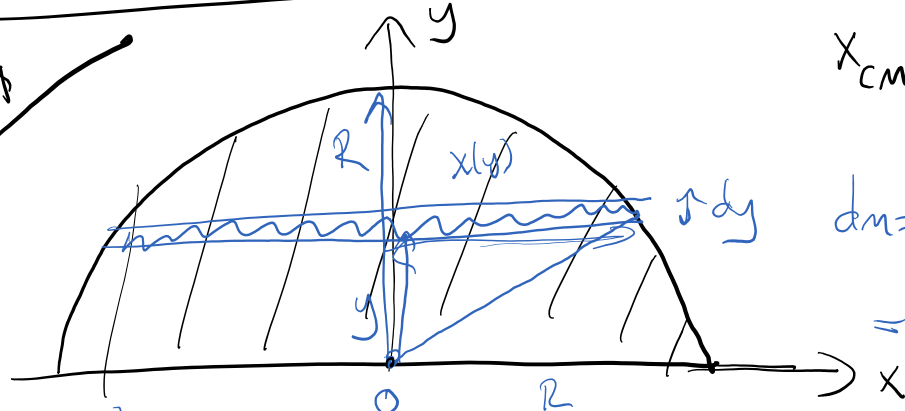
$$X_{cm} = \frac{1}{6}\,\frac{\rho\,\ell^3}{M} = \frac{\cancel{\rho}\,\ell^{\cancel{3}}}{\cancel{6}_{3}\,\cancel{\rho}\,\frac{\cancel{\ell^2}}{\cancel{2}}} = \boxed{\frac{\ell}{3}}$$



$$(x_{cm}, y_{cm}) = \left(\frac{\ell}{3}, \frac{\ell}{3}\right)$$

Ex

$X_{cm} = 0$ by Symmetry



$dm = \rho\, dy\, 2\,x(y)$

$= 2\rho\sqrt{R^2-y^2}\,dy$

$M = \frac{\pi R^2}{2}\rho \qquad x(y) = \sqrt{R^2-y^2}$

$$y_{cm} = \frac{1}{M}\int y\, dm = \frac{1}{M}\int_0^R 2\rho y\sqrt{R^2-y^2}\,dy$$

$$= \frac{1}{\frac{\pi R^2}{2}\cancel{\rho}}\,2\cancel{\rho}\int_0^R \sqrt{R^2-y^2}\;y\,dy$$

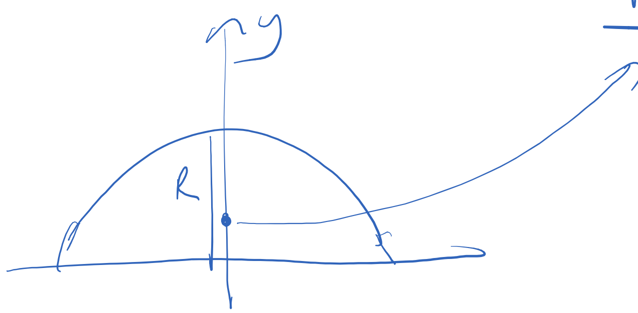
$u = R^2 - y^2$

$$du = -2y\,dy$$

$$= \frac{4}{\pi R^2}\int_{R^2}^{0}\sqrt{u}\left(-\frac{1}{2}\right)du$$

$$= \frac{4}{\pi R^2}\,\frac{1}{2}\int_{0}^{R^2}\sqrt{u}\,du = \frac{2}{\pi R^2}\;\frac{u^{3/2}}{3/2}\Bigg|_{0}^{R^2}$$

$$= \frac{4}{3\pi R^2}\,R^{2\cdot\frac{3}{2}} = \boxed{\frac{4R}{3\pi}}$$



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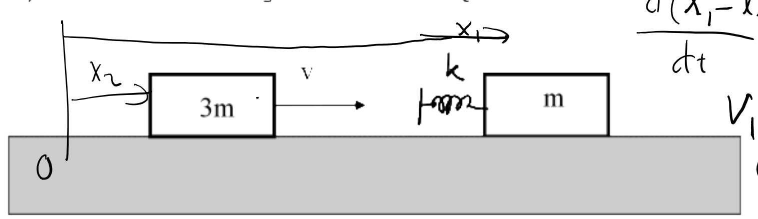


$$\frac{d(x_1 - x_2)}{dt} = 0$$

$$V_1 - V_2 = 0$$

$$V_1 = V_2$$

Initial velocities of the two masses are as shown above, there is no friction. The massless spring has spring constant *k* and is stuck to the second mass (m). Due to the presence of the spring the collision will be elastic.

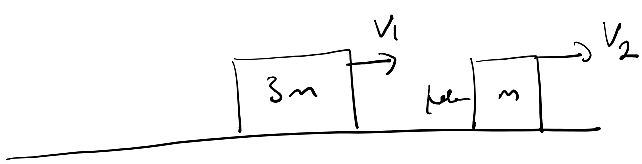
a) Find the direction and magnitude of the velocity of both masses after the collision.

b) Find the **maximum compression** of the spring during the collision.

a) Elastic collision

$P_i = 3mV$

$E_i = \frac{1}{2}3mV^2$



$P_f = 3mV_1 + mV_2$

$E_f = \frac{1}{2}3mV_1^2 + \frac{1}{2}mV_2^2$

$$P_i = P_f \Rightarrow 3\cancel{m}V = 3\cancel{m}V_1 + \cancel{m}V_2 \rightarrow \boxed{V_1 = V - \frac{1}{3}V_2}$$

$$E_i = E_f \qquad \frac{1}{\cancel{2}}3\cancel{m}V^2 = \frac{1}{\cancel{2}}3\cancel{m}V_1^2 + \frac{1}{\cancel{2}}\cancel{m}V_2^2$$

$$3V^2 = 3\left(V - \frac{1}{3}V_2\right)^2 + V_2^2$$

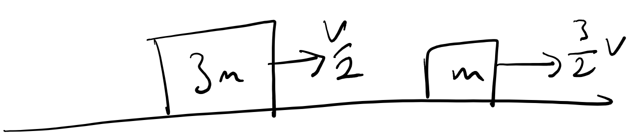
$$\cancel{3V^2} = \cancel{3V^2} - 2VV_2 + \frac{1}{3}V_2^2 + V_2^2$$

$$0 = \left(\frac{4}{3}V_2 - 2V\right)V_2$$

$\cancel{V_2 = 0}$ initial solution

$\boxed{V_2 = \frac{3}{2}V}$ final V_2

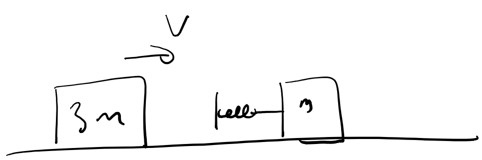
$$V_1 = V - \frac{1}{\cancel{3}}\frac{\cancel{3}}{2}V = \boxed{\frac{1}{2}V}$$



$$P = 3m\frac{V}{2} + m\frac{3}{2}V = 3mV \checkmark$$

$$E = \frac{1}{2}3m\frac{V^2}{4} + \frac{1}{2}m\frac{9V^2}{4} = \frac{1}{2}m\frac{\cancel{12}^{\,3}V^2}{\cancel{4}} = \frac{1}{2}3mV^2 \checkmark$$

b)

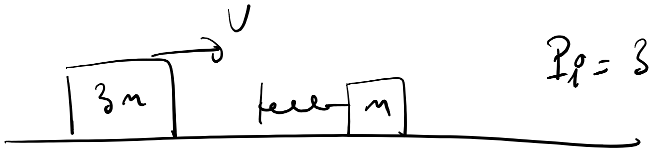


Max compression means the distance $(x_2 - x_1)$ is minimum!

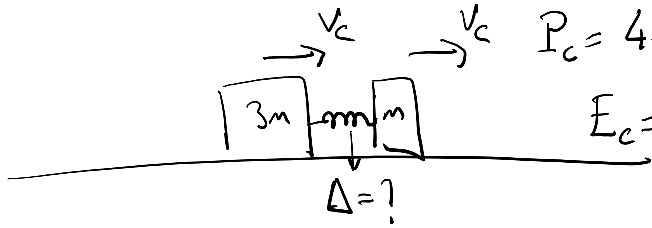
$$\frac{d}{dt}\left(x_1(t) - x_2(t)\right) = 0$$

$$V_1 - V_2 = 0 \Rightarrow V_1 = V_2$$

at max. compression.



$P_i = 3mV \qquad E_i = \frac{1}{2}3mV^2$



$P_c = 4mV_c$

$E_c = \frac{1}{2}3mV_c^2 + \frac{1}{2}mV_0^2 + \frac{1}{2}k\Delta^2$

$P_c = P_i$

$4\cancel{m}V_c = 3\cancel{m}V \Rightarrow V_c = \frac{3}{4}V$

$E_i = E_c$

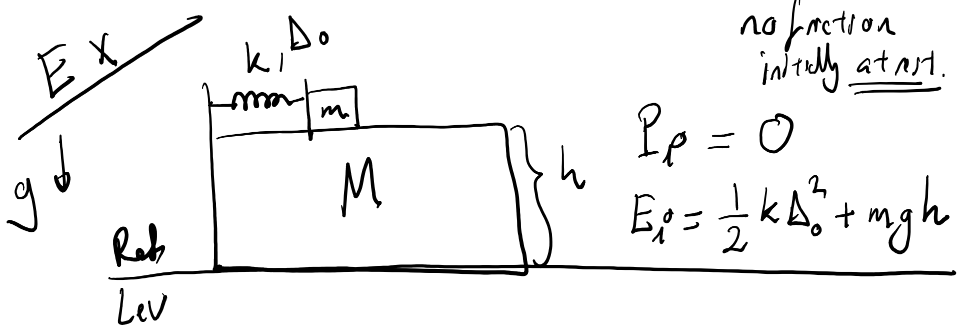
$\frac{1}{\cancel{2}}3mV^2 = \frac{1}{\cancel{2}}4mV_c^2 + \frac{1}{\cancel{2}}k\Delta^2$

$3mV^2 = \cancel{4}m\frac{9}{\cancel{16}4}V^2 + k\Delta^2$

$\underbrace{\left(3 - \frac{9}{4}\right)}mV^2 = k\Delta^2$

$\frac{3}{4}\frac{mV^2}{k} = \Delta^2 \Longrightarrow \boxed{\Delta = \sqrt{\frac{3}{4}m\frac{V^2}{k}}}$

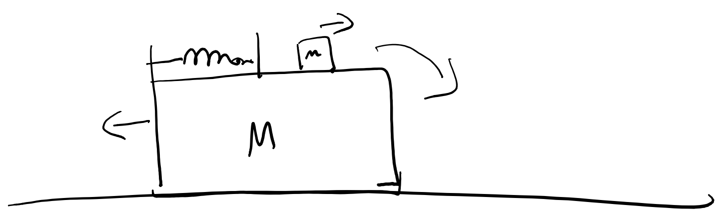
Ex



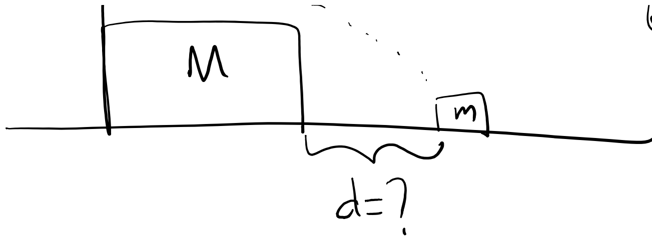
no friction
initially at rest.

$P_i = 0$

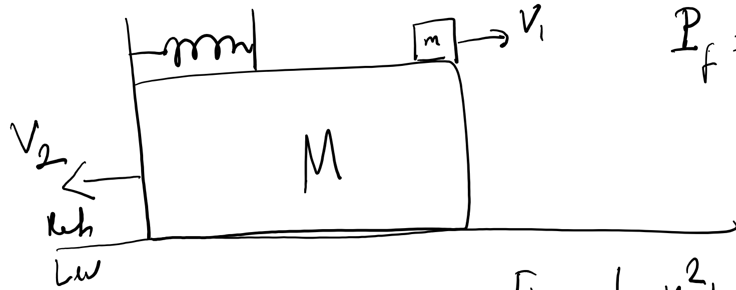
$E_i = \frac{1}{2}k\Delta_0^2 + mgh$



What is the distance between two masses



between two masses when the small mass hits the floor?



$$P_f = m v_1 - M v_2 = 0$$

$$\boxed{v_1 = \frac{M}{m} v_2}$$

$$E_f = \frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2 + mgh$$

$$E_i = E_f$$

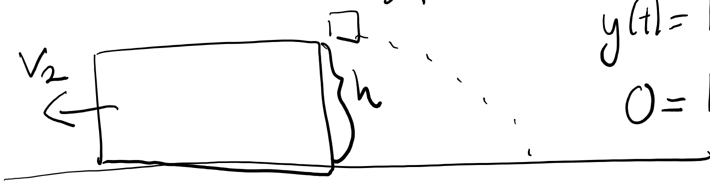
$$\frac{1}{\cancel{2}} k \Delta_0^2 + \cancel{mgh} = \frac{1}{\cancel{2}} m v_1^2 + \frac{1}{\cancel{2}} M v_2^2 + \cancel{mgh}$$

$$k \Delta_0^2 = \cancel{m} \frac{M^2}{m^{\cancel{2}}} v_2^2 + M v_2^2 = M\left(1 + \frac{M}{m}\right) v_2^2$$

$$v_2^2 = \frac{m}{M(m+M)} k \Delta_0^2$$

$$v_2 = \sqrt{\frac{m}{M(m+M)} k \Delta_0^2}$$

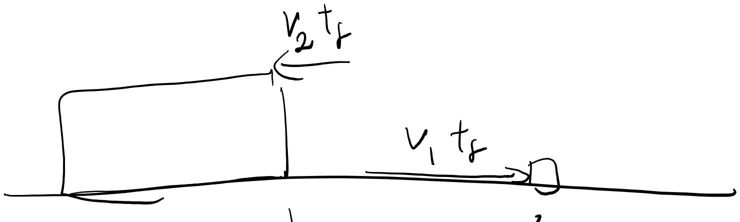
$$v_1 = \frac{M}{m} v_2 = \sqrt{\frac{M}{m(m+M)} k \Delta_0^2}$$



$$y(t) = h - \frac{1}{2} g t^2$$

$$0 = h - \frac{1}{2} g t_f^2$$

$$t_f = \sqrt{\frac{2h}{g}}$$



$$d = (v_1 + v_2)\, t_f$$

$$\boxed{d = \left(\sqrt{\frac{m}{M(m+M)}\, k \Delta_0^2} + \sqrt{\frac{M}{m(M+m)}\, k \Delta_0^2}\right)\sqrt{\frac{2h}{g}}}$$

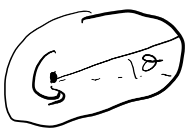
$$\boxed{d = \left(\sqrt{\frac{m}{M}} + \sqrt{\frac{M}{m}}\right)\sqrt{\frac{k\,2h}{(m+M)\,g}}\;\Delta_0}$$

1°) Good ✓

2°) Unit $\quad \sqrt{\dfrac{\cancel{N/m}\;\cancel{m}}{\cancel{kg\cdot m/s^2}}}\; m$ ✓

3°) $M \to \infty$ (check this and find out that our exp. is correct!)

Rotational Motion.



$\theta(t)$; $x(t)$

$\dfrac{d\theta}{dt} = \omega(t) \to$ angular velocity ; $v(t)$

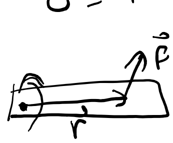
$\dfrac{d^2\theta}{dt^2} = \dfrac{d\omega}{dt} = \alpha(t) \to$ angular accelration. ; $a(t)$

angular acc.

Dynamics: $\quad \vec{\tau} = I\,\vec{\alpha} \quad \Leftrightarrow \quad \vec{F} = m\vec{a}$

torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$



Moment of inertia

$$I = \int r^2\, dm$$

$$I = \begin{cases} I_{ring} = MR^2 \\ I_{Disc} = \dfrac{MR^2}{2} \\ I_{rod} = \dfrac{ML^2}{12} \end{cases}$$

parallel axis theorem

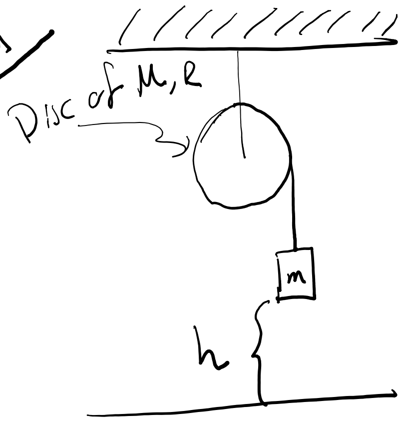
$$I_d = I_{com} + Md^2$$

Kinetic Energy of Rotation

$$K = \frac{1}{2} I \omega^2$$

[But we have to be careful about which I and which ω!]

EX

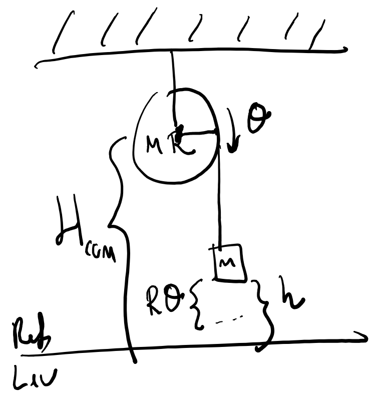


How fast does m hit the floor if it starts from the rest configuration shown?

Soln 1

Energy is conserved

Potential energy depends on the position of COM!

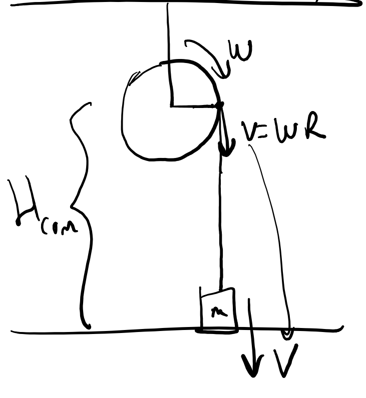


$$E_i = mgh + MgH_{com}$$

$$\frac{d\theta}{dt} = \omega \qquad \frac{d}{dt}(R\theta) = V$$

$$\boxed{R\omega = V}$$

$$I_{DISC} = \frac{MR^2}{2}$$



$$E_f = MgH_{com} + \frac{1}{2}mv^2 + \frac{1}{2} I \omega^2$$

$$= MgH_{com} + \frac{1}{2}mv^2 + \frac{1}{4}MR^2\omega^2$$

$$E_i = E_f \quad (1)$$

(2) $\omega R = V$

$\alpha R = a$

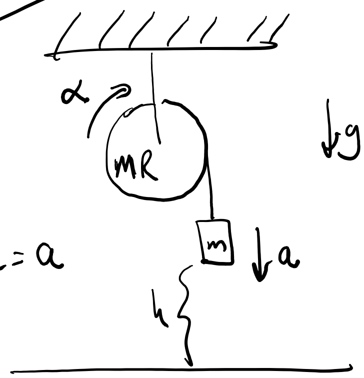
$$E_i = E_f \quad (1) \qquad \alpha R = a$$

$$\cancel{M g H_{com}} + mgh = \cancel{M g H_{com}} + \frac{1}{2} m v^2 + \frac{1}{4} M R^2 \omega^2$$

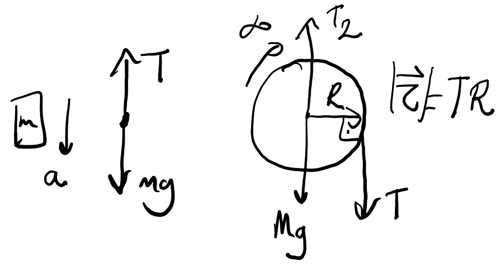
$$mgh = \frac{1}{2} m v^2 + \frac{1}{4} M v^2$$

$$mgh = \frac{1}{2}\left(m + \frac{M}{2}\right) v^2 \Rightarrow \boxed{v = \sqrt{\frac{2mgh}{\left(m + \frac{M}{2}\right)}}}$$

Solution 2



$$\alpha R = a$$



$$ma = mg - T$$

$$I\alpha = TR$$

$$\frac{M R^{\cancel{2}}}{2}\alpha = T \cancel{R}$$

$$\frac{Ma}{2} = T$$

$$ma = mg - \frac{M}{2} a$$

$$\boxed{a = \frac{m}{\left(m + \frac{M}{2}\right)} g}$$

How long will it take for m to hit the floor?

$$\frac{1}{2} a t_f^2 = h$$

$$\boxed{t_f = \sqrt{\frac{2h}{a}}}$$

$$V_f = a t_f = \sqrt{2ha}$$

$$\boxed{V = \sqrt{\frac{2mhg}{\left(m + \frac{M}{2}\right)}}}$$

uniform rod

M

L

$E_i^p = 0$

$\downarrow g$

$V_{com} = ?$

$\frac{L}{2}$

$E_f = -Mg\frac{L}{2} + \frac{1}{2} I_{edge} \omega^2$

$V_{com} = \omega \frac{L}{2}$

ω

Find V_{com} as the rod passes through vertical configuration.

$I_{edge} = I_{com} + M\frac{L^2}{4} = M\frac{L^2}{3}$

$I_{com} = \frac{ML^2}{12}$

$\left(\frac{L}{2}\right)$

Solution: Energy conservation.

$$E_i = E_f$$

$$0 = -Mg\frac{L}{2} + \frac{1}{2} I_{edge} \omega^2$$

$$\cancel{M} g \frac{\cancel{L}}{\cancel{2}} = \frac{1}{\cancel{2}} \frac{\cancel{M} L^{\cancel{2}}}{3} \omega^2$$

$$\sqrt{\frac{3g}{L}} = \omega \implies V_{com} = \omega \frac{L}{2}$$

$$= \sqrt{\frac{3 g L^{\cancel{2}}}{\cancel{L} 4}} = \boxed{\sqrt{\frac{3}{4}} \sqrt{gL}}$$

In general

ω

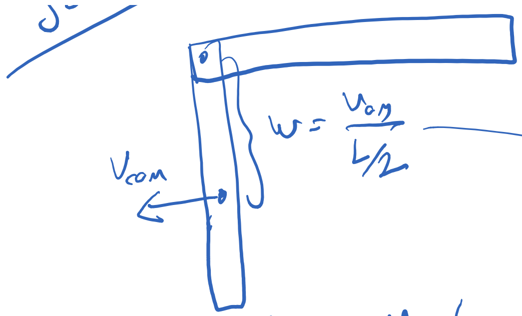
V_{com}

$$\boxed{K = \frac{1}{2} m V_{com}^2 + \frac{1}{2} I_{com} \omega^2}$$

only works for COM.

Solution

$E_i = 0$



$E_f = 0$

$\omega = \frac{v_{com}}{L/2}$

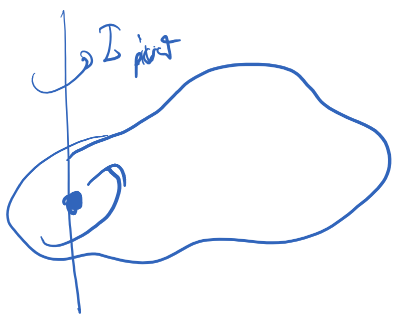
$$E_i = E_f = -Mg\frac{L}{2} + \frac{1}{2} M v_{com}^2 + \frac{1}{2} I_{com}\, \omega^2$$

$$\cancel{M} g \frac{L}{\cancel{2}} = \frac{1}{\cancel{2}} \cancel{M} v_{com}^2 + \frac{1}{\cancel{2}} \frac{\cancel{M} \cancel{L^2}}{12} \frac{v_{com}^2}{\cancel{L^2}/4}$$

$$gL = v_{com}^2\left(1 + \frac{1}{3}\right) \Rightarrow v_{com} = \sqrt{\frac{3}{4} gL}$$

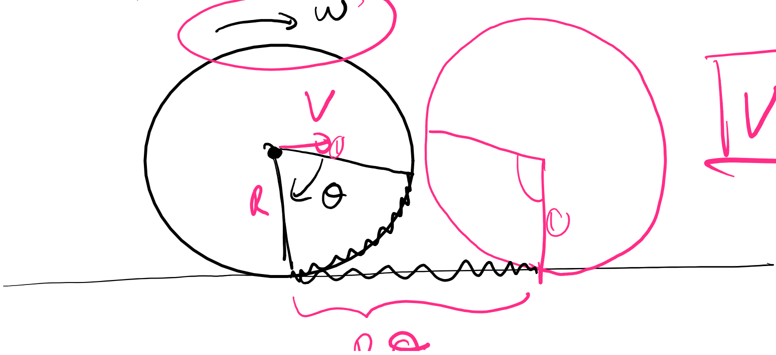
$$\boxed{K = \frac{1}{2} m v_{com}^2 + \frac{1}{2} I_{com}\, \omega^2}$$

Always correct



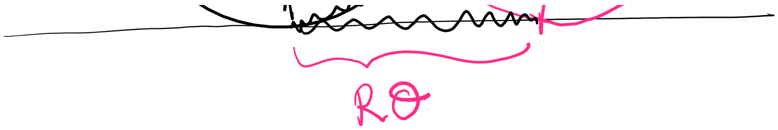
$$K = \frac{1}{2} I_{pivot}\, \omega^2$$

Rolling



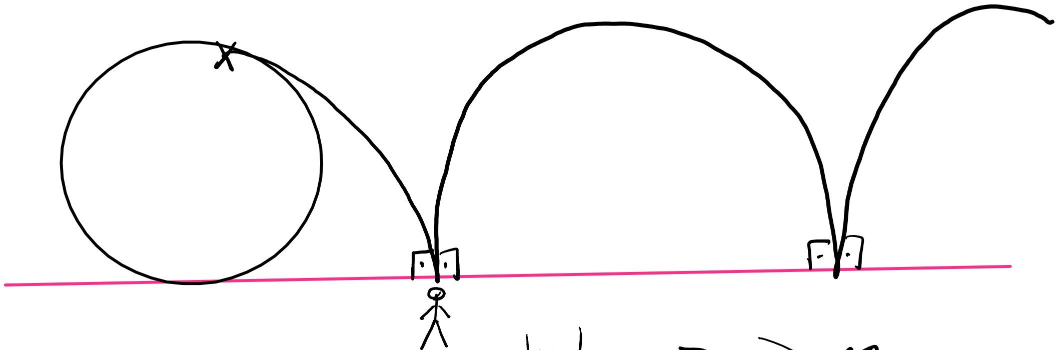
$$\boxed{V = \omega R}$$

rolling condition

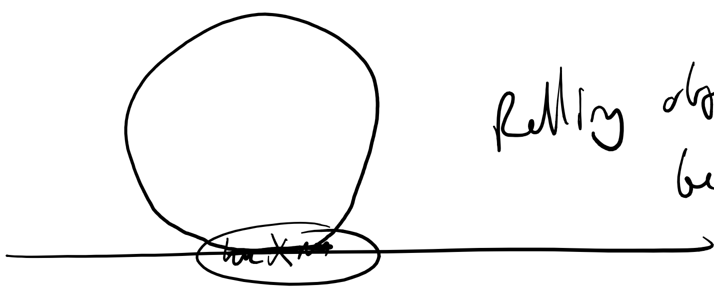


rolling condition

During rolling there may be friction but it does **not** dissipate energy!



no displacement ⇒ no dissipated energy.



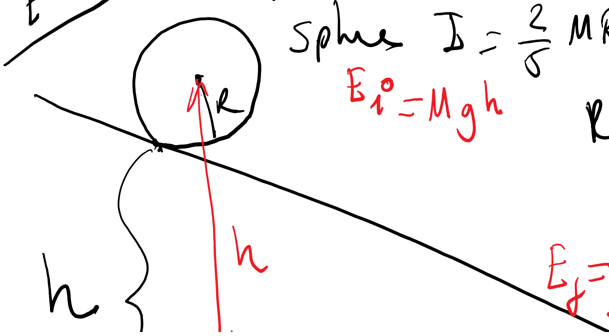
Rolling objects stop because they are not perfect circles.

Flat contact

Ex

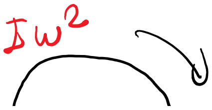
Sphere $I = \frac{2}{5} MR^2$

$E_i^0 = Mgh$



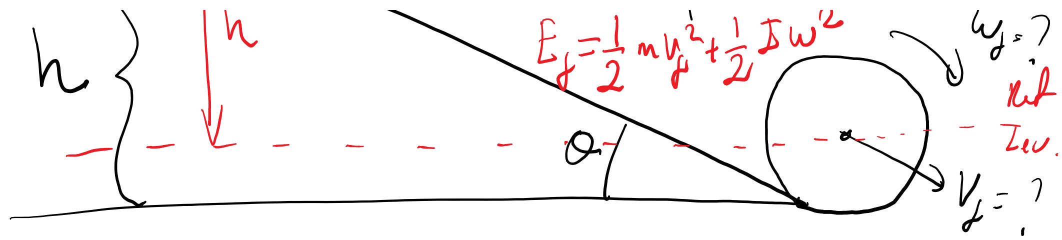
Rolls without slipping
⇓
implies friction.

$E_f = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2$



$\omega_f = ?$

v_f



a) $V_f = ?$ $\quad \omega_f = ?$

b) Find the friction force acting on the sphere during motion.

Although there is friction, during rolling without slipping there is no energy loss, friction force is static friction.

$\Longrightarrow$ Rolling w/o slipping $\Rightarrow$ energy is conserved.

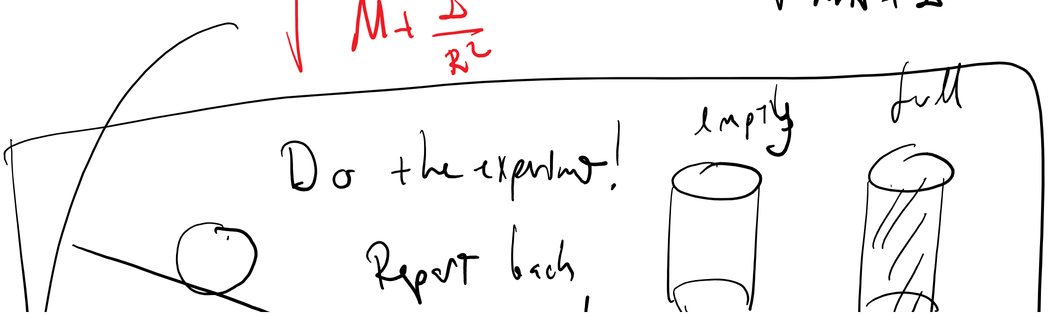
$$Mgh = \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2$$

$V_f = R\omega_f$

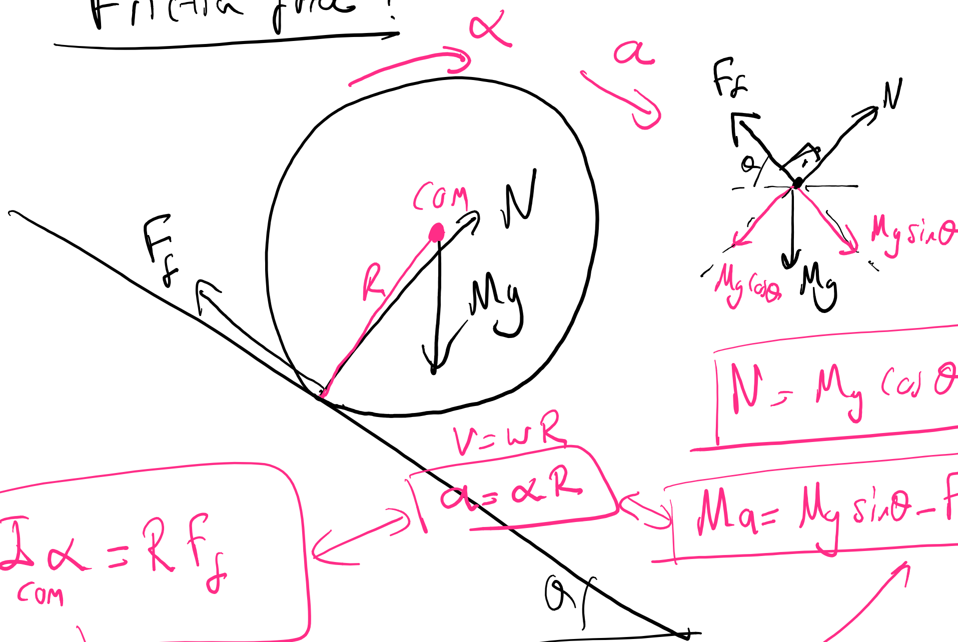
$I = \frac{2}{5} MR^2$

$$Mgh = \frac{1}{2} M v_f^2 + \frac{1}{2}\left(\frac{I}{R^2}\right) v_f^2$$

$$V_f = \sqrt{\frac{2Mgh}{M + \frac{I}{R^2}}} \qquad \omega_f = \sqrt{\frac{2Mgh}{MR^2 + I}}$$



Friction force?



$$\boxed{N = Mg\cos\theta}$$

$$v = \omega R$$

$$\boxed{I_{com}\,\alpha = R F_f} \quad \boxed{a = \alpha R} \quad \boxed{Ma = Mg\sin\theta - F_f}$$

$$I\frac{a}{R} = R F_f \Rightarrow F_f = \frac{I a}{R^2}$$

$$Ma = Mg\sin\theta - \frac{I a}{R^2}$$

$$a\left(M + \frac{I}{R^2}\right) = Mg\sin\theta$$

$$a = \left(\frac{M}{M + \frac{I}{R^2}}\right) g\sin\theta$$

$$\boxed{F_f = \frac{I M}{I + MR^2}\, g\sin\theta}$$

stone from

$$F_f \le \mu_s N$$

$$\frac{I \cancel{M}}{I + MR^2}\cancel{g}\sin\theta \le \mu_s \cancel{Mg}\cos\theta$$

$$\boxed{\frac{I}{(I+MR^2)}\tan\theta \le \mu_s}$$

$I=\frac{2}{5}MR^2$

Rotation

$K=\frac{1}{2}I\omega^2$

$K=\frac{1}{2}Mv_{com}^2+\frac{1}{2}I_{com}\omega^2$

Rolling without slipping.

$\theta,\ \omega,\ \alpha$

$\tau = I\alpha$

$\vec{\tau}=\vec{r}\times\vec{F}$

$I=\int r^2\,dm$

$(I_d = I_{com}+Md^2)$

Momentum Conservation

$$\vec{P}=m\vec{v} \Longrightarrow \frac{d\vec{P}}{dt}=\vec{F} \Longrightarrow \Delta\vec{P}=\underbrace{\int\vec{F}\,dt}_{\text{Impulse}}$$

closed system

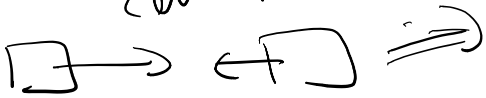
$\vec{P}_{total}=m_1\vec{v}_1+m_2\vec{v}_2+\dots$ is conserved.

$$\vec{P}=M\vec{v}_{com} \qquad M=\sum_i m_i$$

$$\vec{r}_{com}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\dots}{M}$$

for a closed system $\vec{V}_{com}$ is constant.

collision $\Rightarrow$ Momentum is conserved collision

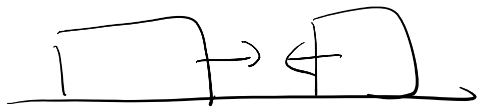


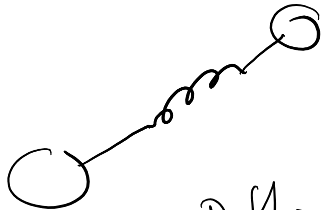
Momentum is conserved

Energy conserved
Elastic

(Inelastic collision)

objects stick together
completely inelastic





Different kinds of collision.

Energy conservation

$$W = \vec{F}\cdot\vec{d} \quad \text{constants}$$

$$\Delta E = W_{n.c}$$

$$= \int_i^f \vec{F}\cdot d\vec{l}$$

$$E_f - E_i \longrightarrow K = \frac{1}{2} m v^2 \left(+ \frac{1}{2} I \omega^2\right)$$

$$U_g = mgh \simeq -G\frac{M_e m}{r}$$

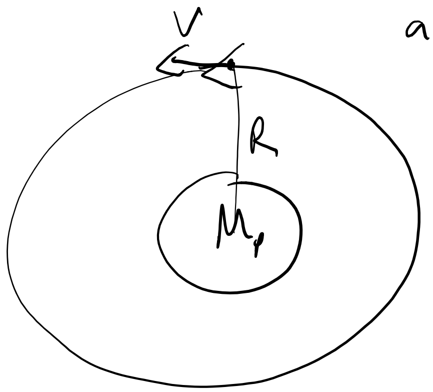
Grav. pot.

$$U_s = \frac{1}{2} k \Delta^2$$

2 pot.

Gravitation $F_G = G\frac{m_1 m_2}{r^2}$

attractive force

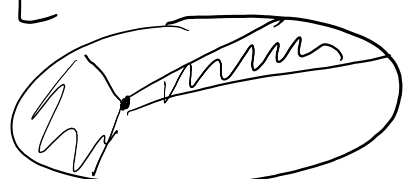


$\cancel{m}\frac{V^3}{\cancel{R}} = F_G = G\frac{M_p \cancel{m}}{R^{\cancel{2}}}$

$V = \sqrt{G\frac{M_p}{R}}$

$T = \frac{2\pi R}{\sqrt{G\frac{M_p}{R}}} = \frac{2\pi}{\sqrt{G M_p}} R^{3/2}$

$\frac{R^3}{T^2} \Longrightarrow$ constant [Keplers Laws]



· Ellipses

· Angular momentum.