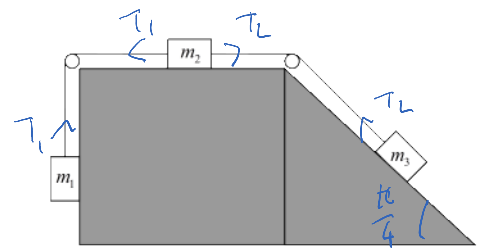
Week 04 (Oct 17 - Oct 21)

Friday, September 23, 2016 2:56 PM

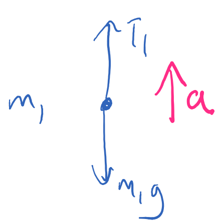
PHYSICS 101- Instructor: M. Özgür OKTEL- 2016

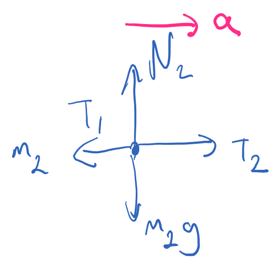
QUIZ-7

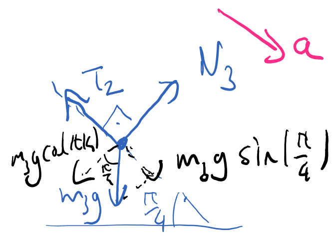
Find the acceleration (indicate direction) of all the masses and the tension in the cables. There is no friction in the system. Use g=10 m/s^2. The masses are m1=1 kg, m2=2 kg, m3=$2\sqrt{2}$ kg. The angle of the inclined plane is 45 degrees.



Acceleration of m2	Which way does m2 move?	Tension on the left cable	Tension on the right cable







① $m_1 a = T_1 - m_1 g$ ② $m_2 a = T_2 - T_1$ ③ $m_3 a = m_3 g \underbrace{\sin\left(\frac{\pi}{4}\right)}_{\frac{1}{\sqrt{2}}} - T_2$

a

T_1

T_2

① + ② + ③ $= (m_1 + m_2 + m_3) a = \cancel{T_1} - m_1 g + \cancel{T_2} - \cancel{T_1} + m_3 g \frac{1}{\sqrt{2}} - \cancel{T_2}$

$$a = \frac{\left(m_3 \frac{1}{\sqrt{2}} - m_1\right) g}{m_1 + m_2 + m_3}$$

$m_3 = 2\sqrt{2}$ kg

$m_1 = 1$ kg

$m_2 = 2$ kg

$g = 10\ m/s^2$

$$a = \frac{(\overbrace{2-1}^{1})\, 10}{1 + 2 + 2\sqrt{2}}$$

$$a = \frac{10}{3+2\sqrt{2}} \; m/s^2 = \frac{10(3+2\sqrt{2})}{9-8} = \boxed{30 - 20\sqrt{2}\ m/s^2}$$

$(3-2\sqrt{2})$

?

> 0

m_1 is moving up!

③ $m_3 a = m_3 g \frac{1}{\sqrt{2}} - T_2$

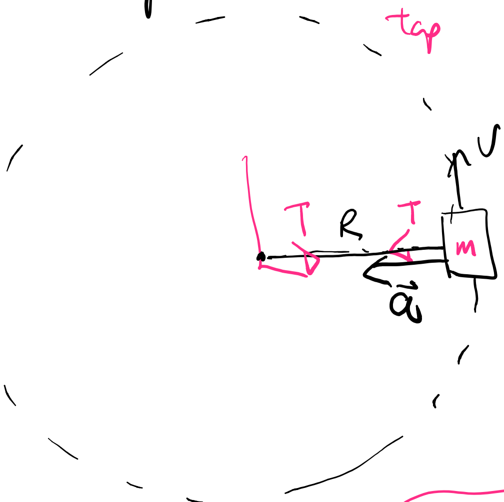
$$\boxed{T_2 = m_3\left(\frac{g}{\sqrt{2}} - a\right)}$$

② $m_2 a = T_2 - T_1 \Rightarrow T_1 = T_2 - m_2 a$

$$\boxed{T_1 = m\left(\frac{g}{\sqrt{2}} - a\right) - m_2 a}$$

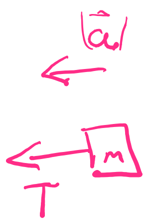
Forces in circular motion.

uniform circular motion



centripetal acceleration

$$|\vec{a}| = \frac{v^2}{R}$$

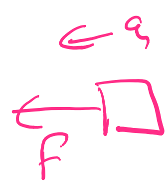


$T = ma$

$$\boxed{T = m\frac{v^2}{R}}$$

~~Centrifugal~~

~~⇓ fictitious force in an accelerating reference frame~~



force & acceleration are centripetal.

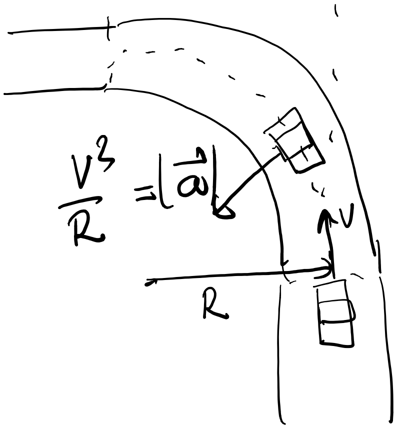
Ex

Banked highway curve

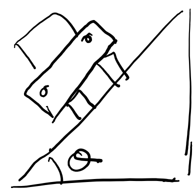
Due to icing the curve has zero friction!

Due to icing the curve has zero friction!

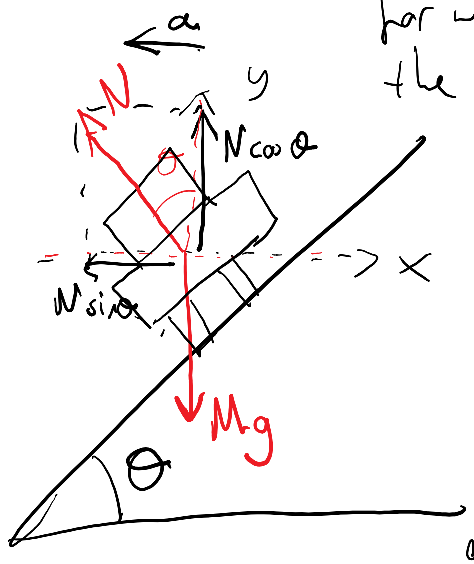
top



side



Find the "safe speed" for which this car goes through the curve without sliding sideways.



y direction no acceln

$$M a_y = 0 = N\cos\theta - Mg$$

$$\boxed{N = \frac{Mg}{\cos\theta}}$$

x direction

$$N\sin\theta = M a$$

$a = \frac{v^2}{R}$

$$\frac{\cancel{M}g}{\cos\theta}\sin\theta = \cancel{M}\frac{v^2}{R}$$

$$v^2 = Rg\tan\theta \Rightarrow \boxed{v = \sqrt{Rg\tan\theta}}$$

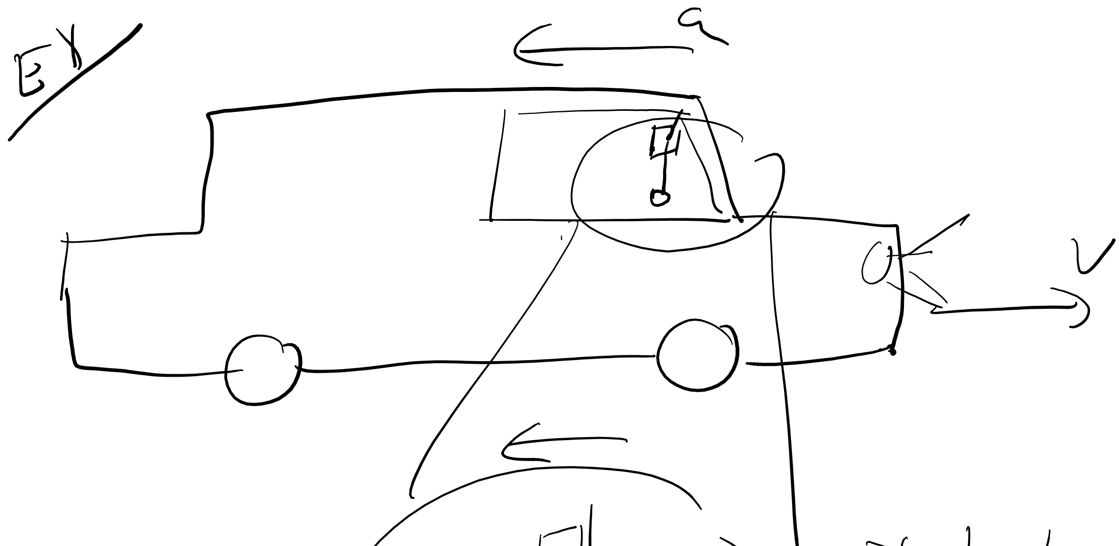
1°) Good?

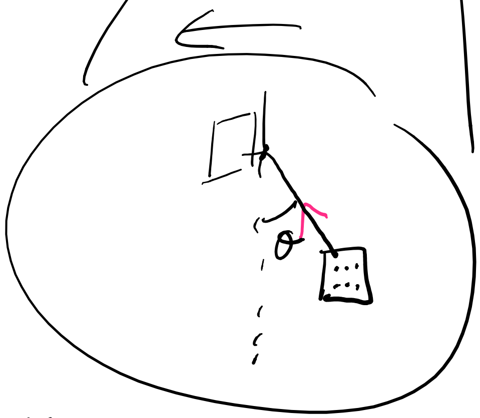
2°) Units $\sqrt{m\, m/s^2} = m/s$ ✓

3°) Lim $R \to \infty$, $v \to \infty$ ✓

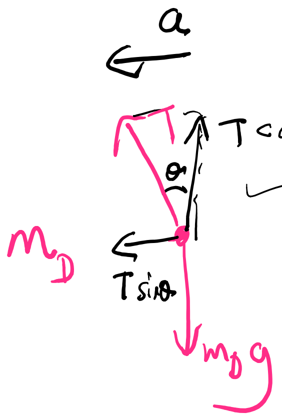
$\theta \to 0, \tan\theta \to 0, v \to 0$

EX





If the dice hangs from your mirror is making an angle θ as shown what is the direction and magnitude of the acceleration of your car?



$T\cos\theta = m_D g$

no acceleration in vertical direction

$$T\sin\theta = m_D a$$

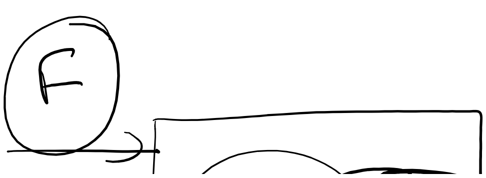
$$\frac{\cancel{m_D} g}{\cos\theta}\sin\theta = \cancel{m_D} a$$

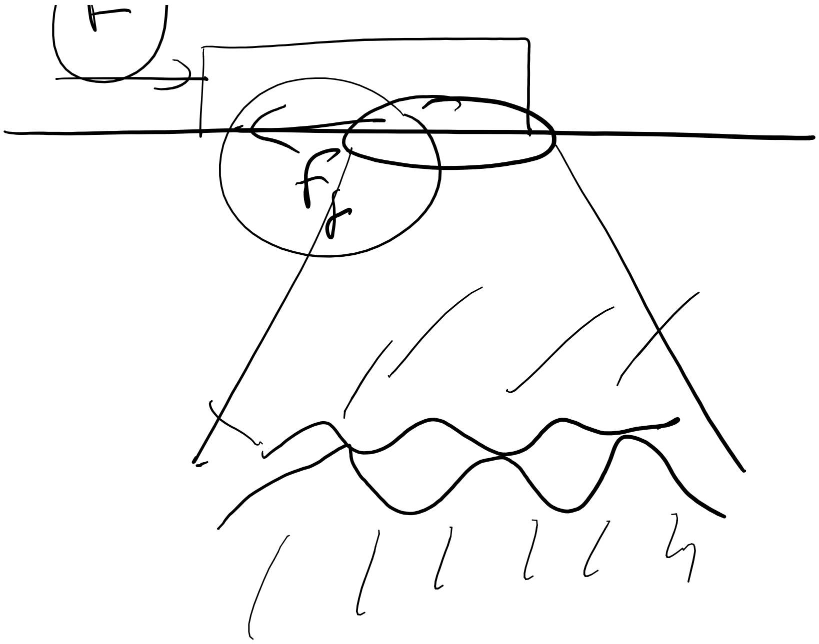
$$\boxed{a = g\tan\theta}$$

to the left

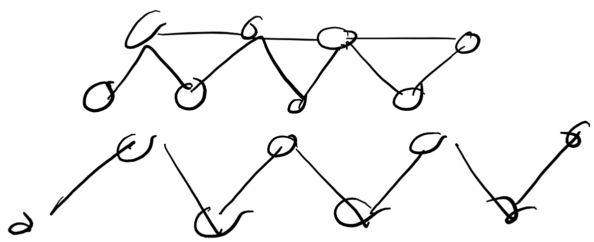
Friction

Even when F is applied m does not move



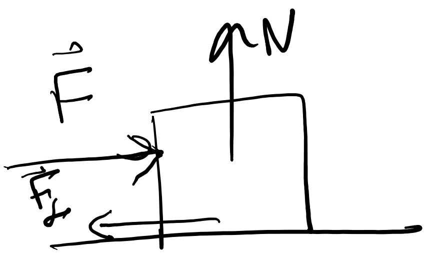


Even small



Simple model for surface friction

1°) Static friction

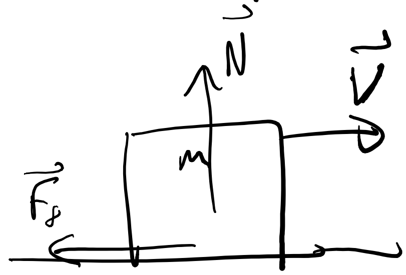


$\vec{P} + \vec{F}_f = 0 \Rightarrow$ no motion

$$|\vec{F}_f| \leq \mu_s |\vec{N}|$$

coefficient of
static friction

2°) Kinetic friction



Friction force is

- directed opposite to the velocity.
- Magnitude

$$|\vec{F}_f| = \mu_k |\vec{N}|$$

coefficient of static friction.

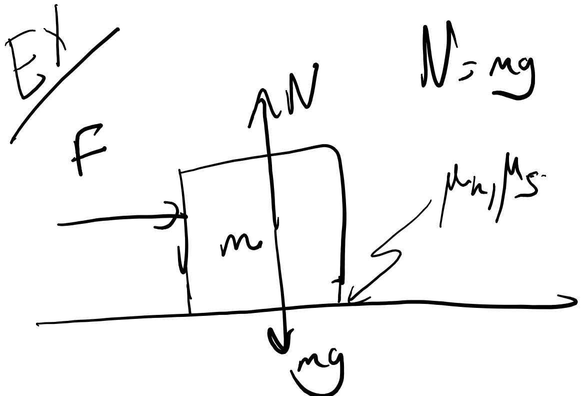
$|f| = \mu_k |N|$

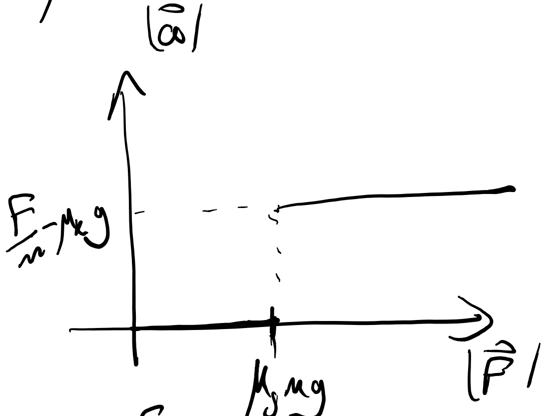
coefficient of kinetic friction.

generally $\mu_k \leq \mu_s \lesssim 1$

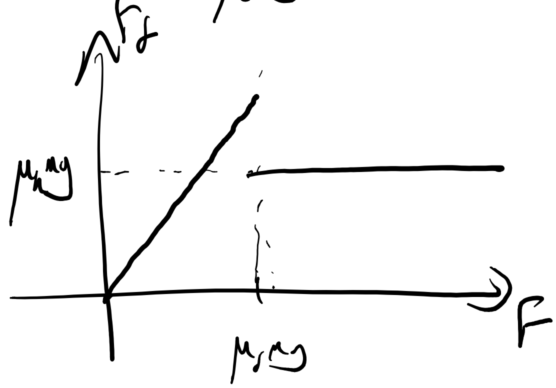
EX

$N = mg$

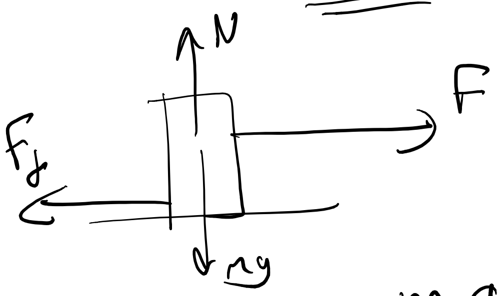




If $F \leq \mu_s mg$ friction will cancel F, there will be no acceleration



If m is moving



$$|\vec{F}_f| = \mu_k mg$$

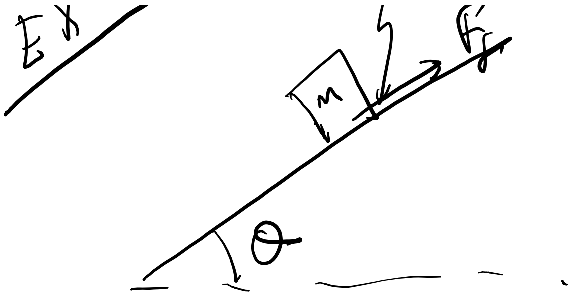
$$ma = |\vec{F}| - |\vec{F}_f|$$

$$a = \frac{F - \mu_k mg}{\cancel{m}}$$

EX

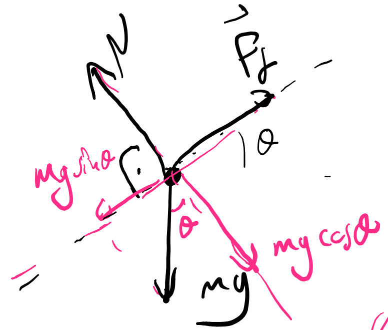
Find the critical angle after which the

Ex



Find the critical angle above which the mass starts to slide.

Free body diagram



No motion
$\vec{a} = 0$

$N = mg\cos\theta$

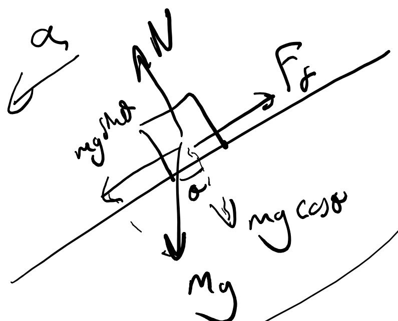
$F_f = mg\sin\theta$

$$|F_f| \le \mu_s N$$

$$\cancel{mg}\sin\theta \le \mu_s \cancel{mg}\cos\theta$$

$$\boxed{\tan\theta \le \mu_s}$$

If we are above that angle what is the acceleration?



$ma = mg\sin\theta - F_f$

$N = mg\cos\theta$

$F_f = \mu_k N$

$F_f = \mu_k mg\cos\theta$

$$\cancel{m}a = \cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta$$

$[\mu_k] \rightarrow$ no units.

$$\cancel{m} a = \cancel{m} g \sin\theta - \mu_k \cancel{m} g \cos\theta$$

$$\boxed{a = g(\sin\theta - \mu_k \cos\theta)}$$

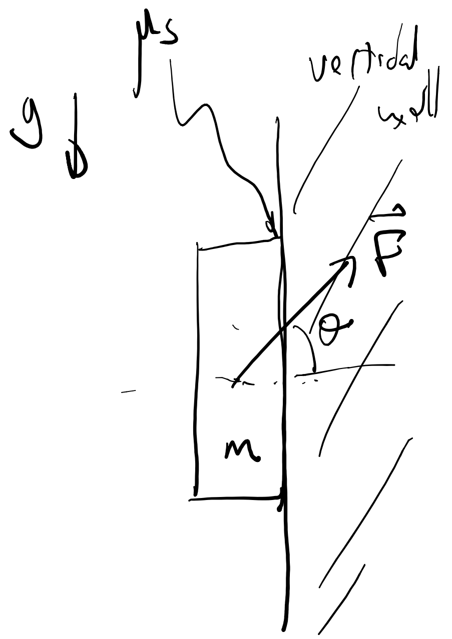
$[\mu_k]$, $[\mu_s]$ → no units.

$[a] = m/s^2 \quad [g] = m/s^2$ ✓

$$\sin\theta - \mu_s \cos\theta > 0$$

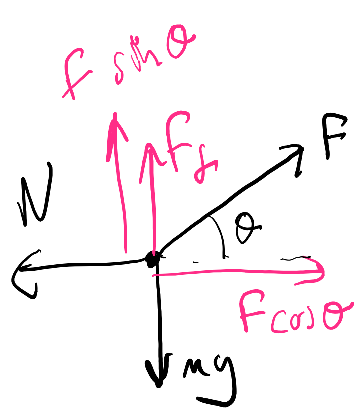
$$\underbrace{\tan\theta > \mu_s}_{\text{sliding condn}} > \mu_k \Rightarrow \boxed{\sin\theta - \mu_k \cos\theta > 0}$$

<u>Ex</u>



- What is the minimum F needed so that the object does not slide down?
- What is the maximum F that can be applied without the object sliding up?

<u>Solution</u>



To find minimum F:

F_f is acting up!

$F_{min}\cos\theta = N$

$F_f + F_{min}\sin\theta = mg$

$$\boxed{F_f = mg - F_{min}\sin\theta}$$

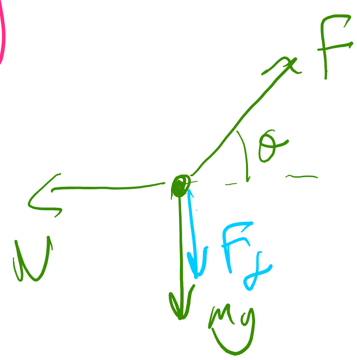
$$|F_f| \leq \mu_s N$$

$$mg - F_{min}\sin\theta \leq \mu_s F_{min}\cos\theta$$

$$mg \leq F_{min}(\sin\theta + \mu_s\cos\theta$$

$$\boxed{F_{min} = \frac{mg}{\sin\theta + \mu_s\cos\theta}}$$

For the max force



$$N = F_{max}\cos\theta$$

$$F_{max}\sin\theta = F_f + mg$$

$$F_f = F_{max}\sin\theta - mg$$

$$|F_f| \leq \mu_s N$$

$$F_{max}\sin\theta - mg \leq \mu_s F_{max}\cos\theta$$

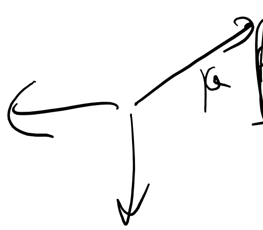
$$F_{max}(\sin\theta - \mu_s\cos\theta) \leq mg$$

$$F_{max} = \frac{mg}{(\sin\theta - \mu_s \cos\theta)}$$

$$\frac{mg}{\sin\theta + \mu_s \cos\theta} \le F \le \frac{mg}{\sin\theta - \mu_s \cos\theta}$$

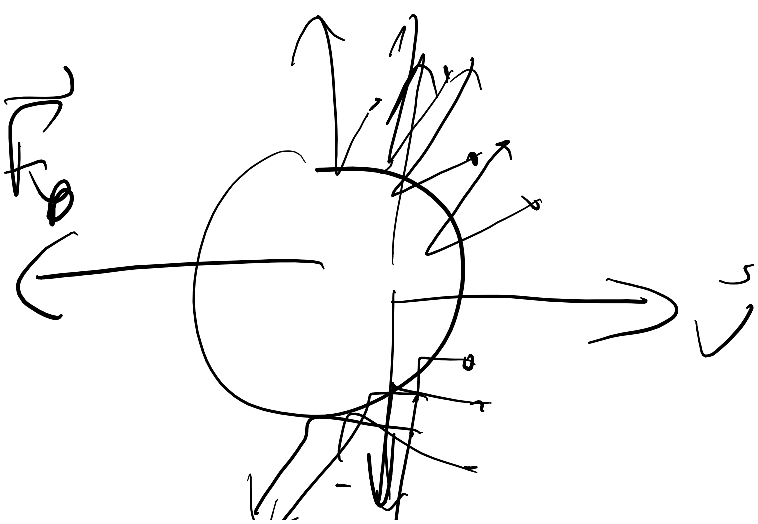
no motion!

If $\mu_s = 0$

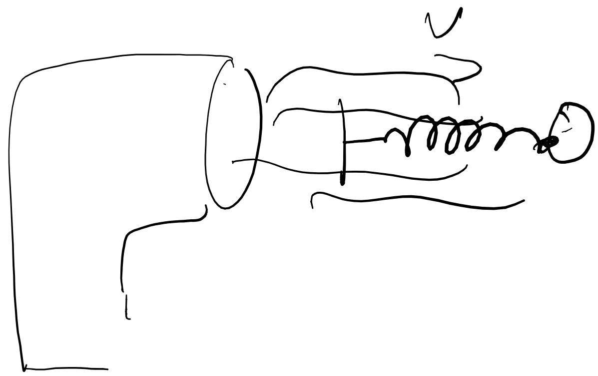


$\boxed{F \sin\theta = mg}$ $\quad F = \frac{mg}{\sin\theta}$

Air (fluid) Friction (Drag force)



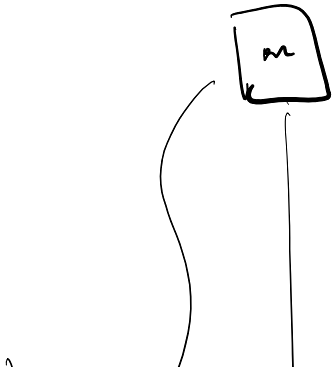
$\vec{F}_D$ is opposite to the velocity $\vec{v}$

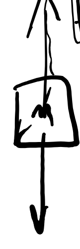


"Low velocity" $\underbrace{\vec{F}_D = -k_D \vec{v}}$

High velocity $|\vec{F}| = \gamma |\vec{v}|^2$

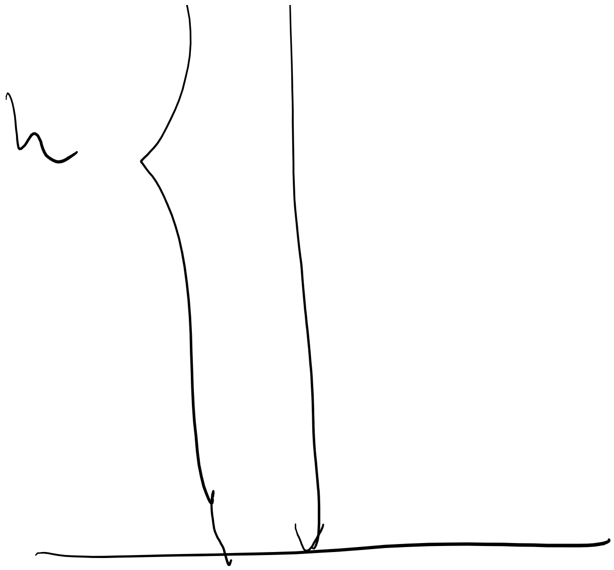
- Find the terminal velocity of an object dropped in a fluid with coefficient k_D.





$|F| = k_D v$ $\qquad k_D v_T = mg$

$\downarrow v \quad \downarrow a \quad \boxed{v_T = \frac{mg}{k_D}}$



mg

$$ma = mg - k_D v$$

$$a = \frac{dv}{dt} \qquad \boxed{v' = v - v_T}$$

$$\frac{dv'}{dt} = \frac{dv}{dt}$$

$$m\left(\frac{dv}{dt} - g\right) = -k_D v$$

$$m\frac{dv'}{dt} = mg - k_D(v' + v_T)$$

$$= \cancel{mg} - k_D v' - \cancel{k_D \frac{mg}{k_D}}$$

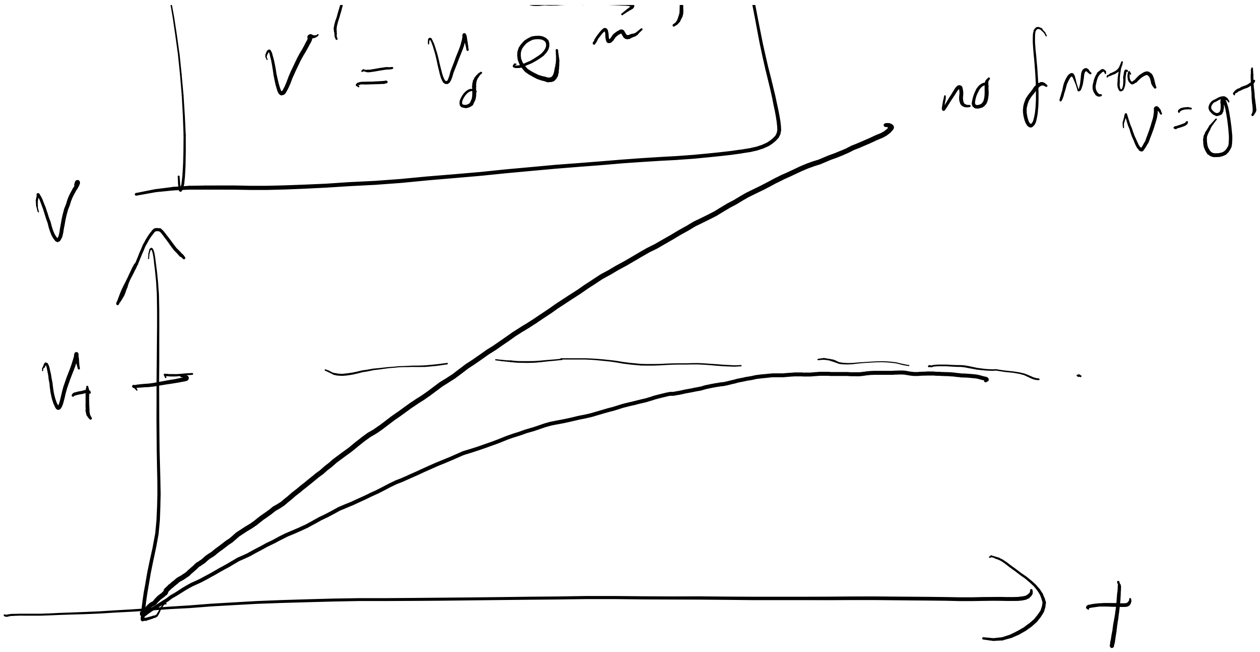
$$\boxed{\frac{dv'}{dt} = -\frac{k_D}{m} v'}$$

$$v' = v_0 e^{At}$$

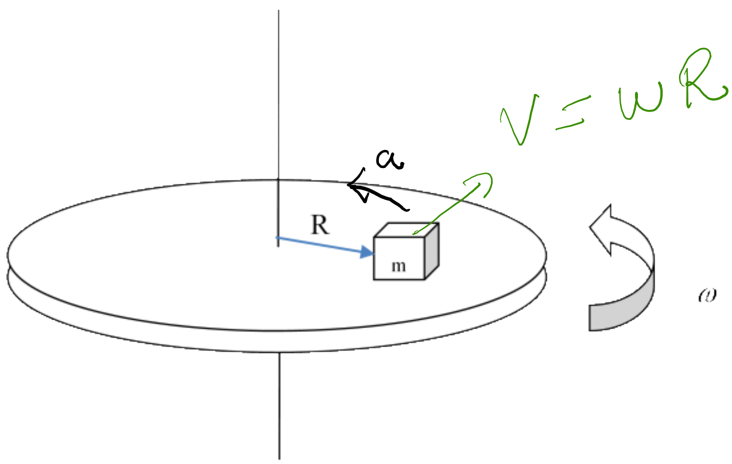
$$\frac{dv'}{dt} = A v_0 e^{At}$$

$$A \cancel{v_0 e^{At}} = -\frac{k_D}{m} \cancel{v_0 e^{At}}$$

$$\boxed{v' = v_0 e^{-\frac{k_D}{m} t}}$$

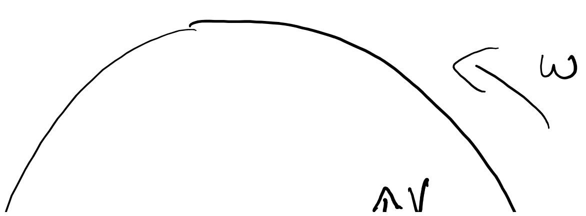


A horizontal disk is rotating around its center with constant angular velocity ω. A mass m is sitting on the disk, and rotating with it. The mass is situated a distance R away from the center of the disk. If the static friction coefficient between the disk and the mass is μ_s, find the maximum possible angular velocity for the mass to be stationary on the disk. (Gravitational acceleration is g)



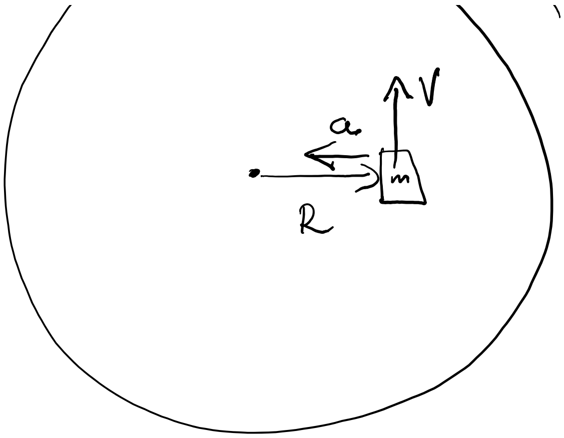
$$|\vec{a}| = \frac{v^2}{R} = \omega^2 R$$

top view



$$V = R\omega$$

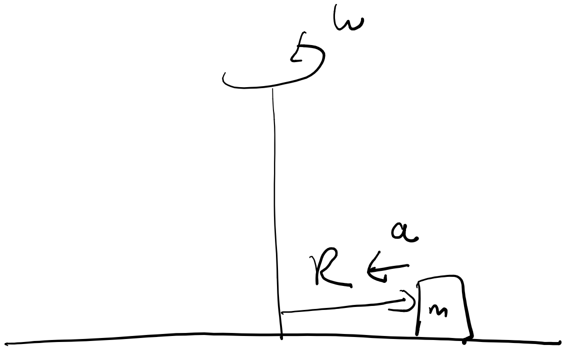
$$a = \frac{v^2}{R} = \omega^2 R$$

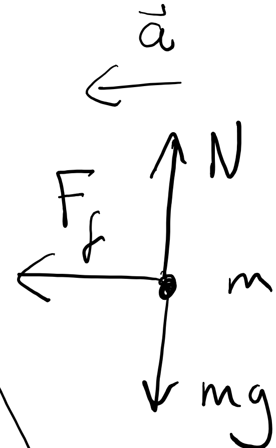


$$a = \frac{v^2}{R} = \omega^2 R$$

$$\vec{F} = m\vec{a}$$

Side view





$$\vec{F} = m\vec{a}$$

$$N = mg$$

$$ma = F_f$$

$$|F_f| \le \mu_s N$$

$$\cancel{m} a \le \mu_s \cancel{m} g$$

$$\omega^2 R \le \mu_s g$$

$$\boxed{\omega \le \sqrt{\mu_s \frac{g}{R}}}$$

$$R\omega = v \qquad \omega_{max} = \sqrt{\mu_s \frac{g}{R}}$$

$$[\omega] = \frac{[v]}{[R]}$$

m/s

1°) Goal?

2°) $\sqrt{[\mu_s]\frac{[g]}{[R]}} = \sqrt{1 \cdot \frac{m/s^2}{m}} = \frac{1}{s}$ ✓

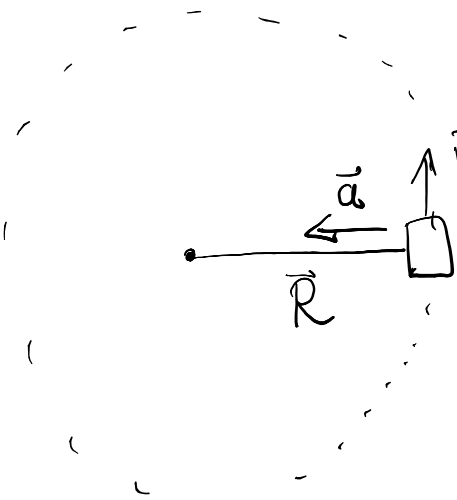
$= \frac{\quad}{m} = 1/s$

3°) Limits $\omega \to 0$ ✓

$\mu_s \to 0$

$R \to \infty \quad \omega \to 0$ ✓

Rotational motion with constant speed



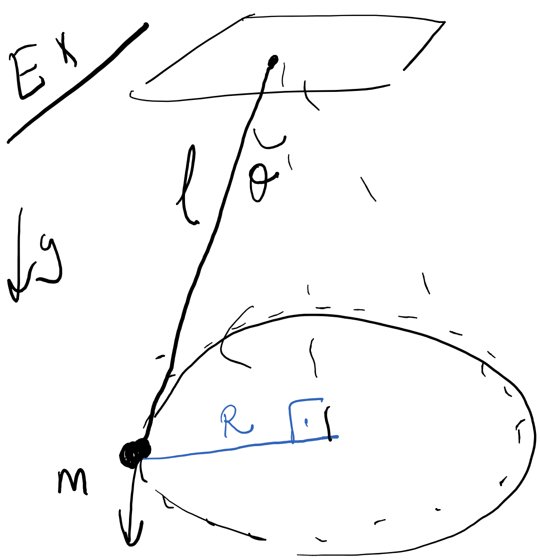
Angular velocity

$$\omega = \frac{v}{R}$$

Period $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$

$|\vec{a}| = \frac{v^2}{R}$, directed towards the center

Ex



Conical pendulum

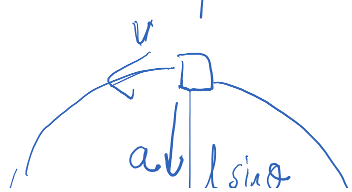
$T = ?$ period?

$F_T = ?$ tension in the rope.

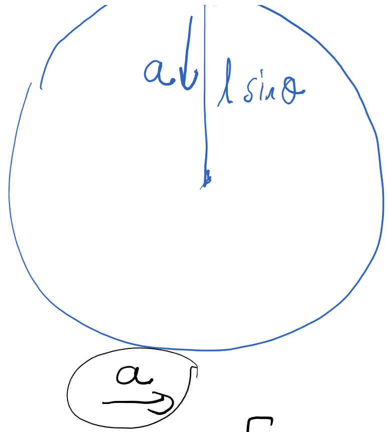
$R = l \sin\theta$

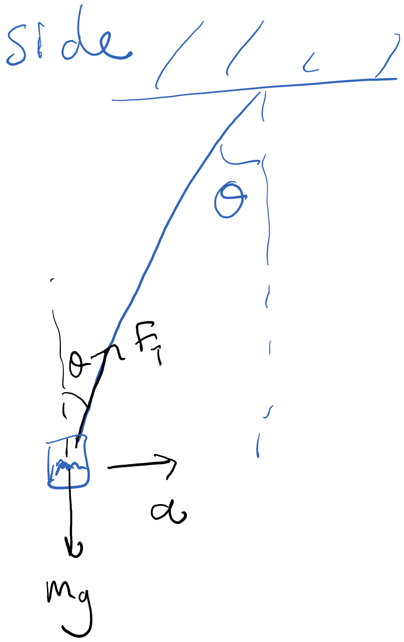
$v^2 \qquad v^2$

top view

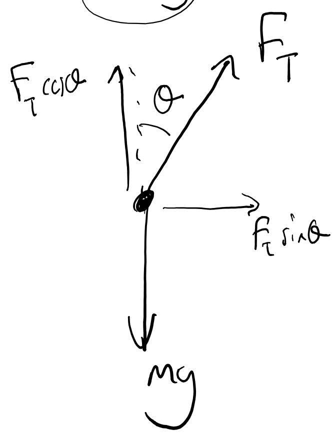


$$a = \frac{v^2}{R} = \frac{v^2}{l\sin\theta}$$





$$a = \frac{v^2}{l\sin\theta}$$



$$F_T\cos\theta = mg \quad (1)$$

$$F_T\sin\theta = ma \quad (2)$$

$$F_T\cos\theta = mg \qquad (1)$$

$$F_T\sin\theta = m\frac{v^2}{l\sin\theta} \qquad (2)$$

$$\frac{(1)}{(2)} \quad \frac{\cancel{F_T}\cos\theta}{\cancel{F_T}\sin\theta} = \frac{\cancel{m}g}{\cancel{m}\frac{v^2}{l\sin\theta}} \Rightarrow \frac{\cos\theta}{\sin\theta} = \frac{l\sin\theta\, g}{v^2}$$

$$\boxed{V = \sqrt{\frac{l\sin^2\theta\, g}{\cos\theta}}}$$

$$T = \frac{2\pi R}{V} = \frac{2\pi l \cancel{\sin\theta}}{\sqrt{\frac{l\cancel{\sin^2\theta}\, g}{\cos\theta}}} \boxed{= 2\pi\sqrt{\frac{l\cos\theta}{g}}}$$

$$[T] = s = \sqrt{\frac{\cancel{m}}{\cancel{m}/s^2}} = s$$

(1 $F\cos\theta = mg$

P) Good

① $F_T \cos\theta = mg$

$$\boxed{F_T = \frac{mg}{\cos\theta}}$$

1°) Goal ✓

2°) Units ✓ V m/s^2

3°) limits $l \to \infty$ ✓

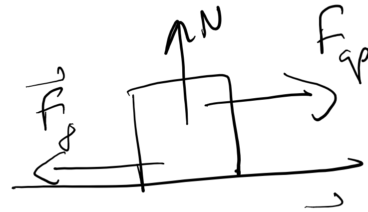
$T \to \infty$ ✓

$m \to \infty$ $F_T \to \infty$ ✓

$\theta = 0$ $F_T = mg$ ✓

Friction

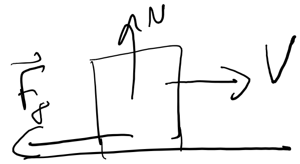
Static



$\vec F_f$ counters $\vec F_{app}$

$|\vec F_f| \le \mu_s N$

Kinetic



$\vec F_f$ is opposite to $\vec v$

$|\vec F_f| = \mu_k N$

$$\mu_k \le \mu_s < 1$$

Fluid friction

$\vec F_{fluid}$ is opposite to $\vec v$

$$\vec F_f = -k_D \vec v \quad \text{or} \quad |\vec F_f| = -C_D v^2$$

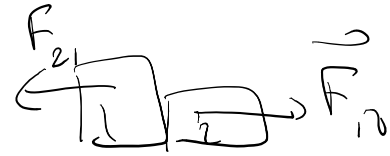
A terminal velocity

Newton's

1st Law $\vec{a} = 0 \Rightarrow \vec{F} = 0$

2nd Law $\vec{F} = m\vec{a}$

3rd Law



• point particles $\Rightarrow$ draw free-body diagrams

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

$\frac{d}{dt}$ $\quad\quad$ $\int dt$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$\frac{d}{dt}$ $\quad\quad$ $\int dt$

$$\vec{a} = \frac{d\vec{v}}{dt} \ldots\ldots$$

• motion with constant velocity $\quad x(t) = vt + x_0$

$a(t) = 0$

• motion with constant acceleration $\quad a(t) = a$

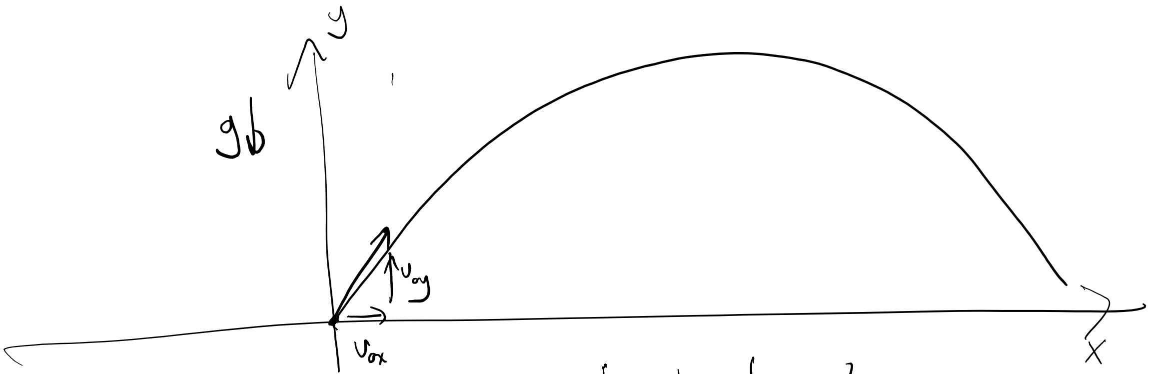
$\int dt$

$v(t) = at + v_0$

$\int dt$

$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$

projectile motion



$$y(t) = v_{0y}\, t - \frac{1}{2} g t^2$$

$$x(t) = v_{0x}\, t$$

Units, Significant figures, $\boxed{\text{Prefixes}}$