Week 03 (Oct 10 - Oct 14)

Friday, September 23, 2016 2:55 PM

$$\vec r(t) = x(t)\,\hat\imath + y(t)\,\hat\jmath + z(t)\,\hat k$$

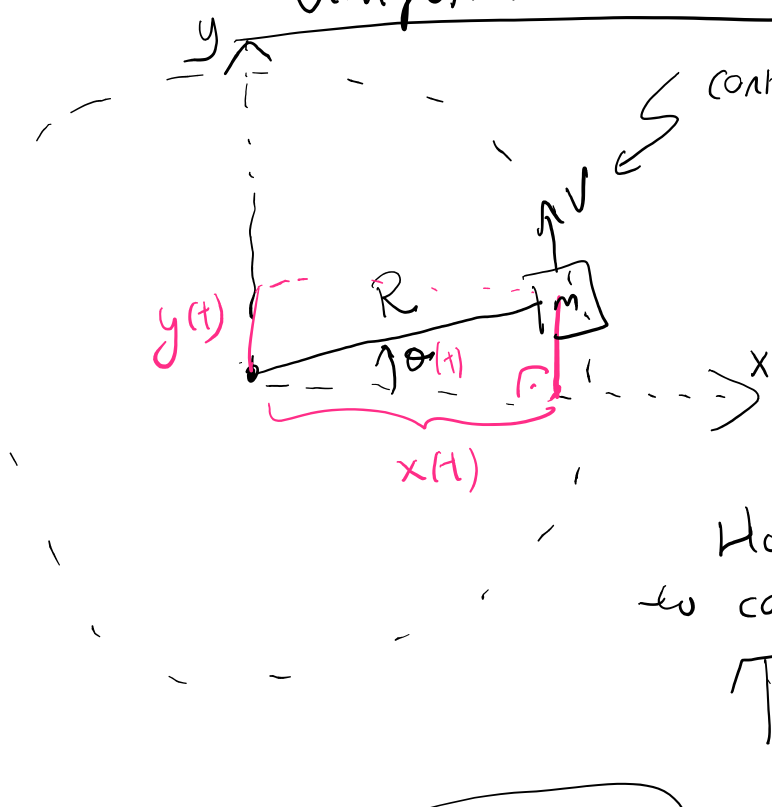
$$\vec V(t) = \frac{d\vec r}{dt} = \frac{dx}{dt}\,\hat\imath + \frac{dy}{dt}\,\hat\jmath + \frac{dz}{dt}\,\hat k$$

$$\vec a = \frac{d\vec v}{dt} = \frac{d^2}{dt^2}\vec r.$$

* projectile motion → y(t) motion is motion w/ constant acceleration

→ x(t) motion is motion w/ constant velocity.

Uniform circular motion



constant speed.

$\vec r(t) = ?$

$\vec v(t) = ?$

$\vec a(t) = ?$

$\theta(t) = ?$

How long would the particle take to complete 1 revolution?

$$T = \frac{2\pi R}{V} \leftarrow \text{period}$$

$$\overline{V}$$

$$\boxed{\theta = 2\pi \frac{t}{T}}$$

$$\theta(t) = \cancel{2\pi} \frac{t}{\frac{\cancel{2\pi} R}{V}} = \frac{V}{R} t$$

angular velocity

$$\omega = \frac{d\theta}{dt} = \frac{V}{R} \frac{d}{dt} t = \frac{V}{R}$$

Figure

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} = \boxed{R\cos(\theta(t))\,\hat{i} + R\sin[\theta(t)]\,\hat{j}}$$

$$\vec{V}(t) = \frac{d}{dt}\vec{r}(t) = \frac{d}{dt}\left(R\cos(\theta(t))\right)\hat{i} + \frac{d}{dt}\left(R\sin[\theta(t)]\right)\hat{j}$$

$$= R\left(-\sin\theta(t)\right)\underbrace{\frac{d\theta}{dt}}_{\omega}\,\hat{i} + R\cos(\theta(t))\underbrace{\frac{d\theta}{dt}}_{\omega}\,\hat{j}$$

$$\vec{V}(t) = -\omega R\sin\theta\,\hat{i} + \omega R\cos\theta\,\hat{j}$$

$$|\vec{V}(t)| = \sqrt{(-\omega R\sin\theta)^2 + (\omega R\cos\theta)^2} = \omega R\underbrace{\sqrt{\sin^2\theta + \cos^2\theta}}_{1}$$

$$|\vec{V}| = \omega R = \frac{V}{\cancel{R}}\cancel{R} = V$$

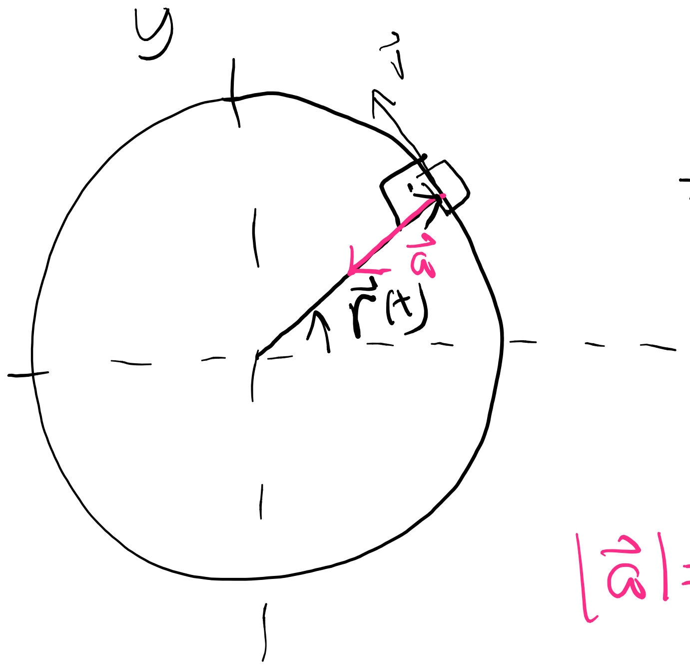
$$\vec{a}(t) = \frac{d\vec{v}}{dt} = -\omega R\frac{d}{dt}\left(\sin(\theta(t))\right)\hat{i} + \omega R\frac{d}{dt}\left(\cos(\theta(t))\right)\hat{j}$$

$$= -\omega R\cos\theta\,\omega\,\hat{i} + \omega R(-\sin\theta)\,\omega\,\hat{j}$$

$$\vec{a}(t) = -\omega^2\underbrace{R\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})}$$

$$\vec{a}(t) = -\omega^2\vec{r} \qquad \vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$\vec{a}(t) = -\omega^2 \vec{r}$



$\vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$

$\vec{v} = -\omega R\sin\theta\,\hat{i} + \omega R\cos\theta\,\hat{j}$

$\vec{v}\cdot\vec{r} = -\cancel{\omega R^2\cos\theta\sin\theta} + \cancel{\omega R^2\cos\theta\sin\theta}$

$= 0$

$\omega = \frac{V}{R}$

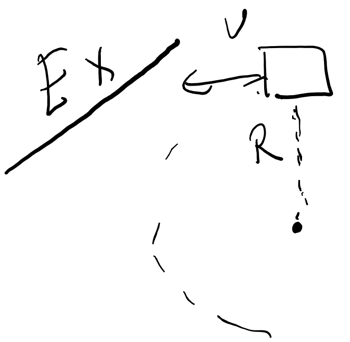
$\vec{a} = -\omega^2\vec{r}$

$$|\vec{a}| = \omega^2|\vec{r}| = \omega^2 R = \frac{V^2}{R^2}R = \boxed{\frac{V^2}{R}}$$

Uniform circular motion:

For radius R and speed V
there is an acceleration $|\vec{a}| = \frac{V^2}{R}$
$\underbrace{\text{directed towards center.}}_{\text{centripetal.}}$

Ex



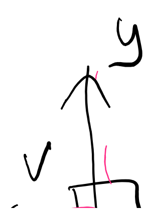
An object on a horizontal table is tied with a string of length R to a nail. After half a revolution the string breaks. Plot the trajectory of the particle!

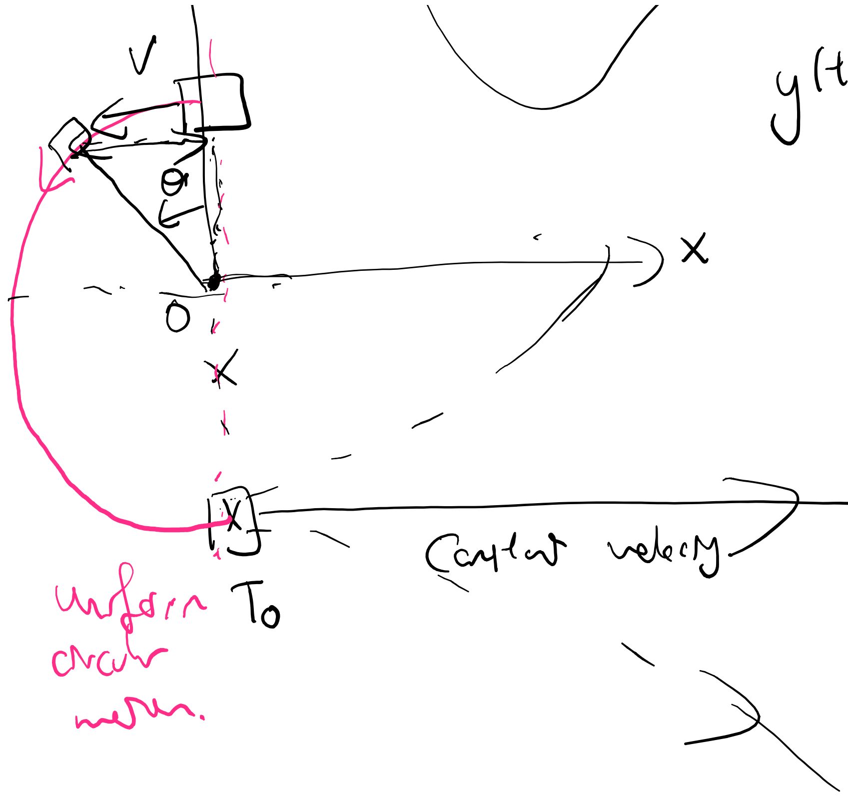
$$\boxed{T_0 = \frac{\pi R}{V}}$$

$\theta(t) = \frac{V}{R}t$

$x(t) = ?$

$y(t) = ?$





$y(t) = ?$

$t < T_0$

$y(t) = R\cos(\theta(t))$

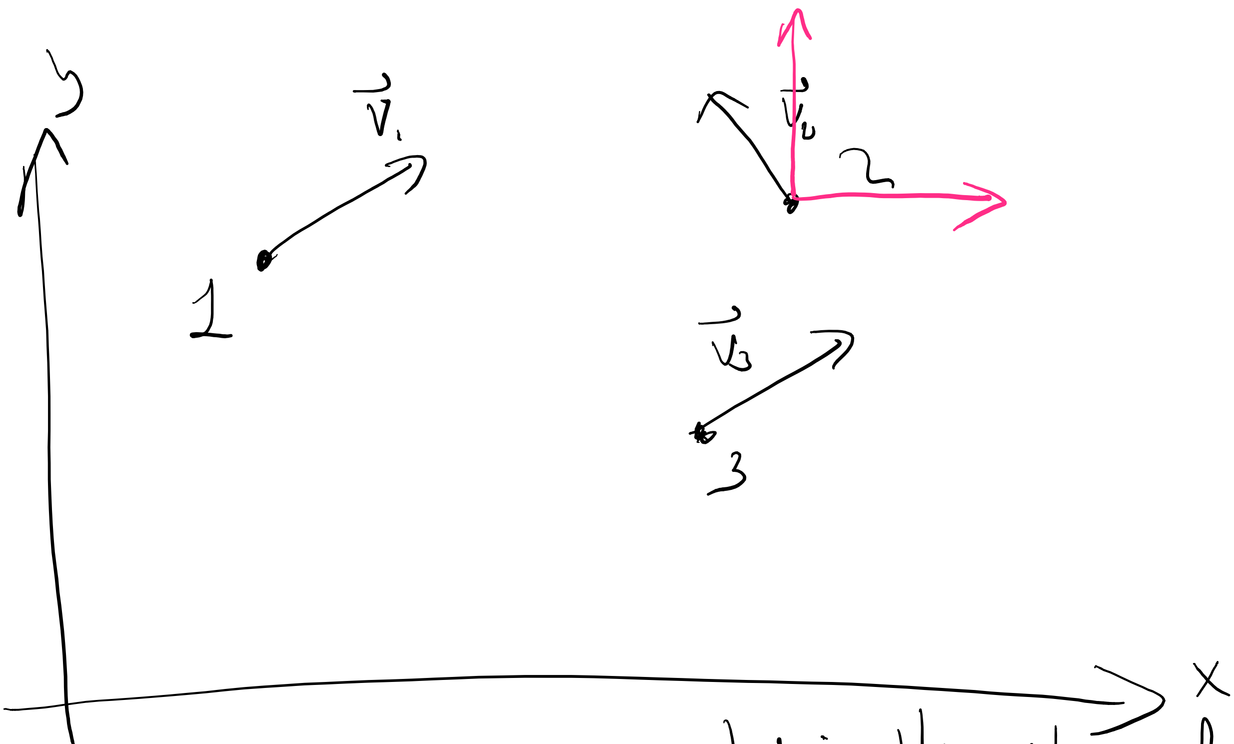
$x(t) = -R\sin(\theta(t))$

$t > T_0$

$y(t) = -R$

$x(t) = V(t - T_0)$

Relative velocity



Fixed "lab" frame

What is the velocity of the 1st object <u>with respect to</u> 2nd object?

$$\boxed{\vec{V}_{1\,rel} = \vec{V}_1 - \vec{V}_2}$$

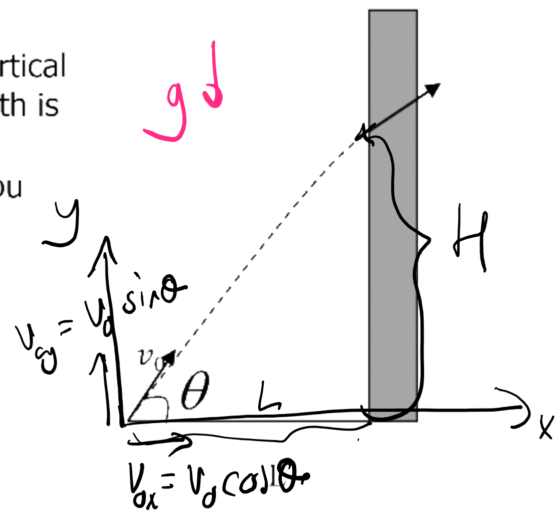
velocity of the n^{th} object with respect to m^{th} object

$$\boxed{\vec{V}_{n,m} = \vec{V}_n - \vec{V}_m}$$

QUIZ-5

You are standing a distance $L = 10\,m$ away from a vertical wall. If the maximum speed you can throw a stone with is $v_0 = 10\sqrt{3}\,m/s$,

a) What is the height H of the highest point on you can hit on the wall? (Assume gravitational acceleration $g = 10\,m/s^2$.)
b) What is the angle of the throw θ that gives the maximum height?



$$x(t) = V_0 \cos\theta \, t$$

$$y(t) = V_0 \sin\theta \, t - \frac{1}{2} g t^2$$

time of flight

$$\left.\begin{aligned} x(t_f) &= L \\ V_0 \cos\theta \, t_f &= L \end{aligned}\right\} \quad t_f = \frac{L}{V_0 \cos\theta}$$

$$H = y(t_f) = v_0 \sin\theta \, t_f - \frac{1}{2} g t_f^2$$

$$= \cancel{v_0} \sin\theta \frac{L}{\cancel{v_0}\cos\theta} - \frac{1}{2} g \frac{L^2}{v_0^2 \cos^2\theta}$$

$$H(\theta) = \tan\theta \, L - \frac{1}{2} g \frac{L^2}{v_0^2} \cos^{-2}\theta$$

$$\text{Max} \Rightarrow \boxed{\frac{dH}{d\theta} = 0}$$

$$\frac{d}{d\theta}(\tan\theta) = \frac{d}{d\theta}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{\cos\theta\cos\theta - (-\sin\theta)\sin\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta}$$

$$\frac{d}{d\theta}\left[L \tan\theta - \frac{1}{2} g \frac{L^2}{v_0^2} \cos^{-2}\theta\right]$$

$$= L \frac{1}{\cos^2\theta} - \frac{1}{\cancel{2}} g \frac{L^2}{v_0^2} (-\cancel{2}) \cos^{-3}\theta \frac{d(\cos\theta)}{d\theta}$$

$$\frac{d}{dt} t^n = n t^{n-1}$$

$$= L \frac{1}{\cos^2\theta} + g \frac{L^2}{v_0^2} \frac{(-\sin\theta)}{\cos^3\theta} = 0$$

$$\cancel{L} \frac{1}{\cancel{\cos^2\theta}} = g \frac{L^{\cancel{2}}}{v_0^2} \frac{\sin\theta}{\cancel{\cos^3\theta}}$$

$\cos\theta$

$v_0 = 10\sqrt{3}$ m/s

$g = 10$ m/s² $L = 10$ m

$$\boxed{\tan\theta = \frac{v_0^2}{gL}}$$

$$\theta = \tan^{-1} \frac{v_0^2}{gL}$$

$$\theta = \tan^{-1} \frac{3 \cancel{100}}{\cancel{10}\,\cancel{10}}$$

$(m/s)^2$

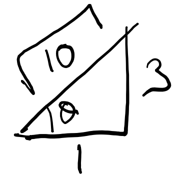
$\theta = \tan^{-1} 3$

$$\frac{\cancel{\left(\frac{m}{s}\right)^2}}{\cancel{m/s^2\, m}} = 1$$

$$\boxed{\theta = \tan^{-1} 3}$$

$$H_{max} = ? \qquad H(\theta) = (\tan\theta) L - \frac{1}{2} g \frac{L^2}{v_0^2} \frac{1}{\cos^2\theta}$$

$\tan\theta = 3$



$\Rightarrow \cos\theta = \frac{1}{\sqrt{10}}$

$$H_{max} = 3\; 10 - \frac{1}{2}\, 10 \frac{10\cancel{0}}{30\cancel{0}} \frac{1}{\frac{1}{10}}$$

$$= 30 - \frac{50}{3} = \boxed{\frac{40}{3}\ m}$$

Newton's Laws of Motion

1st Law

A body at rest stays at rest, and a body in motion keeps moving with constant velocity unless a net Force acts on them.

if $F_{net} = 0$, $\vec{a} = 0$

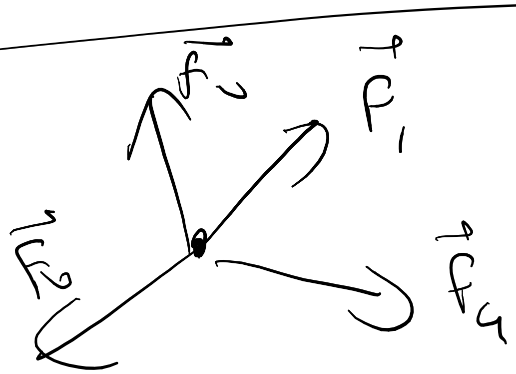
2^{nd} Law

The acceleration of a particle is proportional to the net force acting on it and inversely proportional to its mass.

$$\boxed{\vec{F}_{net} = m\vec{a}}$$

3^{rd} Law

If an object is acting a force $\vec{F}$ on another object, the acted upon object applies an equal but opposite force $-\vec{F}$ to the first object.



$$\vec{F}_{net} = \overbrace{\underbrace{\vec{F}_1 + \vec{F}_2 + \dots}_{\text{all forces acting upon the object!}}}^{\text{vector sum}}$$

All forces arise from

Weak

Strong

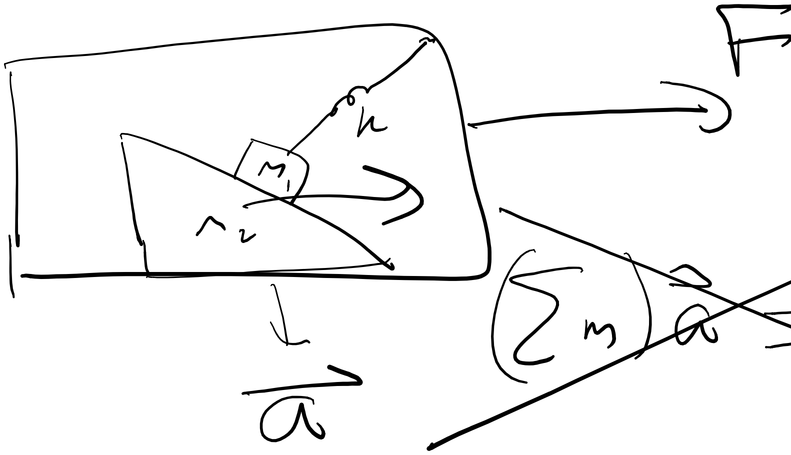
Gravity

Electromagnetism

Weak Nuclear

Strong Nuclear force

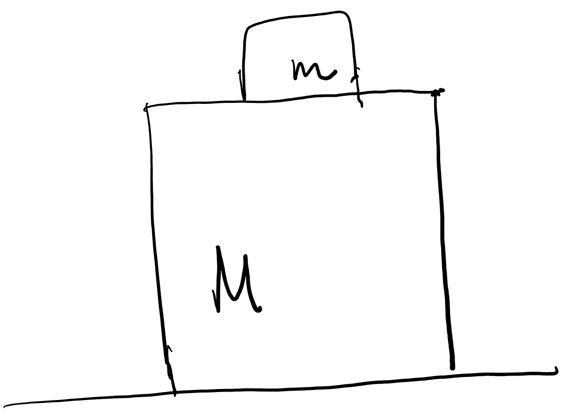
Newton's laws apply for $\boxed{\text{point particles}}$



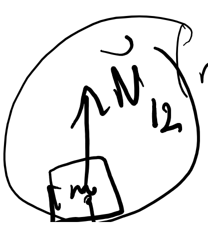
$$\cancel{\left(\sum m\right)\vec{a} = \vec{F}}$$

Examples

$\downarrow g$

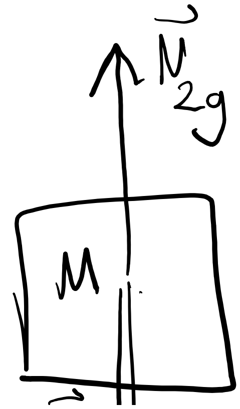


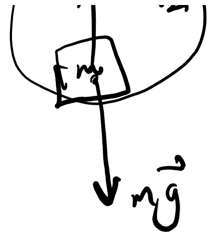
Show all the forces acting on m and M.



(normal force of M acting on m)

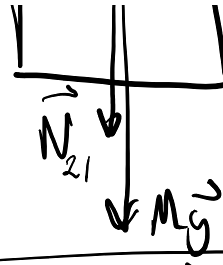
$\vec{N}$ … $= 0$





$$\boxed{\vec{N}_{12} + m\vec{g} = 0}$$

$$\boxed{\vec{N}_{12} = -\vec{N}_{21}}$$



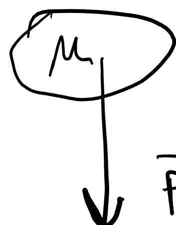
$$\boxed{M\vec{g} + \vec{N}_{21} + \vec{N}_{2g} = 0}$$

Force ⇒ mass

$$[\vec{F}] = [m][\vec{a}]$$

$$N = [kg\ m/s^2]$$

newton.



$$\vec{F} = m\vec{g}$$

weight

$$[mass] = kg$$

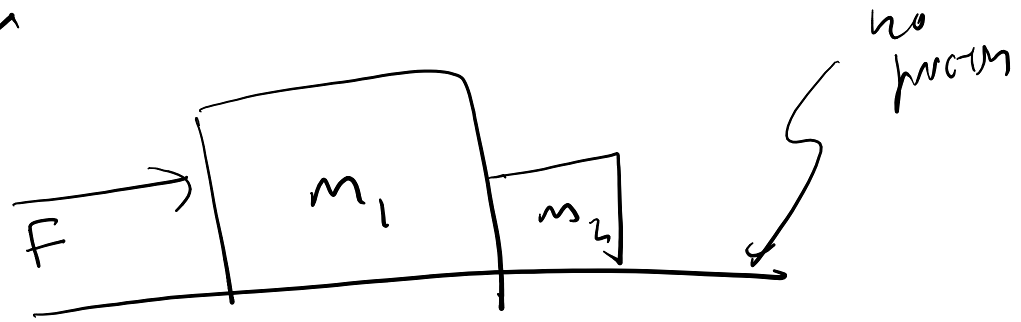
balance ⇒ measures the Normal force

it's acting on to whatever is on it

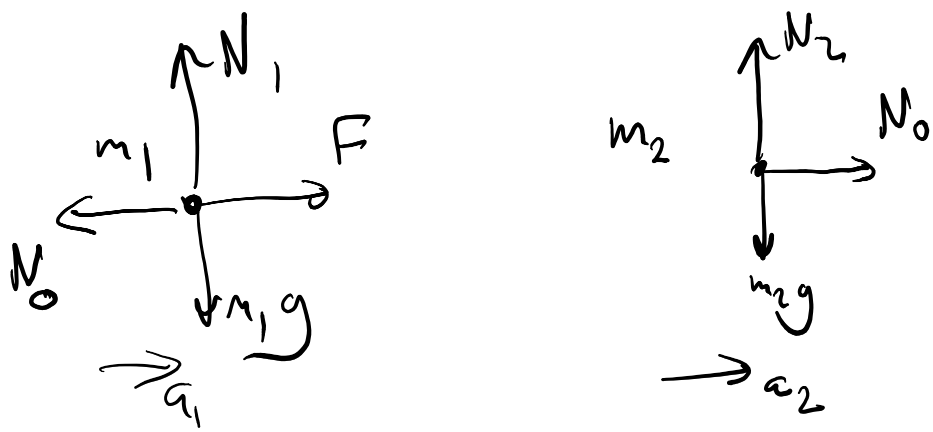
$$N = M_2 g + F_{21}$$

$$\boxed{N = M_2 g + m_1 g}$$

Ex



• Find the accelerations and the force m_1 acts on m_2.



$$m_1 a_1 = F - N_0 \qquad m_2 a_2 = N_0$$

$$a = a_1 = a_2$$

$$+ \quad m_1 a = F - N_0 \qquad m_2 a = N_0$$

$$(m_1 + m_2)\, a = F - \cancel{N_0} + \cancel{N_0}$$

$$\boxed{a = \frac{F}{m_1+m_2}} \qquad \boxed{N_2 = m_2\frac{F}{m_1+m_2}}$$

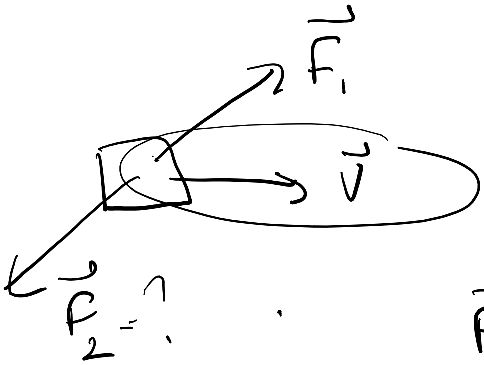
PHYSICS 101- Instructor: M. Özgür OKTEL- 2016

QUIZ-6

While two forces are acting on it, a particle is to move at the constant velocity **v** ~~= (3 m/s) **i** − (4 m/s) **j**~~. One of the forces is

$$\mathbf{F_1} = (2\text{ N})\,\mathbf{i} + (-6\text{ N})\,\mathbf{j}.$$

What is the other force?



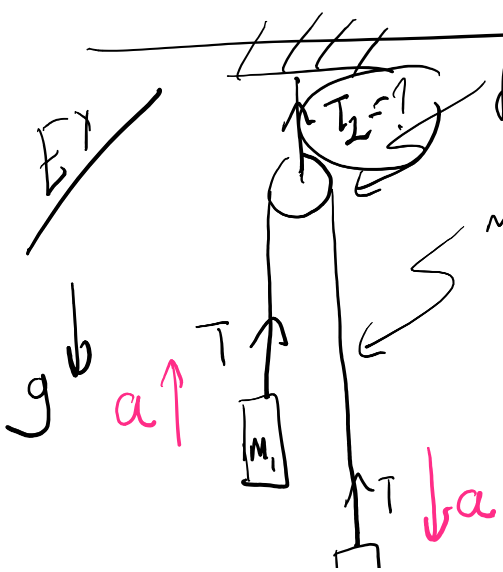
$$\sum \vec{F} = \vec{F}_{net} = 0$$

Newton's first Law.

$$\vec{F}_1 + \vec{F}_2 = 0$$

$$\boxed{\vec{F}_2 = -\vec{F}_1 = -2\text{ N}\,\hat{i} + 6\text{ N}\,\hat{j}}$$

Ex



frictionless massless pulley

massless, inextensible string

what are the acceleration of the masses if the system is let go

$m_1 = 2.0\,kg$

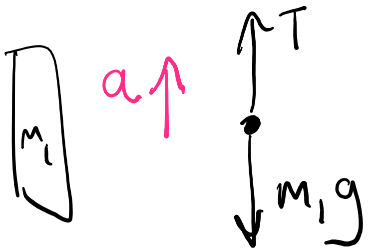
$m_2 = 1.0\,kg$

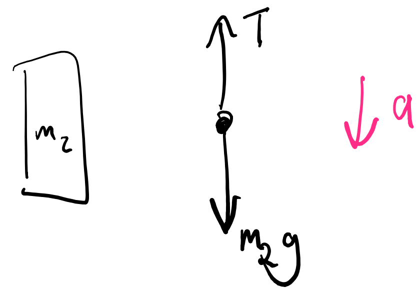
if the system is let go from the position shown in the figure.

$a = ?$

what is the tension in the string tying the pulley to the ceiling

Free body diagrams





① $\boxed{m_1 a = T - m_1 g}$

② $m_2 a = m_2 g - T$

①+② $m_1 a + m_2 a = \cancel{T} - m_1 g + m_2 g - \cancel{T}$

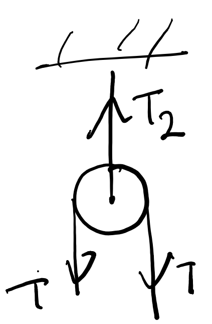
$(m_1 + m_2)\, a = (m_2 - m_1)\, g$

$g = 10\ m/s^2$

$$\boxed{a = \frac{m_2 - m_1}{m_2 + m_1}\, g}$$

$$a = \frac{1.0 - 2.0}{3.0}\, 10$$

$$\boxed{a = -3.3\ m/s^2}$$



$a = 0$

$ma = 0 = T_2 - 2T$

$\Rightarrow T_2 = 2T$

① - ② $\Rightarrow$ $(m_1-m_2)a = T - m_1 g - m_2 g + T$

$$2T = (m_1-m_2)a + (m_1+m_2)g$$

$$T_2 = 2T = (m_1-m_2)\frac{(m_2-m_1)}{(m_1+m_2)}g + (m_1+m_2)g$$

$$= \left[-\frac{(m_2-m_1)^2}{(m_1+m_2)} + (m_1+m_2)\right]g$$

$$= \frac{(m_1+m_2)^2-(m_2-m_1)^2}{m_1+m_2}g$$

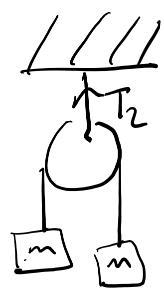
$$= \frac{\cancel{m_1^2}+2m_1m_2+\cancel{m_2^2}-\cancel{m_2^2}+2m_1m_2-\cancel{m_1^2}}{m_1+m_2}g$$

$$\boxed{a = \frac{m_2-m_1}{m_1+m_2}g} \qquad \boxed{T_2 = 4\frac{m_1m_2}{m_1+m_2}g}$$

1°) Sad?
2°) Units ✓
3°) Limits

$$[T_2] = \frac{kg^{\cancel{2}}}{\cancel{kg}} m/s^2 = N \checkmark$$

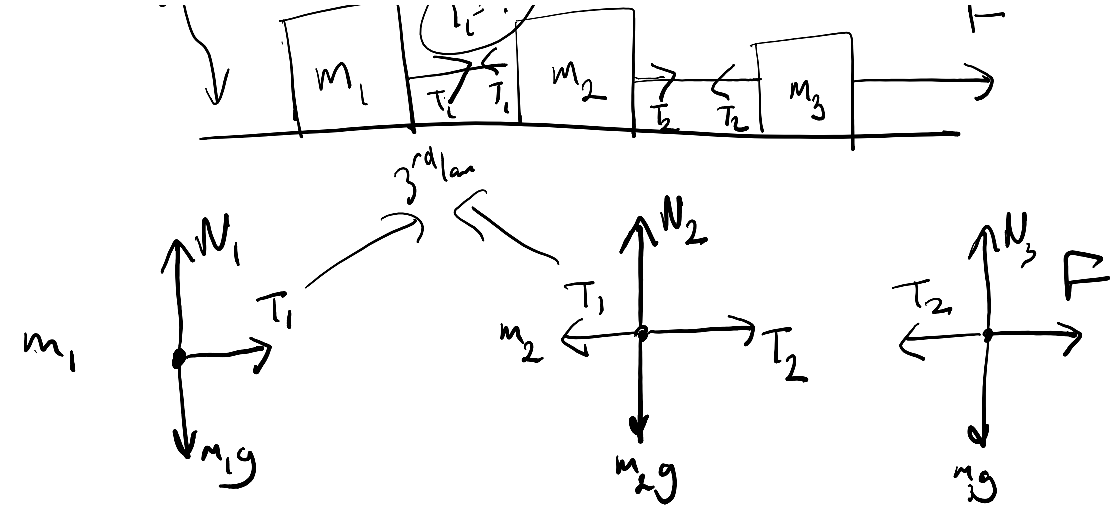
What happens if $m_1 = m_2$



$a = 0$ ✓

$$T_2 = 4\frac{m^{\cancel{2}}}{2\cancel{m}}g = 2mg \checkmark$$

Ex



$\longrightarrow a$ $\qquad$ $\longrightarrow a$ $\qquad$ $\longrightarrow a$

(1) $m_1 a = T_1$ $\qquad$ (2) $m_2 a = T_2 - T_1$ $\qquad$ (3) $m_3 a = F - T_2$

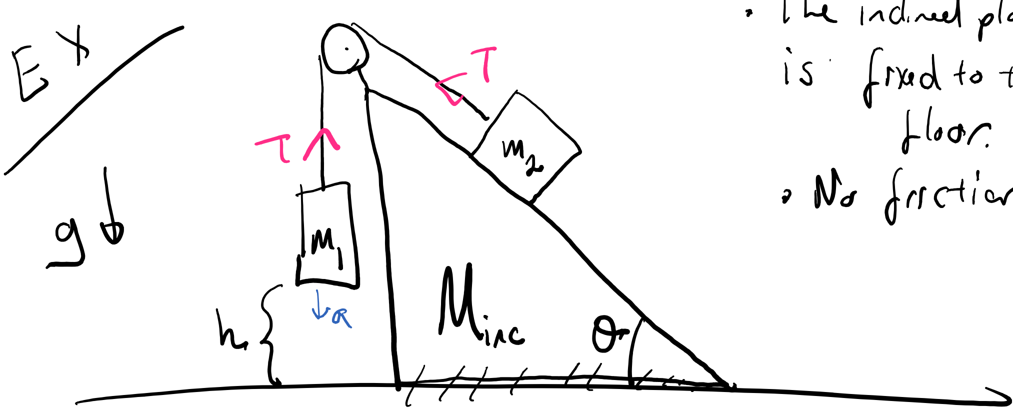
(1) + (2) + (3) $= (m_1 + m_2 + m_3)a = \cancel{T_1} + \cancel{T_2} - \cancel{T_1} + F - \cancel{T_2}$

$$a = \frac{F}{m_1 + m_2 + m_3}$$

$$\boxed{T_1 = m_1 a = \frac{m_1 F}{m_1 + m_2 + m_3}}$$

$m_2 = 0$
$m_3 = 0$
$T \to F$ ✓

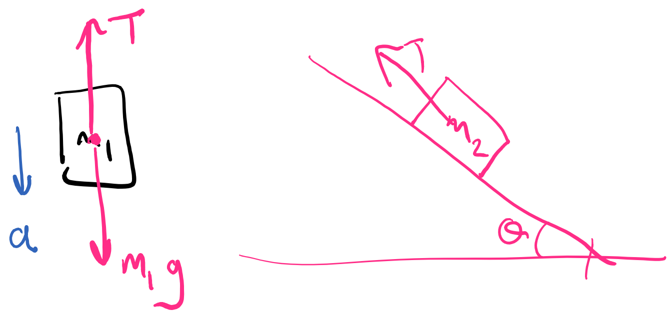
Ex

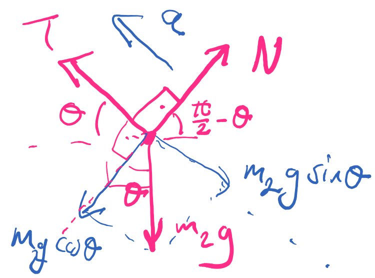


- The inclined plane is fixed to the floor.
- No friction

1°) Find the time it takes for m_1 to hit the floor.

2°) What is the magnitude of the force the floor is applying to the inclined plane.





$$N = m_2 g \cos\theta$$

① $m_1 a = m_1 g - T$

② $m_2 a = T - m_2 g \sin\theta$

① + ② $\Longrightarrow$ $(m_1 + m_2)\, a = m_1 g - \cancel{T} + \cancel{T} - m_2 g \sin\theta$

$$(m_1 + m_2)\, a = (m_1 - m_2 \sin\theta)\, g$$

$$a = \frac{(m_1 - m_2 \sin\theta)}{(m_1 + m_2)}\, g$$

$m_1 > m_2 \sin\theta$

if $m_2 \sin\theta > m_1$

m_1 does not reach the floor.

motion w/ constant a

$$\frac{1}{2} a t^2 = h$$

$$t = \sqrt{\frac{2h}{a}} = \boxed{\sqrt{\frac{2h}{(m_1 - m_2 \sin\theta)} \frac{(m_1 + m_2)}{g}}}$$

1°) Goal
2°) Unit
3°) Limit

$$\sqrt{\frac{m\ kg}{kg\ m/s^2}} = \sqrt{s^2} = s \checkmark$$

2_c ... $\cancel{kg}\ \cancel{m}/s^2$

3_c) Limit

$m_2 \sin\theta > m_1 \Rightarrow t = \sqrt{-}$

imaginary number!!

$\Rightarrow$ no such time!