Week 07 (Nov 7 - Nov 11)

Friday, September 23, 2016 2:57 PM

PHYSICS 101 - Instructor: M. Özgür OKTEL- 2016

QUIZ-13

A 1000 kg car goes from 0 to 100 km/h in 6 seconds. $\sim 100\ Hp$

a) How much work does the engine do?

b) What is the average power in this period. Give your answer in Horsepowers (Hp). 1 Hp = 745.7 W.

a) $K_i = 0$ $\quad K_f = \frac{1}{2} m v_f^2$

$$\Delta K = \boxed{W = \frac{1}{2} m v_f^2}$$

$$v_f = 100\ km/h = \frac{10^5}{60 \times 60}\ m/s = 27.8\ m/s$$

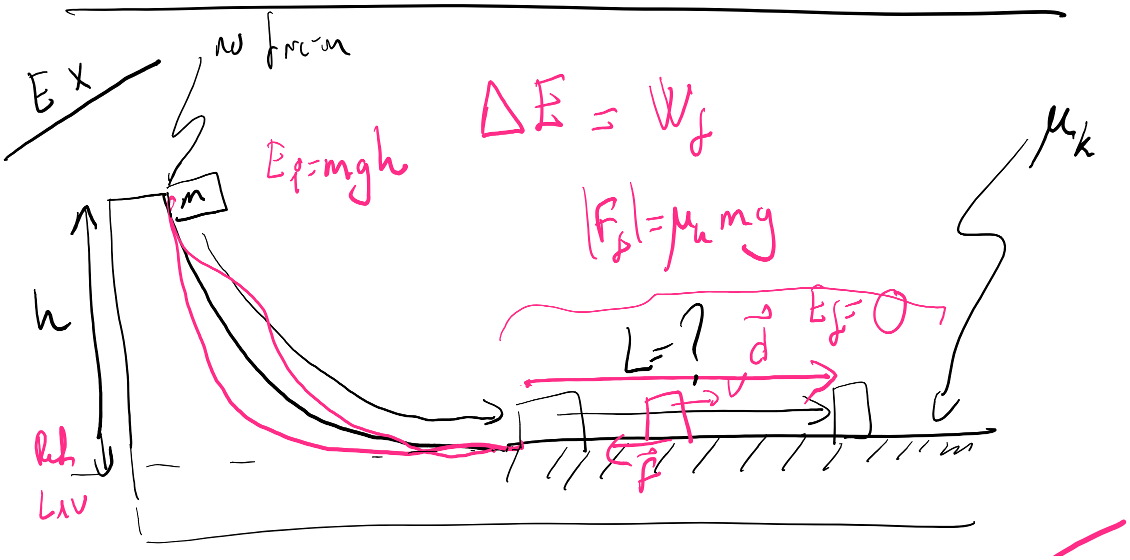
b) $$\boxed{P_{avg} = \frac{W}{\Delta t}}$$

$$W = \frac{1}{2} 10^3 (27.8)^2 = 3.86\, 10^5 \text{ Joules}$$

$$P_{avg} = \frac{W}{\Delta t} = \frac{3.86\, 10^5}{6} = 6.4\, 10^4 \text{ Watts}$$

$$P_{Hp} = \frac{6.4\, 10^4}{745.7} = \boxed{86.4\ Hp}$$

Ex



$$\Delta E = W_f$$

$$|F_f| = \mu_k mg$$

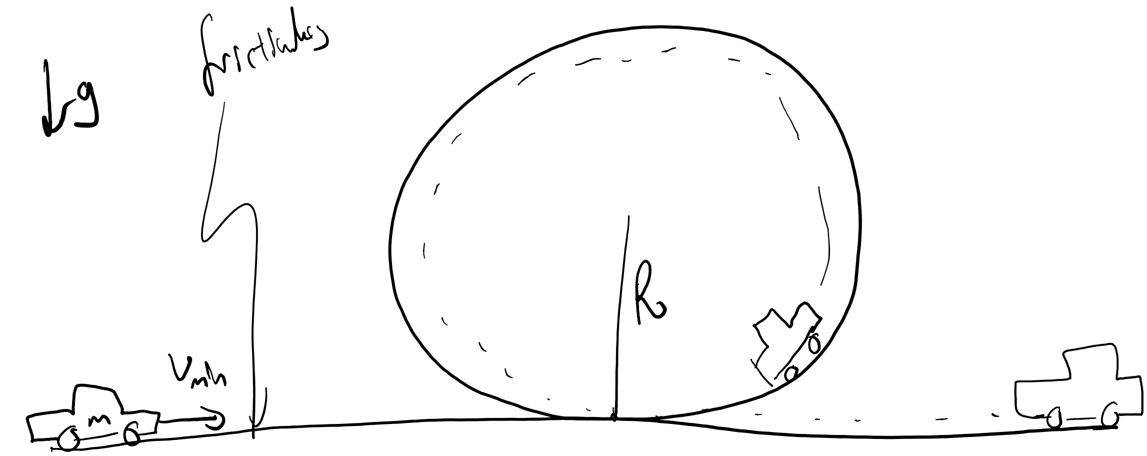
- Length traveled on the track with friction.
- $t = ?$ How long does the whole motion take?

$$\Delta E = E_f - E_i = 0 - mgh = -mgh$$

$$F_f = mg\mu_k \Rightarrow W_f = \vec{F}_f \cdot \vec{d} = -\mu_k mg L$$

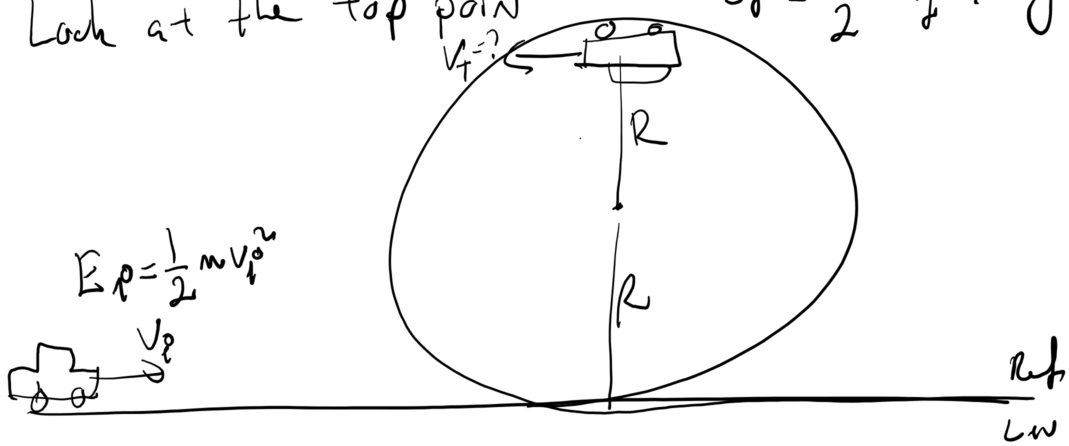
$$\Delta E = W_f$$

$$-\cancel{m}\cancel{g}h = -\mu_k \cancel{m}\cancel{g} L \Rightarrow \boxed{L = \frac{h}{\mu_k}}$$



Look at the top point

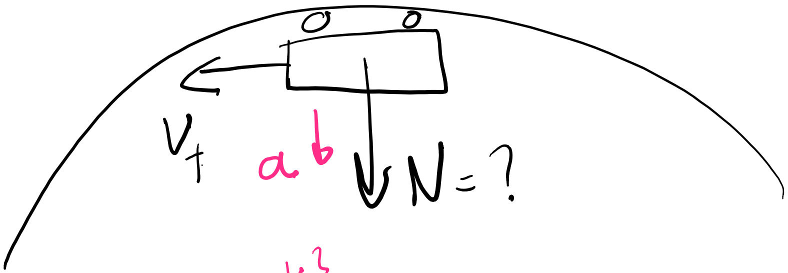
$$E_f = \frac{1}{2} m v_f^2 + mg2R$$

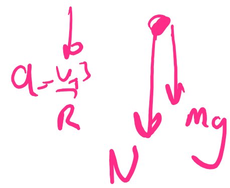


$$E_i = \frac{1}{2} m v_i^2$$

$$\frac{1}{2}\cancel{m}v_i^2 = \frac{1}{2}\cancel{m}v_T^2 + 2\cancel{m}gR$$

$$\boxed{v_T = \sqrt{v_i^2 - 4gR}}$$





$$a = \frac{v_T^2}{R}$$

$$ma = N + mg$$

$$N = ma - mg$$

$$\boxed{N = m\frac{v_T^2}{R} - mg}$$

Leaving the track at the top

$$N = 0 = \cancel{m}\frac{v_T^2}{R} - \cancel{m}g$$

$$\boxed{V_T = \sqrt{Rg}}$$

not to leave the track

$$V_T > \sqrt{Rg}$$

$$V_T = \sqrt{V_i^2 - 4Rg} > \sqrt{Rg}$$

$$V_i^2 - 4Rg > Rg$$

$$\boxed{V_i > \sqrt{5Rg}}$$

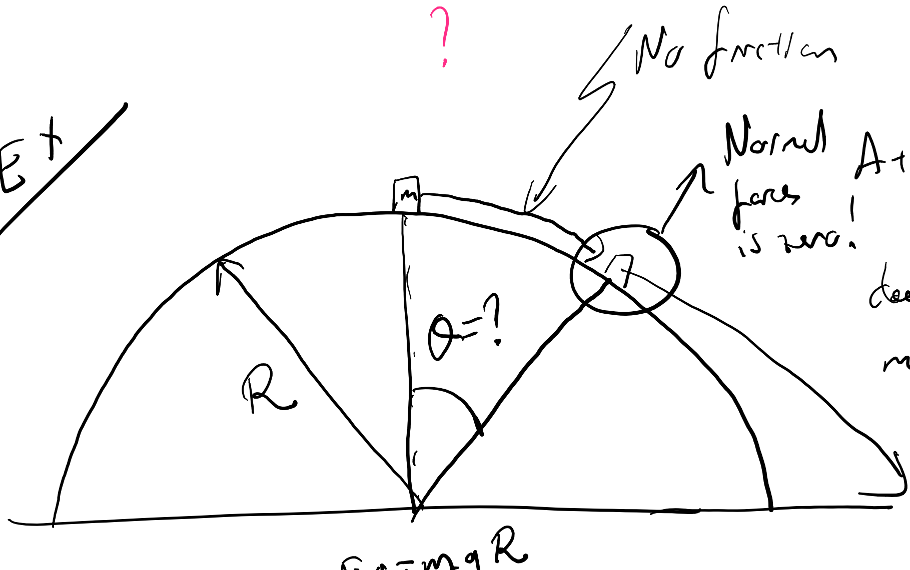
1°) Goal ✓

2°) $\sqrt{m\ m/s^2} = m/s$ ✓

3°) Limits $g \to 0$ $v_i \to 0$

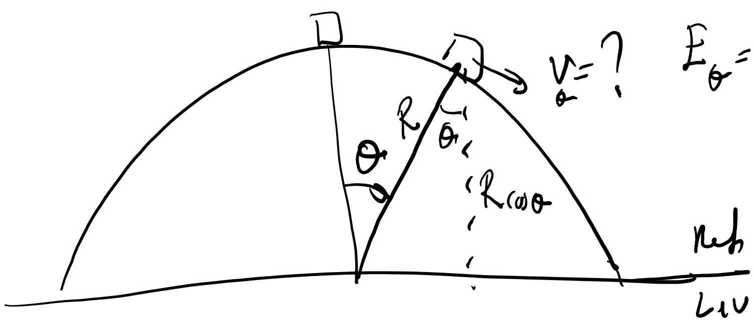
3) Lines
$g \to 0 \quad v_i = 30$

Ex



At which point does the mass leave the block

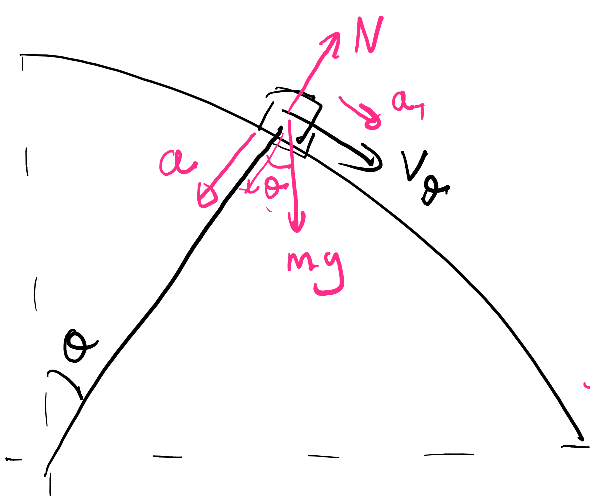
$E_i = mgR$



$E_\theta = \frac{1}{2} m v_\theta^2 + mgR\cos\theta$

$$\cancel{m} g R = \frac{1}{2} \cancel{m} v_\theta^2 + \cancel{m} g R\cos\theta$$

$$\sqrt{2gR(1-\cos\theta)} = v_\theta$$



$$a = \frac{v_\theta^2}{R}$$

$$ma = mg\cos\theta - N$$

$$\boxed{N = mg\cos\theta - ma}$$

$$N = mg\cos\theta - m\frac{v_\theta^2}{R}$$

$$N = \cancel{m}g\cos\theta - \frac{\cancel{m}}{\cancel{R}}\,2\cancel{g}\cancel{R}(1-\cos\theta) = 0$$

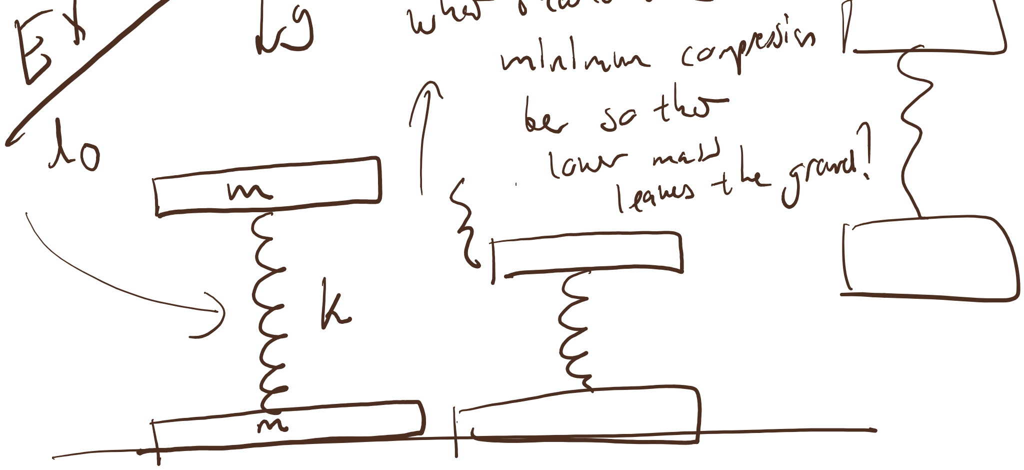
$$\cos\theta - 2 + 2\cos\theta = 0$$

$$3\cos\theta = 2$$

$$\boxed{\theta_c = \cos^{-1}\frac{2}{3}}$$

Ex kg

what should the minimum compression be so that lower mass leaves the ground?



critical point

initial point

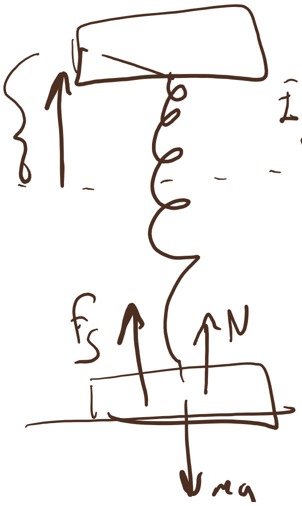
$$E_i = \frac{1}{2}k\Delta_i^2 - mg\Delta_i - \cancel{mgl_0}$$

Ref

Δ_f

$v = 0$

$$E_f = \frac{1}{2}k\Delta_f^2 + mg\Delta_f - \cancel{mgl_0}$$



If $N = 0$ Lower mass leaves the floor

leaves the floor

$$F_s = mg$$

$$\frac{1}{2}k\Delta_i^2 - mg\Delta_i = \frac{1}{2}k\Delta_f^2 + mg\Delta_f$$

$$k\,\Delta_f = mg$$

$$\boxed{\Delta_f = \frac{mg}{k}}$$

$$\frac{1}{2}k\Delta_i^2 - mg\,\Delta_i = \frac{1}{2}k\frac{(mg)^2}{k^2} + \frac{(mg)^2}{k} = \frac{3}{2}\frac{(mg)^2}{k}$$

$$\frac{1}{2}\Delta_i^2 - \left(\frac{mg}{k}\right)\Delta_i = \frac{3}{2}\left(\frac{mg}{k}\right)^2$$

$$\Delta_i^2 - 2\left(\frac{mg}{k}\right)\Delta_i - 3\left(\frac{mg}{k}\right)^2 = 0$$

$$\Delta_i = \frac{mg}{k} \mp \frac{1}{2}\sqrt{\underbrace{4\left(\frac{mg}{k}\right)^2 + 4\times 3\left(\frac{mg}{k}\right)^2}_{16\left(\frac{mg}{k}\right)^2}}$$

$$= \frac{mg}{k} \mp \frac{1}{\cancel{2}}\,\cancel{4}^{2}\,\frac{mg}{k} \xrightarrow[\text{positive root}]{} \boxed{\Delta_i = 3\,\frac{mg}{k}}$$

1°) Goal ✓

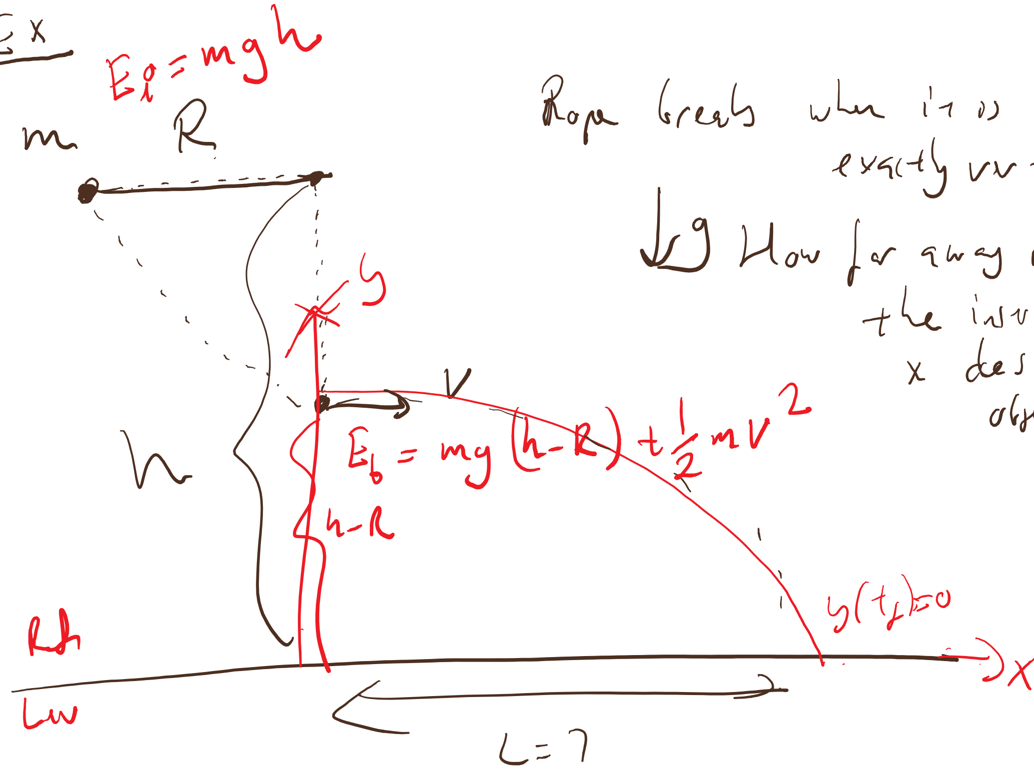
2°) Units $[\Delta_i] \Rightarrow m = \frac{\cancel{kg\,m/s^2}}{\cancel{kg/s^2}}$ m ✓

3°) $k \to 0$ $\quad \Delta_i \to \infty$ ✓

$m \to \infty$ $\quad \Delta_i \to \infty$ ✓

3°) $h \to 0 \quad \Delta_i \to \infty$ ✓
$m \to \infty \quad \Delta_f \to \infty$ ✓

Ex



Rope breaks when it is exactly vertical. How far away from the initial x does the object fall?

1°) $E_i = E_b$

$$\cancel{m}g\cancel{h} = \cancel{m}g(\cancel{h}-R) + \frac{1}{2}\cancel{m}v^2$$

$$gR = \frac{1}{2}v^2 \Longrightarrow v = \sqrt{2gR}$$

2°) Projectile motion

$$x(t) = \underbrace{x(t=0)}_{0} + vt$$

$$y(t) = \underbrace{y(t=0)}_{h-R} + \cancel{v_y t}_{0} - \frac{1}{2}gt^2$$

$$x(t) = \sqrt{2gR}\,t \qquad y(t) = (h-R) - \frac{1}{2}gt^2$$

$$y(t_f) = (h-R) - \frac{1}{2}gt_f^2 = 0$$

$$L = \sqrt{2gR}\sqrt{\frac{2}{g}(h-R)} \qquad t_f = \sqrt{\frac{2}{g}(h-R)}$$

$$L = \sqrt{2\cancel{g}R}\sqrt{\frac{2}{\cancel{g}}(h-R)}$$

$$\boxed{L = 2\sqrt{R(h-R)}}$$

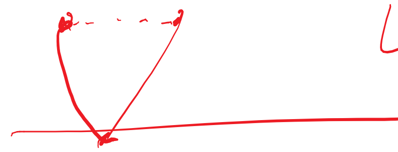
$$t_f = \sqrt{\frac{2}{g}(h-R)}$$

1°) Goal

2°) Units

3°) $h < R$ $L = \sqrt{(-)}$

$h = R$ $L = 0$



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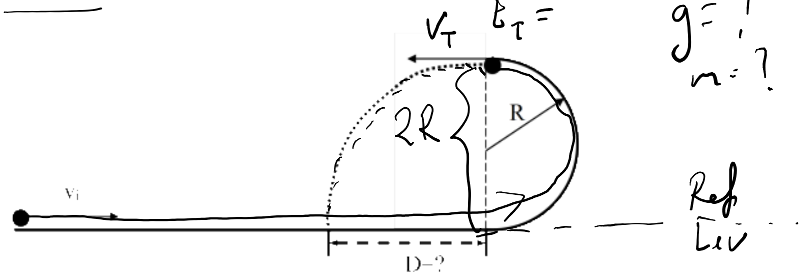
QUIZ-14

A particle is shot with an initial speed of $v_i = (5\, g\, R)^{1/2}$, on a horizontal surface toward a semi-circular track of radius R. How far from the edge of the semicircle does it fall (see figure below) ? (There is no friction)



$g = ?$

$m = ?$

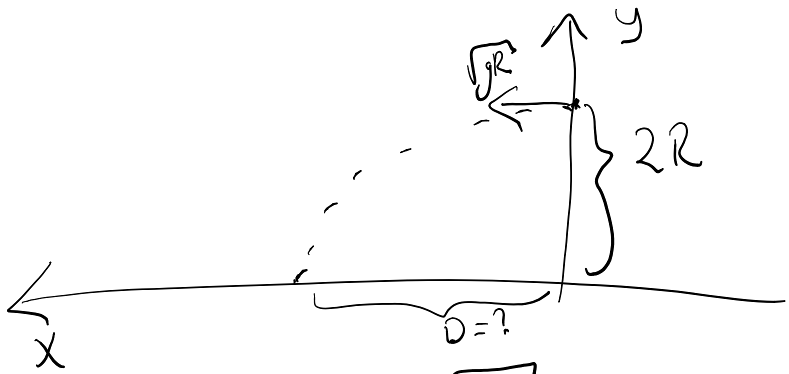
1°) Calculate v_T Energy is conserved!

$$E_i = \frac{1}{2} m v_i^2 \qquad E_T = \frac{1}{2} m v_T^2 + mg2R$$

$$\frac{1}{2}\cancel{m} v_i^2 = \frac{1}{2}\cancel{m} v_T^2 + \cancel{m}g2R$$

$$v_T^2 = \underbrace{v_i^2}_{5gR} - 4gR = gR \Rightarrow v_T = \sqrt{gR}$$

2°) Projectile motion with initial height 2R initial horizontal velocity $\sqrt{gR}$



$$x(t) = \sqrt{gR}\, t$$

$$y(t) = 2R - \frac{1}{2} g t^2$$

time of flight

$$y(t_f) = 0 = 2R - \frac{1}{2} g t_f^2$$

$$\sqrt{\frac{4R}{g}} = t_f$$

$$D = x(t_f) = \sqrt{\cancel{g}R}\sqrt{\frac{4R}{\cancel{g}}} = \boxed{2R}$$

1°) Goal ✓
2°) Units ✓
3°) Limits ?

Energy | Momentum | Angular Momentum

Electric charge | . . . .

Momentum

$\vec{p} \longrightarrow$ vector

$\vec{p} = m\,\vec{v}$ $\qquad$ $[p] = kg\, m/s$

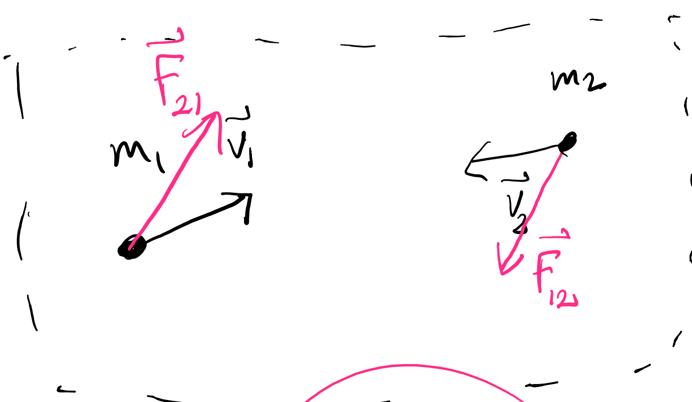
$$\vec{p} = m\vec{v} \qquad [p] = kg\,m/s$$

Let's look at time derivative of momentum

$$\frac{d}{dt}(\vec{p}) = m \underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}} = m\vec{a} = \vec{F}$$

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}} \Rightarrow$$

Holds if not only $\vec{v}$ but also m is changing with time!



Closed system (no external forces)

$$\vec{F}_{21} = -\vec{F}_{12}$$

Newton's third Law

$$\vec{P} = \vec{p}_1 + \vec{p}_2 = (m_1\vec{v}_1 + m_2\vec{v}_2)$$

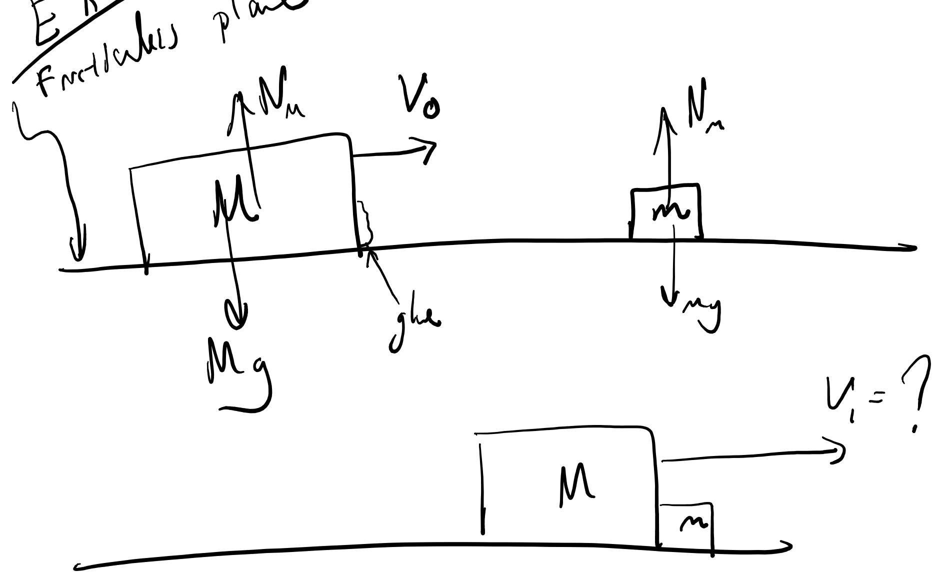
$$\frac{d\vec{P}}{dt} = \underbrace{\frac{d\vec{p}_1}{dt}}_{\vec{F}_{21}} + \underbrace{\frac{d\vec{p}_2}{dt}}_{\vec{F}_{12}} = \vec{F}_{21} + \vec{F}_{12} = 0$$

No net external force.

Total momentum of a closed system of particles

Total momentum of a closed system of particles is conserved!

EX



* Closed system $\Rightarrow$ momentum is conserved.

$$P_i = M V_0 + \cancel{m \cdot 0} = M V_0 \quad \text{to the right}$$

$$P_f = (M+m) V_1 = (M+m) V_1 \quad \text{to the right}$$

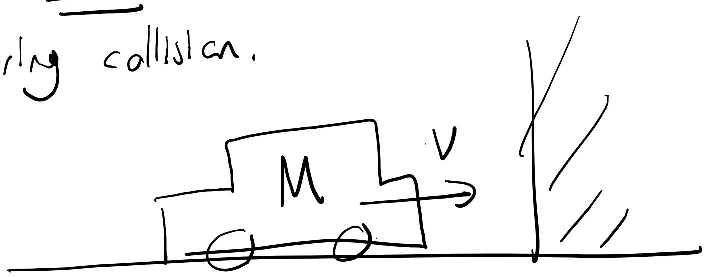
$$M V_0 = (M+m) V_1 \Longrightarrow \boxed{V_1 = \frac{M}{(M+m)} V_0}$$

$$\frac{d\vec{P}}{dt} = \vec{F} \Longrightarrow \int^f d\vec{p} = \int^f \vec{F}\, dt$$

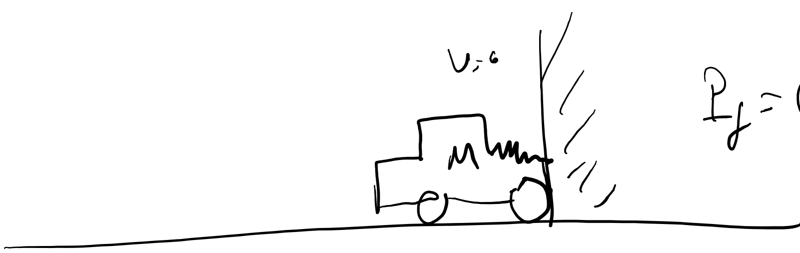
$$\frac{dp}{dt} = F \implies \int_i dp = \int_i F\,dt$$

$$\boxed{\Delta \vec{P} = \vec{P}_f - \vec{P}_i = \underbrace{\int_{t_i}^{t_f} \vec{F}(t)\,dt}_{\text{Impulse}}}$$

Ex: $M = 1000\text{ kg}$ car hits a wall with $V = 10\text{ m/s}$. The collision with the wall ends in 0.01 sec. Find the average force the wall acts on the car during collision.



$P_i = MV$



$P_f = 0$

$$|\Delta P| = MV = \underbrace{\int F(t)\,dt} \qquad F_{avg} = \frac{\int F\,dt}{\Delta t}$$

$$MV = F_{avg}\,\Delta t$$

$$\boxed{F_{avg} = \frac{M\,v}{\Delta t}} = \frac{1000 \; 10}{0.01}$$

$$= \underline{\underline{10^6 \; N}}$$