Week 15 Notes (May 2-3)

Wednesday, January 27, 2016 4:49 PM

QUIZ-28

Let

$$\vec{E} = E_0 \sin\left[2\pi 10^8 t + \frac{2\pi}{3} z\right](3\hat{\imath} + 4\hat{\jmath})$$

where t is in seconds, z in meters and E_0 is a given constant, be the electric field in an EM wave travelling in empty space.

(a) ~~What is the frequency and the direction of propagation of this EM wave?~~
(b) Find the magnetic field.
(c) ~~Find the magnitude and the direction of the Poynting vector.~~

$$\vec{E} = \vec{E}_0 \cos(kz - \underbrace{\omega t}) \quad +\hat{z}$$

$$= \vec{E}_0 \cos\left(k\,(\underbrace{z - \tfrac{\omega}{k} t}_{z - z_0(t)})\right)$$

$$\vec{E} = E_0 (3\hat{\imath} + 4\hat{\jmath}) \sin\left(\frac{2\pi}{3} z + \underbrace{2\pi 10^8}_{\omega} t\right)$$

$$\boxed{\nu = 10^8\ Hz}$$

$$= E_0 (3\hat{\imath} + 4\hat{\jmath}) \sin\left(\frac{2\pi}{3}\left(z + \underbrace{3\,10^8}\, t\right)\right)$$

$$\underline{(z + z_0(t))}$$

Moving toward $-\hat{z}$

Direction of motion is $\boxed{-\hat{k}}$

b) $$\vec{E} = \underbrace{E_0 (3\hat{\imath} + 4\hat{\jmath})}_{\vec{E} = 5E_0} \sin\left(\frac{2\pi}{3} z + 2\pi 10^8 t\right)$$

$$\vec{B} = \vec{B}_0 \sin\left(\frac{2\pi}{3} z + 2\pi 10^8 t\right)$$

$\hat{E} \times \hat{B}$ = direction of propagation $= -\hat{k}$

$\hat{E} \times \hat{B}$ = direction of propagation $= -k$

$\vec{B} \perp -\hat{k}$ $\qquad \vec{B}_0 = B_x \hat{\imath} + B_y \hat{\jmath}$

$\vec{B} \perp \vec{E}$ $\qquad \vec{B}_0 \cdot \vec{E} = 0$

$$(B_x \hat{\imath} + B_y \hat{\jmath}) \cdot (3E_0 \hat{\imath} + 4E_0 \hat{\jmath}) = 0$$

$$3\cancel{E_0} B_x + 4\cancel{E_0} B_y = 0$$

$$B_x = -\frac{4}{3} B_y$$

$$\vec{B} = \left(-\frac{4}{3} B_y \hat{\imath} + B_y \hat{\jmath}\right) \Rightarrow |\vec{B}| = \sqrt{1 + \frac{16}{9}}\, B_y = \frac{5}{3} B_y$$

$$\vec{E} \times \hat{B} = (E_0 3 \hat{\imath} + E_0 4 \hat{\jmath}) \times \left(-\frac{4}{3} B_y \hat{\imath} + B_y \hat{\jmath}\right)$$

$-\hat{k}$ $\qquad 0 \qquad 0$

$$= 3 E_0 B_y \hat{k} + \frac{16}{3} E_0 B_y \hat{k}$$

$$= \frac{25}{3} E_0 B_y \hat{k}$$

$$\boxed{B_y < 0}$$

$|\vec{B}|$

$$\frac{|\vec{B}|}{|\vec{E}|} = \frac{1}{c} = \frac{\cancel{5}/3\,|B_y|}{\cancel{5}\,|E_0|} \Rightarrow \boxed{|B_y| = \frac{3}{c}|E_0|}$$

$$\boxed{B_y = -\frac{3}{c}E_0}$$

$$\vec{B}(t) = \left(-\frac{4}{3}B_y\,\hat{i} + B_y\,\hat{j}\right)\sin(kz + \omega t)$$

$$= -\frac{3}{c}E_0\left(-\frac{4}{3}\hat{i} + \hat{j}\right)$$

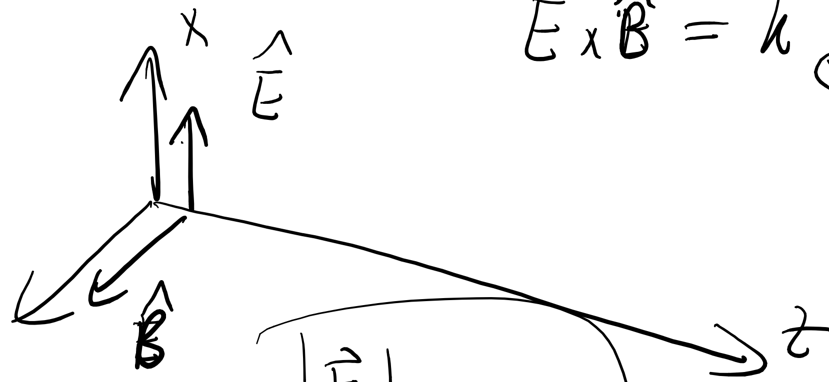
$$\boxed{\vec{B}(t) = -\frac{E_0}{c}\left(-4\hat{i} + 3\hat{j}\right)\sin(kz + \omega t)}$$

c) $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \boxed{\frac{1}{\mu_0}\,\frac{25}{3}|E_0||B_y|\,(-\hat{k})}$

$\cancel{4}$

Review

EM waves



$$\hat{E} \times \hat{B} = \hat{k}$$ ← propagation

$$\boxed{\frac{|\vec{E}|}{|\vec{B}|} = c} \rightarrow c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{r} - \omega t)$$
$$\vec{B} = \vec{B}_0 \cos(\vec{k}\cdot\vec{r} - \omega t)$$

$$|\vec{k}| = \frac{2\pi}{\lambda}$$

$$\boxed{\lambda \nu = c}$$

$$\omega = 2\pi \nu$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad [\vec{S}] \Rightarrow \frac{J}{s\,m^2}$$

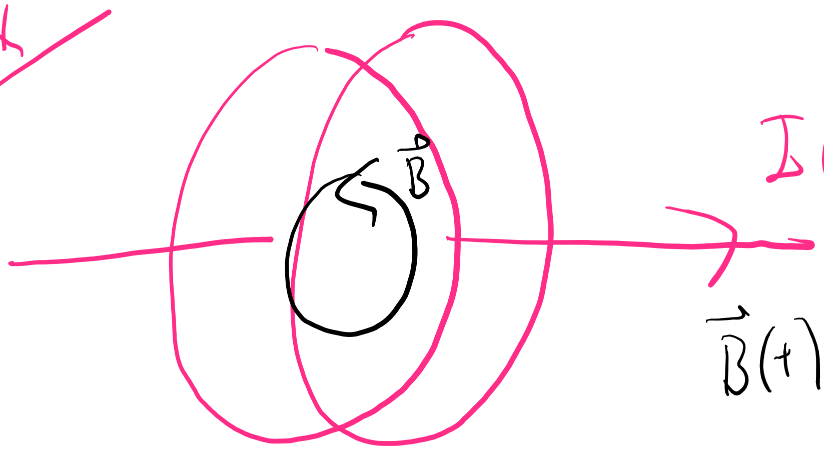
Maxwell's Eqns

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

1 ±

$$\oint \vec{E}\cdot\vec{dl} = -\frac{d\Phi_B(t)}{dt} \qquad \oint \vec{B}\cdot\vec{dl} = \mu_0 I_{in} + \underbrace{\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}_{\text{Displacement current.}}$$

Ex



$\vec{B}(t)$ inside the capacitor?

AC circuits

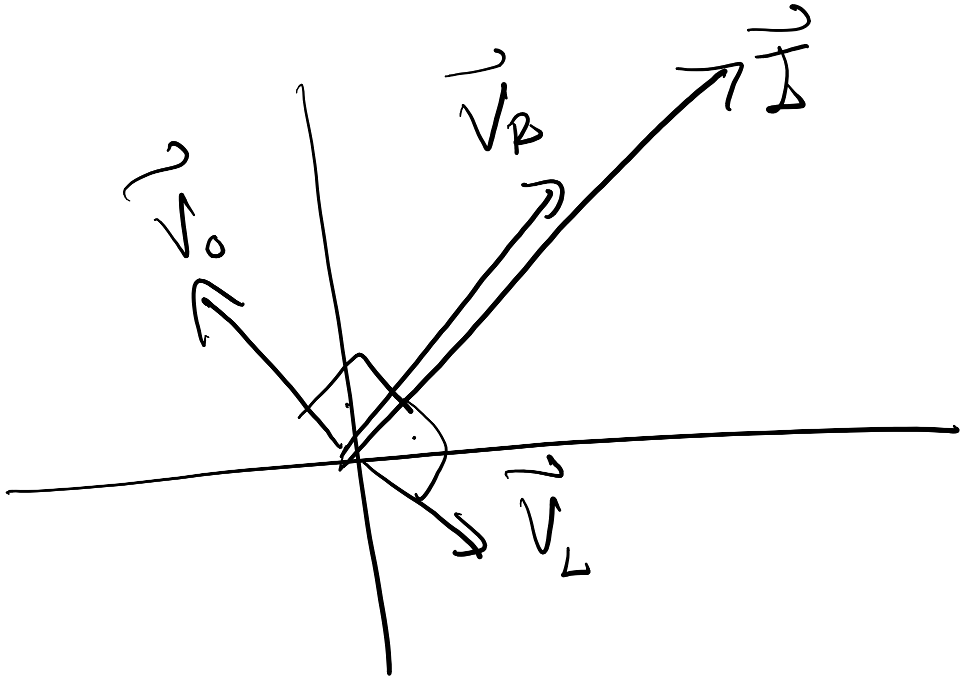
$V(t) = V_0 \cos(\omega t + \varphi_V)$
$I(t) = I_0 \cos(\omega t + \varphi_I)$

$V = IR$ $\qquad I = C\frac{dV}{dt}$ $\qquad V = L\frac{dI}{dt}$

$Z = \frac{|V|}{|I|}$ $\quad Z_R = R$ $\qquad Z_C = \frac{1}{\omega C}$ $\qquad Z_L = \omega L$

$V = V_0 \cos(\omega t)$

$I = C\frac{dV}{dt} = C V_0 \underbrace{-\sin(\omega t)}_{+\cos(\omega t + \frac{\pi}{2})}$



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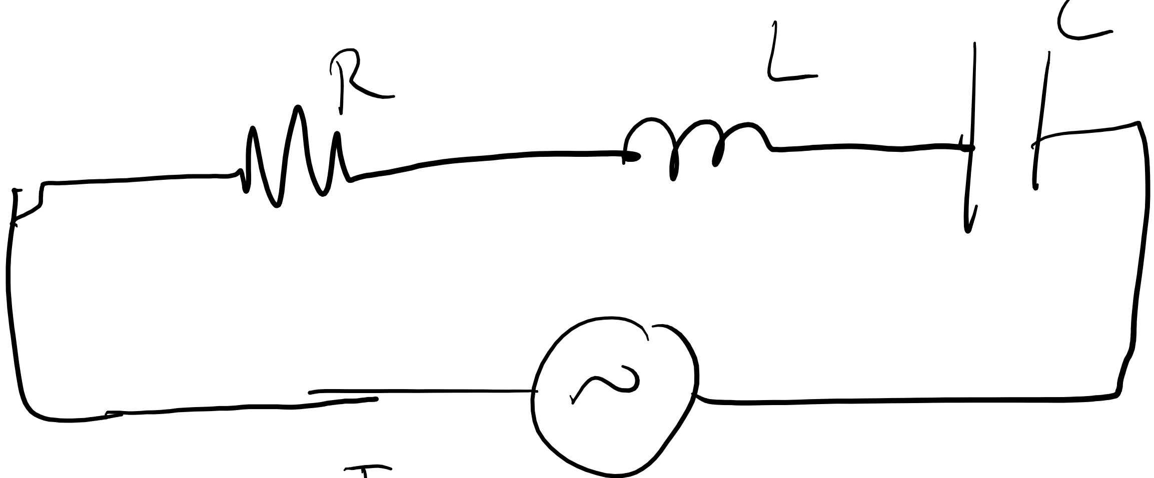
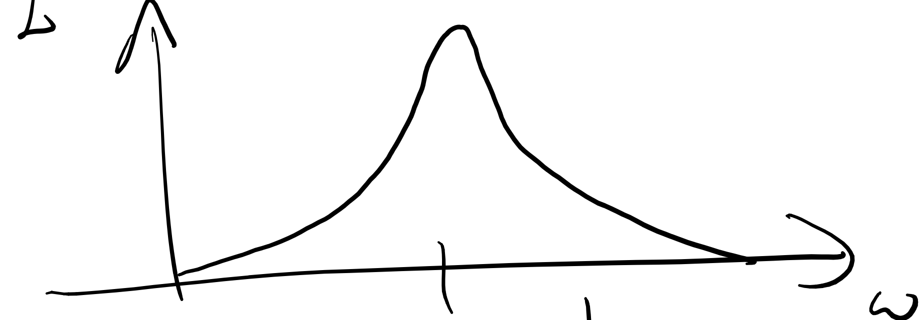
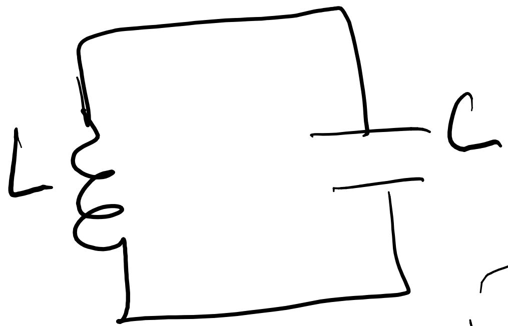


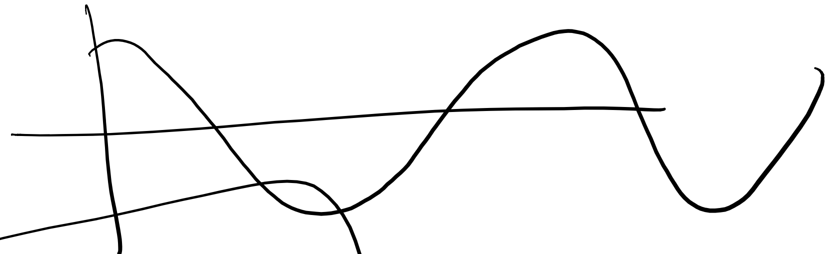
Resonance



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

LC and RLC circuits





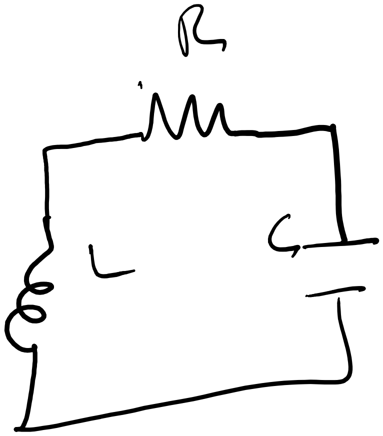
$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

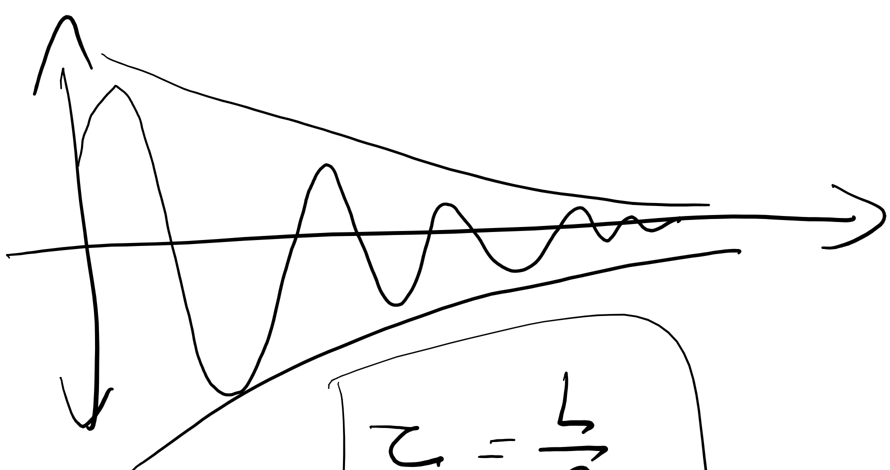
$U_C = \frac{1}{2}\frac{Q^2}{C}$

$U_L = \frac{1}{2} L I^2$

$$\frac{d^2 f(t)}{dt^2} = -\omega^2 f(t)$$

$$f(t) = A\cos(\omega t) + B\sin(\omega t)$$

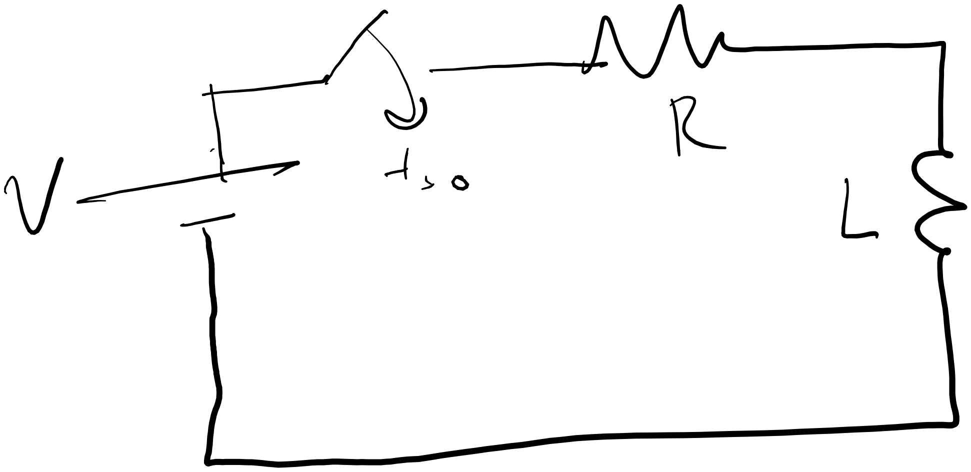




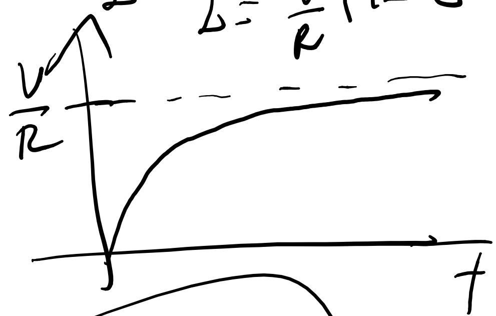
$$\boxed{\tau = \frac{L}{R}}$$

LR

$$V = L\frac{di}{dt}$$



$$I = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$



$$\boxed{\tau = \frac{L}{R}}$$

$$\boxed{\tau = RC}$$

$$[\tau]^2 = LC$$

$$\left[\frac{1}{\tau}\right] = Hz = \frac{1}{\sqrt{LC}}$$

Induction

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

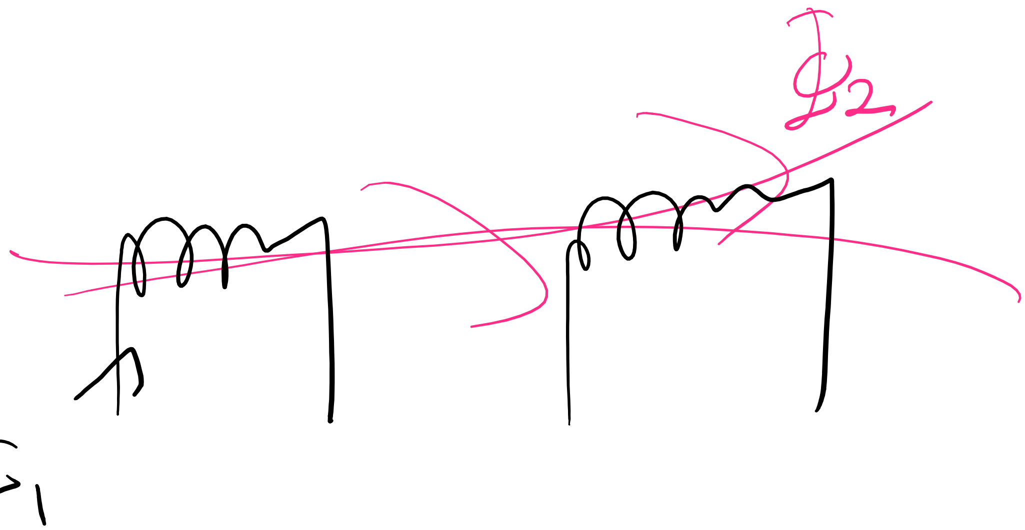
T

self inductance

inductance

$$\Phi = L I$$

$$\varepsilon = -\frac{d\Phi}{dt} = L\frac{dI}{dt}$$

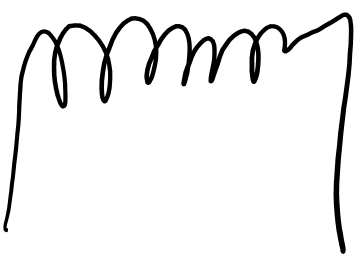


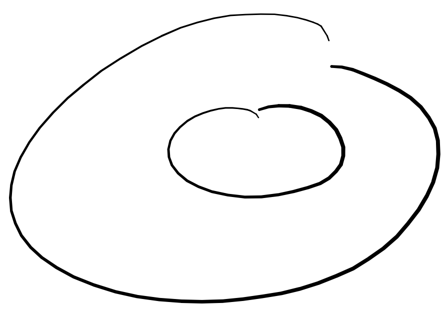
$$\Phi_2 = M i_1$$

mutual inductance

Ex

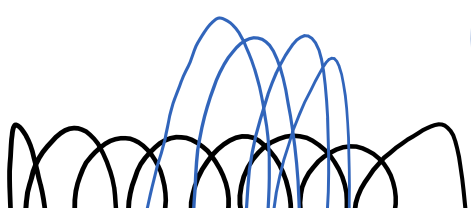
$L_{solenoid} = ?$



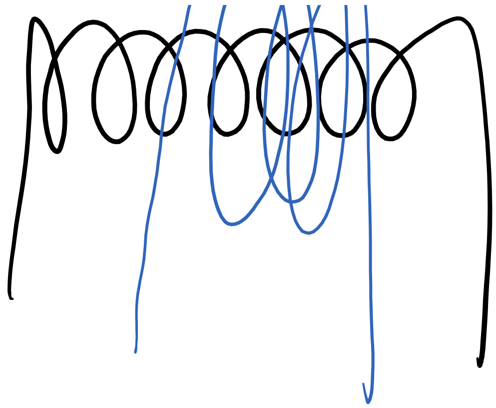


L_{torus}

toroid



$M = ?$



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Lenz's Law

use this to find the direction of current.

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$|\vec{B}|$ change

$|\vec{B}||\vec{A}|\underline{\cos\theta}$ change

Area change