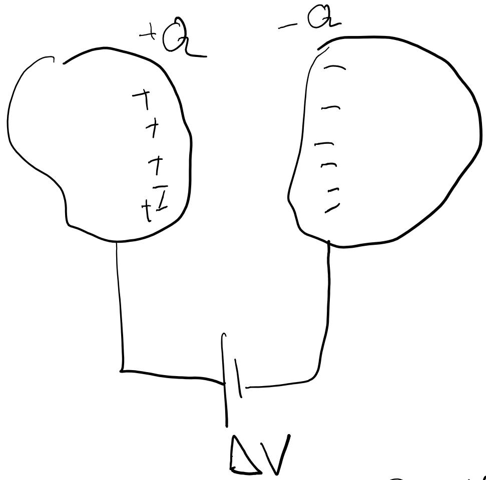
Week 6 Notes (Feb 29-Mar 4)

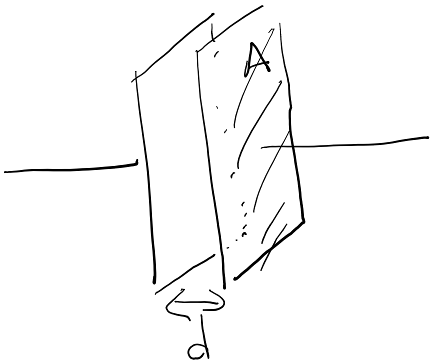
Wednesday, January 27, 2016 4:45 PM

Capacitors



$$\boxed{\frac{Q}{\Delta V} = C}$$

Parallel plate capacitor

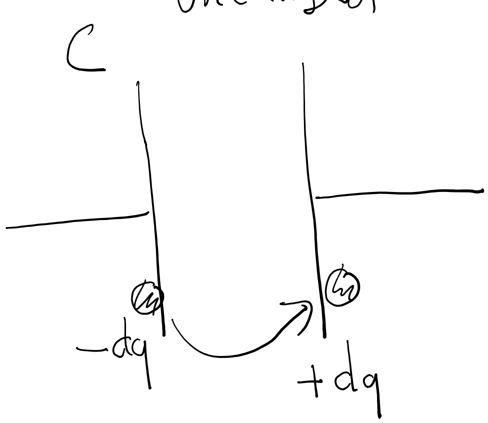


$$C = \varepsilon_0 \frac{A}{d}$$

If we want to store more charge make $A \uparrow$ or $d \downarrow$

Storing charge $\Longrightarrow$ storing energy!

uncharged



$$dW = \Delta V \, dq$$

$$\Delta V = \frac{q}{C}$$

$$W = \int dW = \int^{Q} \frac{q}{C} \, dq$$

$$W = \int dW = \int_0^Q \frac{q}{C}\, dq$$

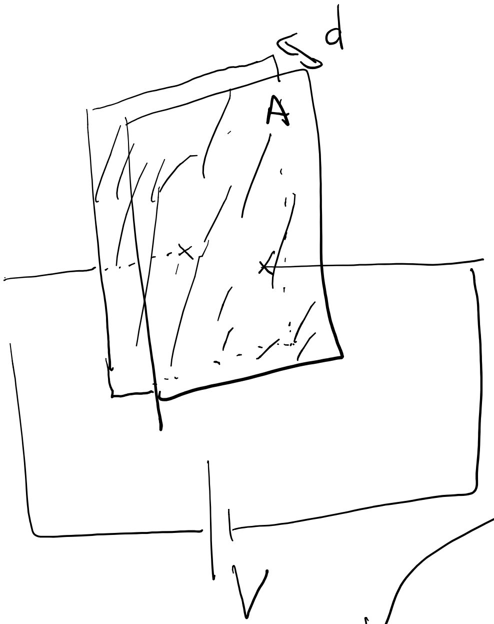
$$= \frac{1}{C}\int_0^Q q\, dq = \boxed{\frac{Q^2}{2C}}$$

$$W = \frac{Q^2}{2C} = \boxed{\frac{1}{2} C (\Delta V)^2}$$

Stored energy in a capacitor.

$$W \Rightarrow E_{\text{stored}} = \frac{1}{2} C (\Delta V)^2$$

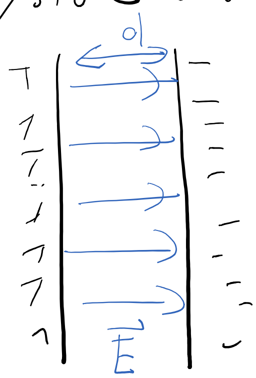
Where is this energy stored.



$$C = \varepsilon_0 \frac{A}{d} \qquad V$$

$$U = \frac{1}{2}\varepsilon_0 \frac{A}{d} V^2$$

side view



$$\left|\int \vec{E} \cdot d\vec{l}\right| = V$$

$$|\vec{E}| \underbrace{\int dl}_{d} = V$$

$$|\vec{E}| = \frac{V}{d}$$

$$U = \frac{1}{2}\varepsilon_0 \frac{A}{d} V^2$$

$$= \frac{1}{2}\varepsilon_0 (Ad)\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 (E)^2 \underbrace{(Ad)}$$

Volume

If you create electric field E
in Volume $d\Omega$

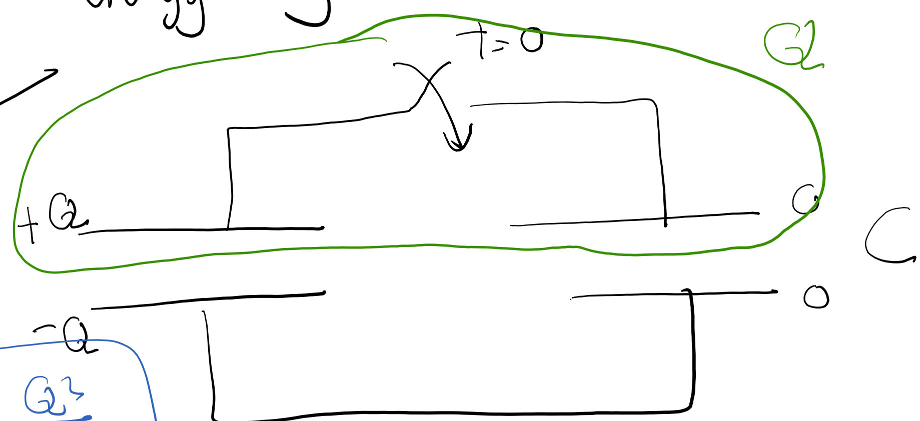
$$dU = \frac{1}{2}\varepsilon_0 E^2 \, d\Omega$$

energy

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

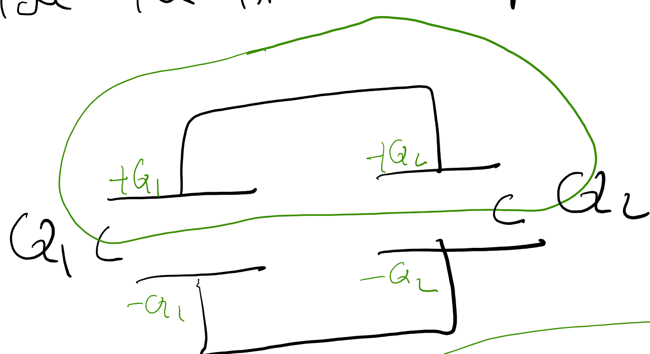
energy density

EX



$$\boxed{E_{i} = \frac{1}{2}\frac{Q^2}{C}}$$

Find the final charges on the capacitors
Calculate the initial and final energies.



Finally

$$\frac{Q_1}{\cancel{C}} = \frac{Q_2}{\cancel{C}}$$

$$Q_1 = Q_2$$

$$Q_1 + Q_2 = Q$$

$$\boxed{Q_1 = Q_2 = \frac{Q}{2}}$$

$$Q_1 + Q_2 = Q$$

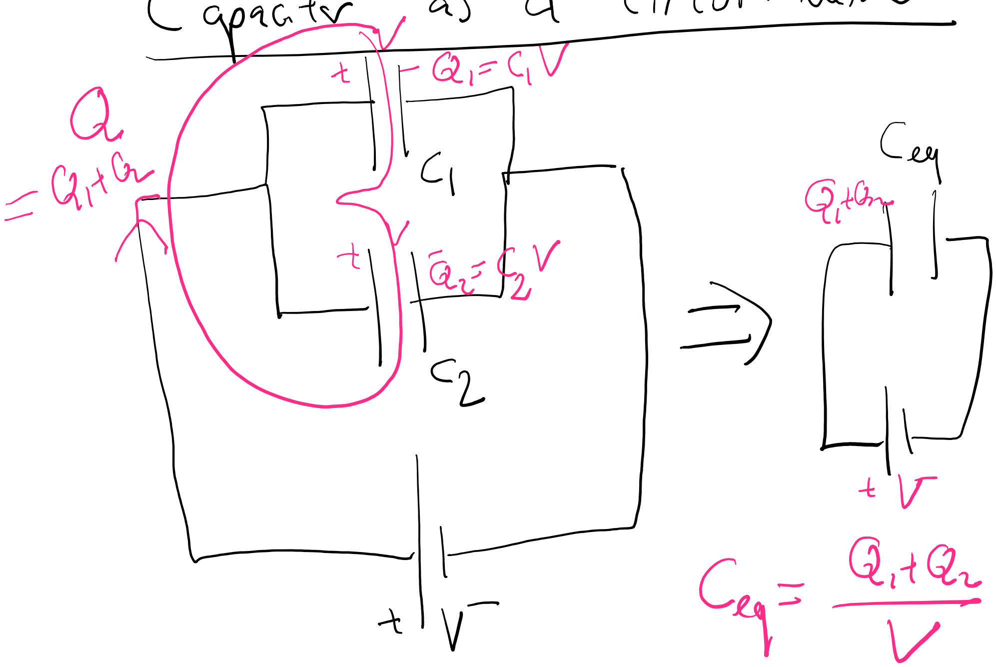
$$2Q_1 = Q \qquad \boxed{Q_1 = Q_2 = \frac{Q}{2}}$$

$$E_f = \frac{1}{2}\frac{Q_1^2}{C} + \frac{1}{2}\frac{Q_2^2}{C} = \frac{1}{2}\frac{(Q/2)^2}{C} + \frac{1}{2}\frac{(Q/2)^2}{C}$$

$$\boxed{E_f = \frac{1}{4}\frac{Q^2}{C}} < E_i = \frac{1}{2}\frac{Q^2}{C}$$

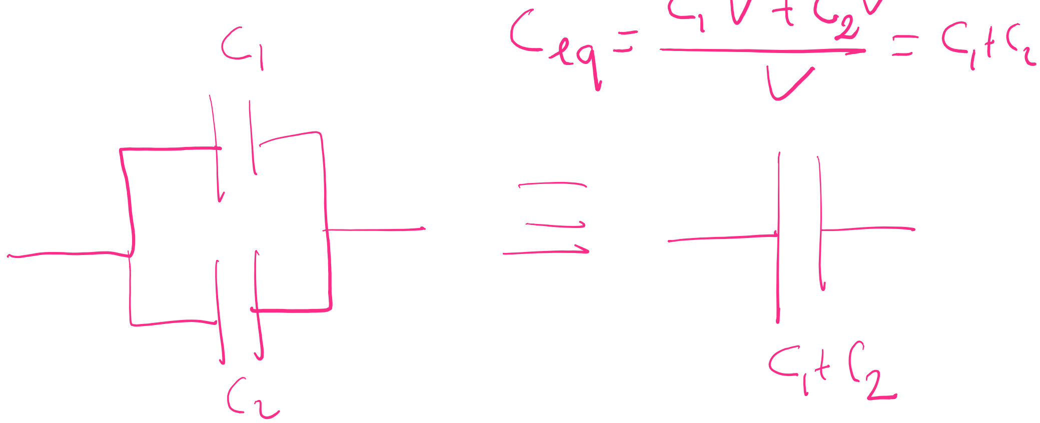
Radiated as heat or other kind of radiation!

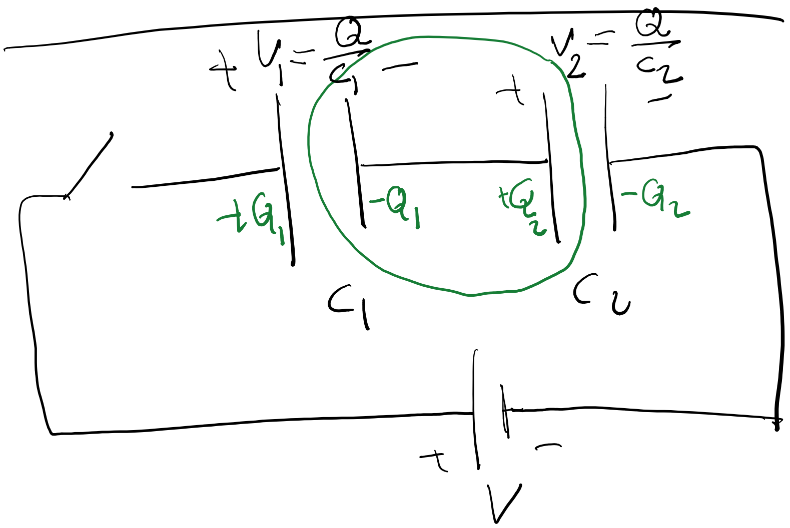
Capacitor as a circuit element



$$C_{eq} = \frac{Q_1 + Q_2}{V}$$

$$C_{eq} = \frac{C_1 V + C_2 V}{V} = C_1 + C_2$$





$-Q_1 + Q_2 = 0$

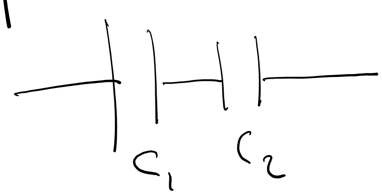
$Q_1 = Q_2 = Q$

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

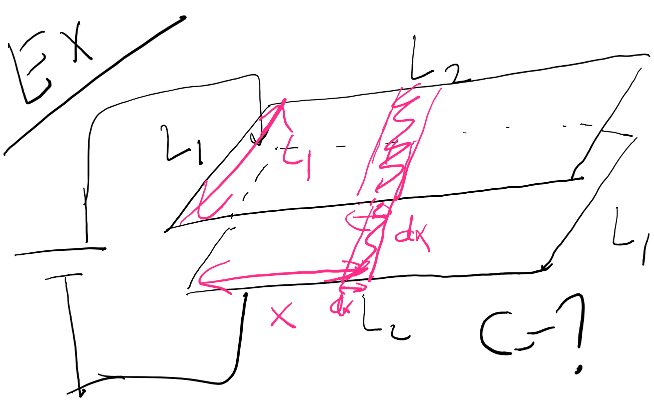
$$V = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q$$

$$Q = \frac{1}{\underbrace{\frac{1}{C_1} + \frac{1}{C_2}}_{C_{eq}}} V$$

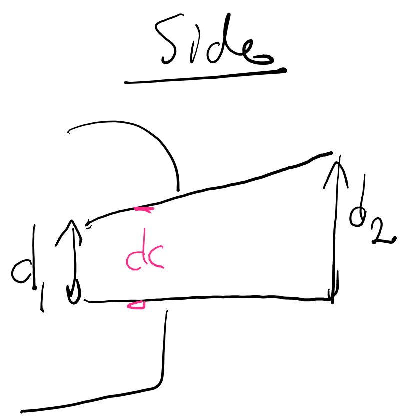
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$



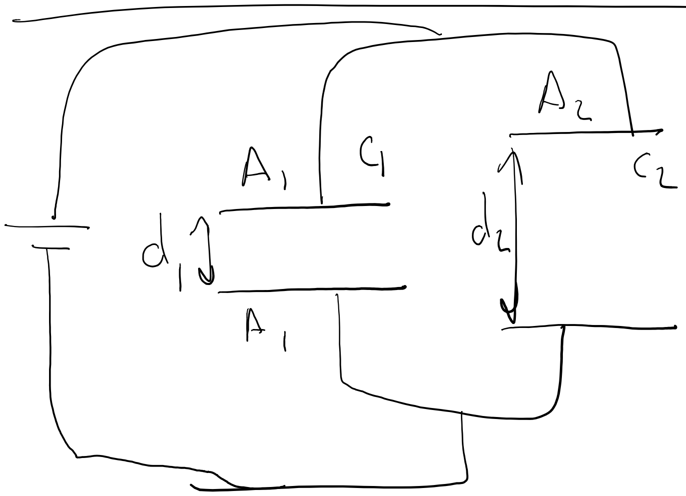
EX



$C = ?$

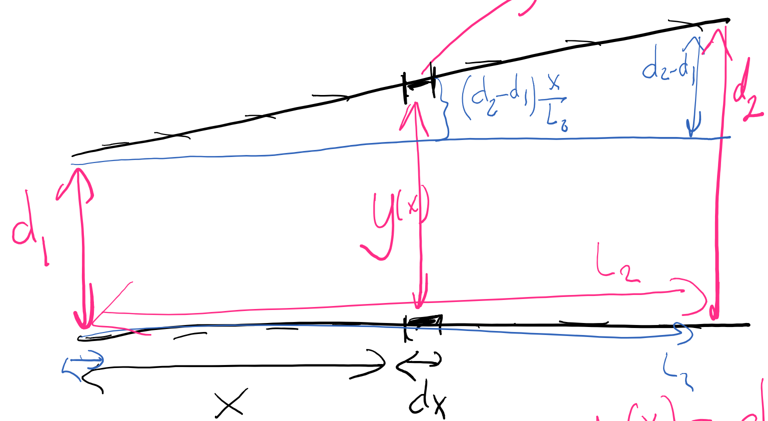


$$|d_1-d_2| \ll d_1, d_2 \ll L_1, L_2$$



$$C_{eq} = ? = C_1 + C_2$$
$$= \epsilon_0 \frac{A_1}{d_1} + \epsilon_0 \frac{A_2}{d_2}$$

dC



Surface area of this small capacitor

$dA = L_1\, dx$

distance between the plates for this small capacitor

$$y(x) = d_1 + (d_2-d_1)\frac{x}{L_2}$$

$$y(x) = d_1 + \frac{x}{L_2}(d_2-d_1)$$

$y(x=0) = d_1 \qquad y(x=L_2) = d_2$

$$dC = \epsilon_0 \frac{dA}{y(x)} = \epsilon_0 \frac{L_1\, dx}{d_1 + \frac{(d_2-d_1)}{L_2}x}$$

$$C = \int dC = \int_0^{L_2} \epsilon_0 \frac{L_1 L_2}{d_1 L_2 + (d_2-d_1)x}\, dx$$

$$\int^{L_2} \quad 1 \quad dx$$

$$= \varepsilon_0 L_1 L_2 \int_0^{L_2} \frac{1}{(d_2-d_1)x + d_1 L_2}\, dx$$

$$u = (d_2-d_1)x + d_1 L_2$$

$$du = (d_2-d_1)\, dx$$

$$= \varepsilon_0 L_1 L_2 \int_{d_1 L_2}^{d_2 L_2} \frac{1}{u} \frac{du}{(d_2-d_1)}$$

$$= \varepsilon_0 \frac{L_1 L_2}{(d_2-d_1)} \int_{d_1 L_2}^{d_2 L_2} \frac{1}{u}\, du$$

$$C = \varepsilon_0 \frac{L_1 L_2}{d_2-d_1} \ln\left(\frac{d_2 L_2}{d_1 L_2}\right) = \boxed{\varepsilon_0 \frac{L_1 L_2}{d_2-d_1} \ln\left(\frac{d_2}{d_1}\right)}$$

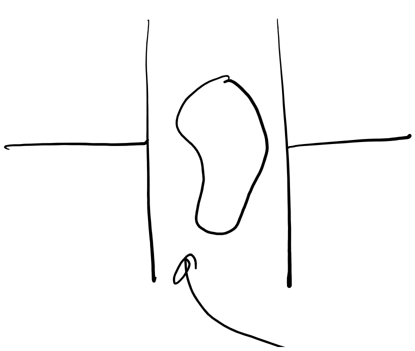
$d_1 \to d_2$!!

Exam info

Saturday 5th of March 2016 10:00 AM

BZ01 Abbasoğlu-Gül

BZ02 Gömen-Yücel



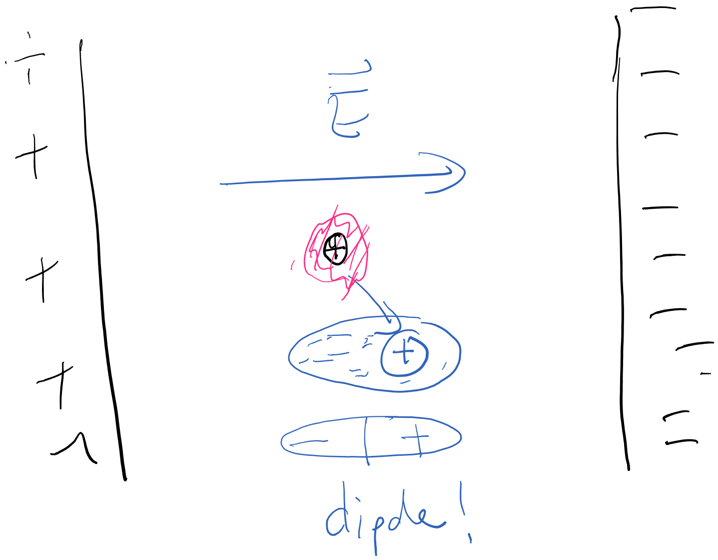
More capacitance

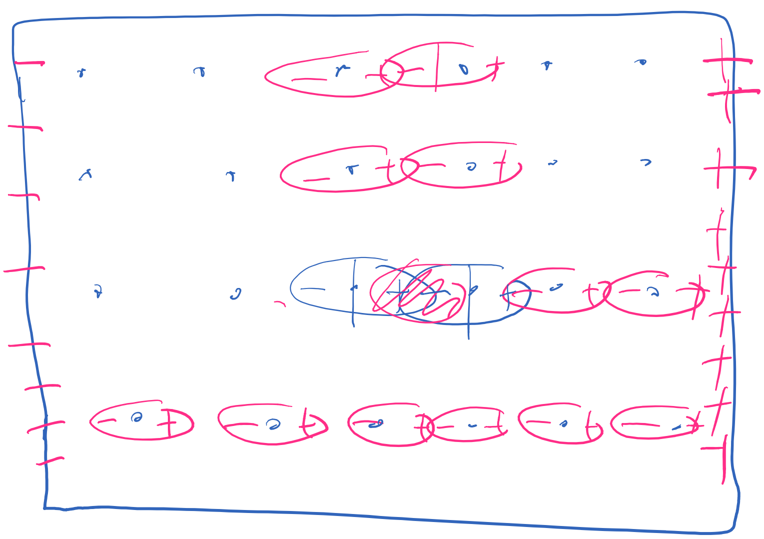
How do we do it?

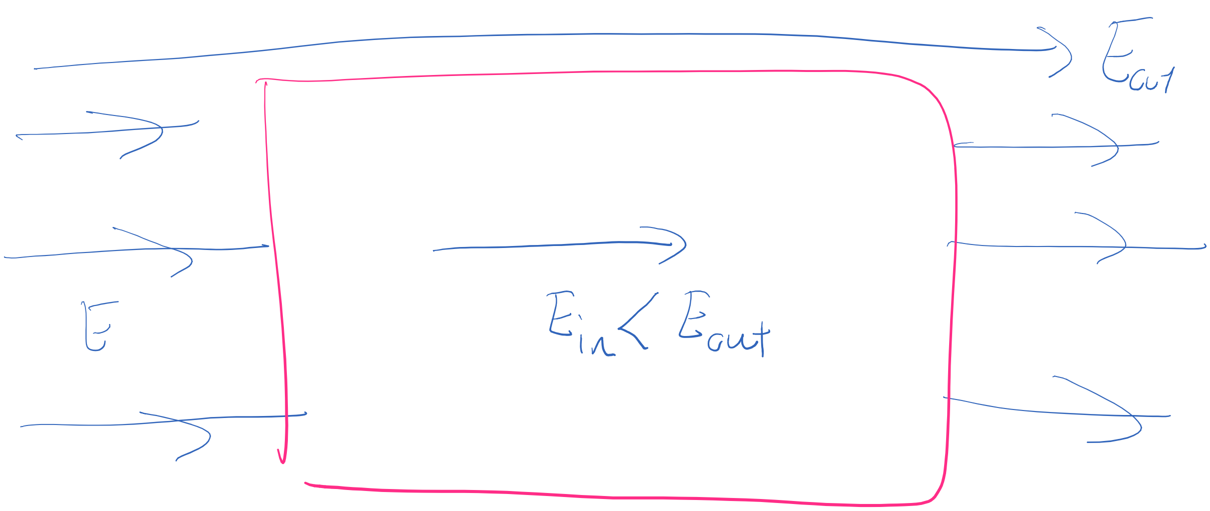
Put any non conductor inside

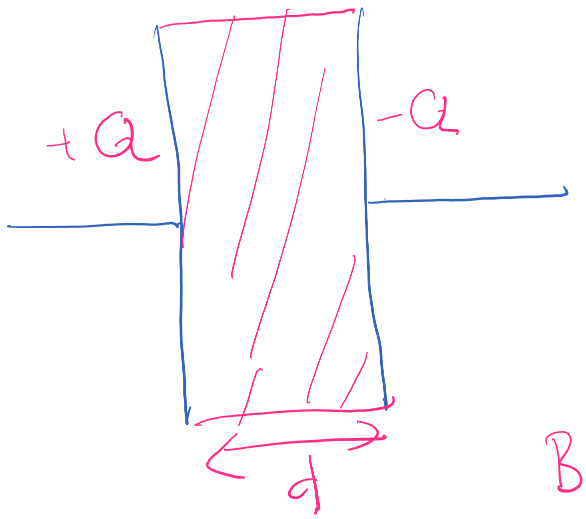
inside

Capacitance increases!







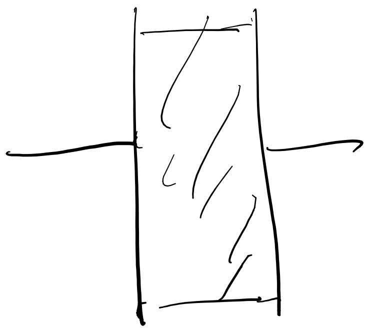


$\Delta V = E_{in} d$

if $E_{in} < E_{out}$

$$C = \frac{Q}{\Delta V} = \frac{Q}{E_{in} d} > \frac{Q}{E_{out} d} = C_0$$

Because of dipoles and bound charges capacitance increases.



$C = K C_0$

$\downarrow$

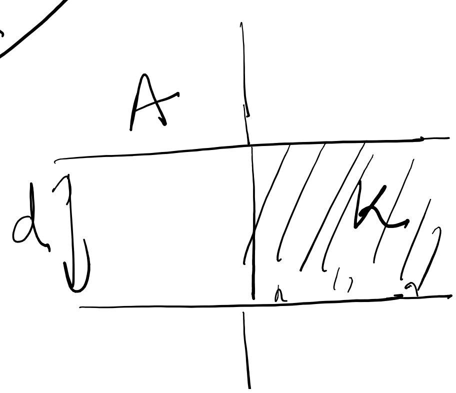
Dielectric constant of the material.

$K_{Si} \sim 10$

$K_{H_2O} \sim 80$

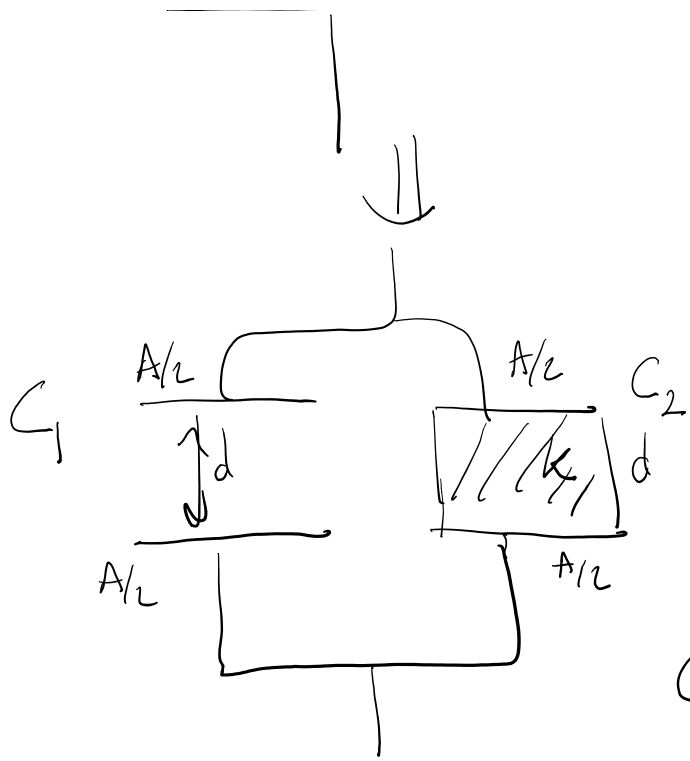
But this is good if there is no dielectric breakdown $\Longrightarrow$ Material properties are very important!

EX



What is the capacitance of the capacitor when only half of the volume is filled with dielectric K.

dielectric K.



$$C_1 = \varepsilon_0 \frac{A/2}{d}$$

$$C_2 = K \varepsilon_0 \frac{A/2}{d}$$

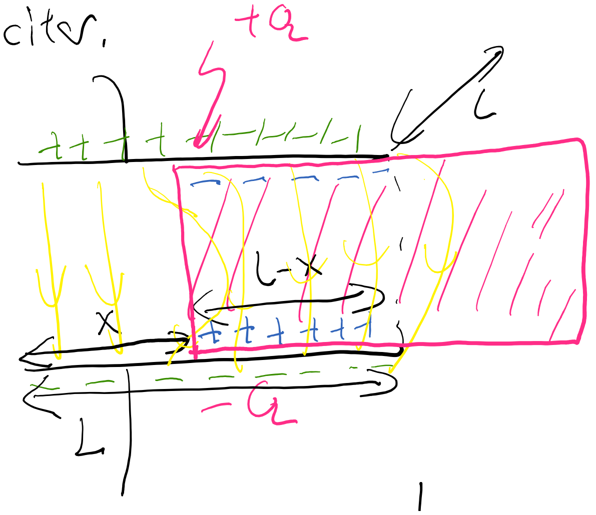
$$C = C_1 + C_2$$

$$= \varepsilon_0 \frac{A}{2d} + K \varepsilon_0 \frac{A}{2d}$$

$$\boxed{C = \varepsilon_0 \frac{A}{d} \frac{1+K}{2}}$$

Check $K = 1$ ✓

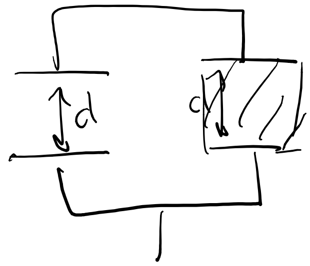
Ex. Same question! But assume that a total charge Q is on the isolated capacitor.



What is the force on the dielectric slab?

$$F = -\frac{\partial}{\partial x} U(x)$$

Capacitance



$$C_1 = \varepsilon_0 \frac{L x}{d}$$

$$C_2 = \varepsilon_0 K \frac{L(L-x)}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C_{eq}} = \frac{1}{2}\frac{Q^2}{C_1 + C_2}$$

$$= \frac{1}{2}\frac{Q^2}{\varepsilon_0 \frac{L}{d} x + \varepsilon_0 \frac{L}{d} K(L-x)}$$

$$U(x) = \frac{Q^2}{2\varepsilon_0 \frac{L}{d}}\frac{1}{[x + KL - Kx]} = \frac{Q^2}{2\varepsilon_0 \frac{L}{d}}\frac{1}{[KL + (1-K)x]}$$

$$F = -\frac{\partial}{\partial x} U(x) = -\frac{Q^2}{2\varepsilon_0 \frac{L}{d}}\frac{\partial}{\partial x}\frac{1}{(KL + (1-K)x)}$$

$$= -\frac{Q^2}{2\varepsilon_0 \frac{L}{d}}(-1)\frac{(1-K)}{(KL + (1-K)x)^2}$$

$$F = \frac{Q^2}{2\varepsilon_0 \frac{L}{d}}\frac{(1-K)}{(KL + (1-K)x)^2}$$

Force for $\vec{x} = \frac{L}{2}$ $\quad F = \frac{Q^2}{2\varepsilon_0 \frac{L}{d}}\frac{(1-K)}{(KL + \frac{L}{2} - K\frac{L}{2})^2}$

$$F = \frac{Q^2}{\quad}\frac{(1-K)}{\quad}$$

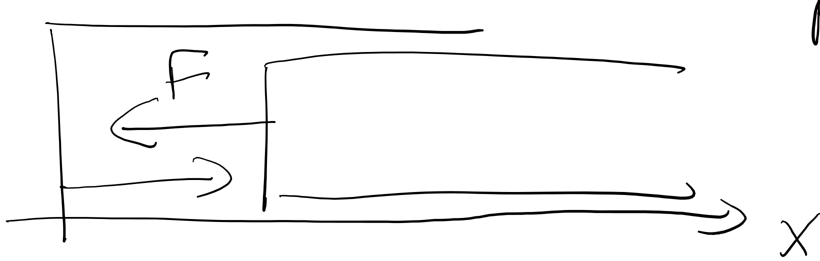
$$2\varepsilon_0 \frac{L}{d} L^2 \left(\frac{K}{2}+\frac{1}{2}\right)^2$$

$$\boxed{F = \frac{2Q^2}{\varepsilon_0 \frac{L^3}{d}} \frac{(1-K)}{(1+K)^2}}$$

$K > 1$

$(1-K) < 0$

pulling the slab



$$|F| = \frac{2Q^2}{\varepsilon_0 \frac{L^3}{d}} \frac{(K-1)}{(K+1)^2}$$

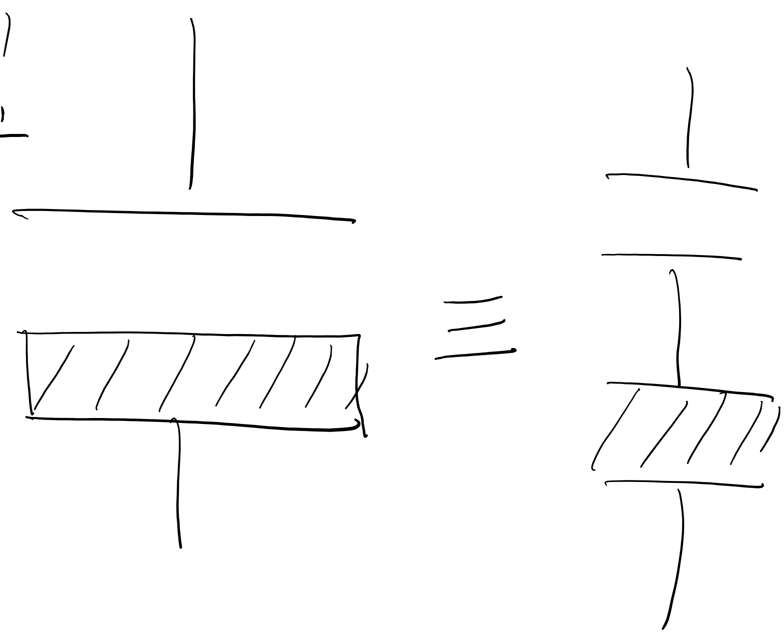
$$\frac{C^2}{F} = \frac{Q^2}{C} \rightarrow J$$

$$[F] = \frac{C^2}{\frac{F}{\cancel{m}} \frac{\cancel{m^3}}{\cancel{m}}} = \frac{J}{m} = N \checkmark$$

$C = \varepsilon_0 \frac{A}{d}$ $[K] \rightarrow 1$

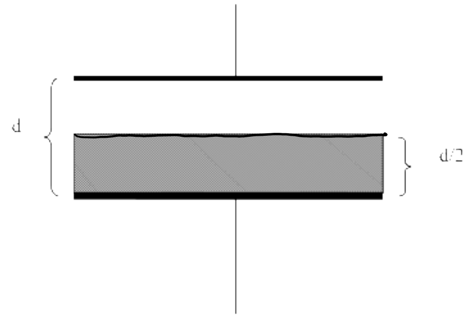
$[\varepsilon_0] \rightarrow F/m$

Quiz!

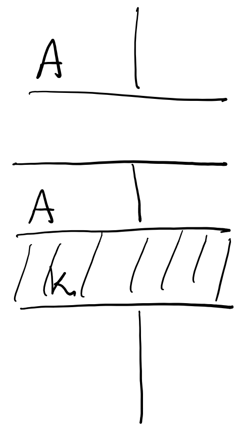


PHYSICS 102- - Instructor: M. Özgür OKTEL- Spring 2016
QUIZ-10

A parallel plate capacitor is constructed from two metal plates of area A, which are separated by a distance d. If the space in between the plates is filled halfway with a dielectric of dielectric constant κ, calculate the capacitance.



$\Rightarrow$



$$C_1 = \varepsilon_0 \frac{A}{d/2}$$

$$C_2 = K \varepsilon_0 \frac{A}{d/2}$$

$$C_{eq}^{-1} = C_1^{-1} + C_2^{-1}$$

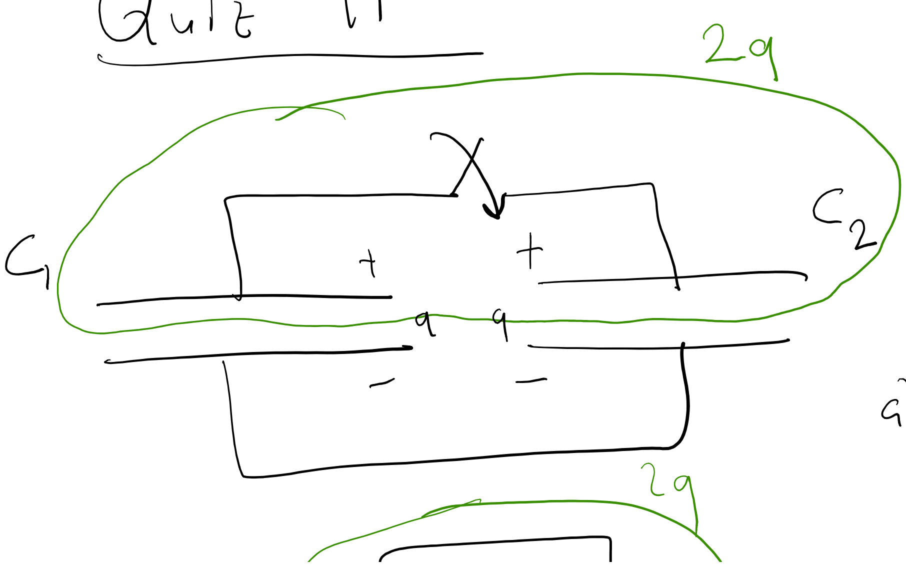
$$\frac{1}{C_{eq}} = \frac{d}{2\varepsilon_0 A} + \frac{d}{2K\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(1 + \frac{1}{K}\right)$$

$$\boxed{C_{eq} = 2\varepsilon_0 \frac{A}{d} \frac{K}{K+1}}$$

Units ✓

$$K = 1 \quad C_{eq} = \varepsilon_0 \frac{A}{d} \checkmark$$

Quiz 11



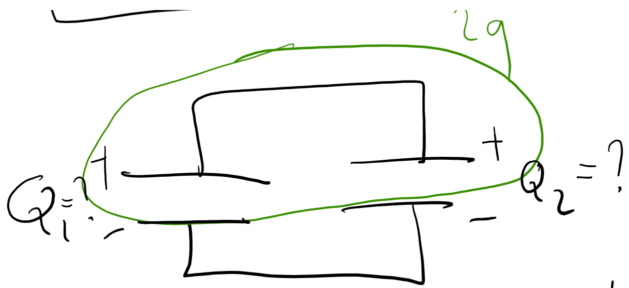
a) $Q_1 = ?$

$Q_2 = ?$

b) Energy lost!

$Q_2 = ?$

b) Energy lost!



Charges will flow until potentials accross the capacitors is the same

$$V_1 = \boxed{\frac{Q_1}{C_1} = \frac{Q_2}{C_2}} = V_2 \qquad Q_1 + Q_2 = 2q$$

$$Q_1 = \frac{C_1}{C_2} Q_2 \implies \frac{C_1}{C_2} Q_2 + Q_2 = 2q$$

$$Q_2 \left(\frac{C_1 + C_2}{C_2} \right) = 2q$$

$$\boxed{Q_1 = q \frac{2C_1}{C_1 + C_2}} \qquad \boxed{Q_2 = q \frac{2C_2}{C_1 + C_2}}$$

b) $\Delta E = ?$

$$E_i = \frac{1}{2} \frac{q^2}{C_1} + \frac{1}{2} \frac{q^2}{C_2} = \frac{q^2}{2} \left(\frac{C_1 + C_2}{C_1 C_2} \right)$$

$$E_f = \frac{1}{\cancel{2}} \frac{1}{\cancel{C_1}} \frac{\overset{2}{\cancel{4}} \cancel{C_1^{2}} q^2}{(C_1 + C_2)^2} + \frac{1}{\cancel{2}} \frac{1}{\cancel{C_2}} \frac{\overset{2}{\cancel{4}} \cancel{C_2^{2}} q^2}{(C_1 + C_2)^2}$$

$$= \frac{2 C_1 q^2}{(C_1 + C_2)^2} + \frac{2 C_2 q^2}{(C_1 + C_2)^2} = 2q^2 \frac{1}{(C_1 + C_2)}$$

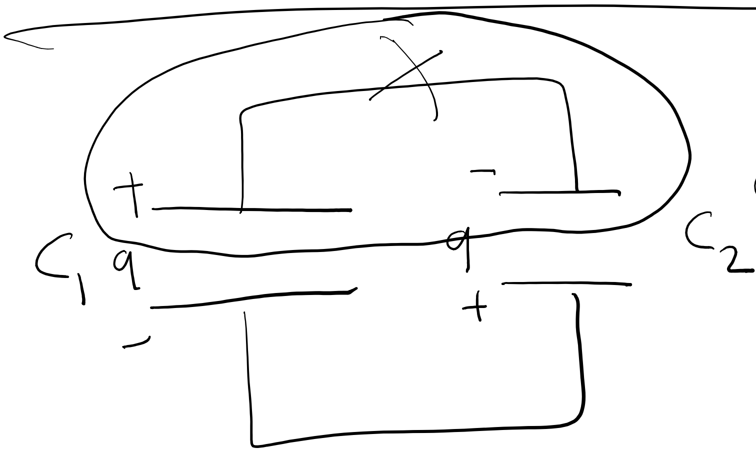
$$\Delta E = E_f - E_i = q^2 \left[\frac{2}{C_1 + C_2} - \frac{1}{2} \frac{C_1 + C_2}{C_1 C_2} \right]$$

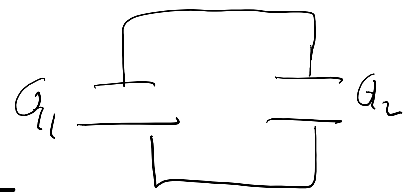
$$\Delta E = E_f - E_i = q^2\left[\frac{2}{C_1+C_2} - \frac{1}{2}\frac{C_1+C_2}{C_1C_2}\right]$$

$$= q^2\left[\frac{4C_1C_2 - (C_1+C_2)^2}{2C_1C_2(C_1+C_2)}\right]$$

$$\Delta E = -q^2\frac{(C_1-C_2)^2}{2C_1C_2(C_1+C_2)}$$

$$[\Delta E] = C^2\frac{\cancel{F}}{F^{\cancel{2}}} = J$$ ✓



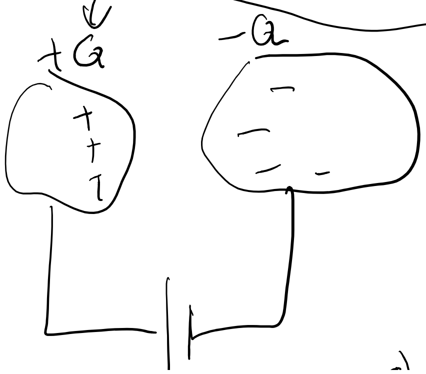


$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad \frac{Q_1}{C_1} = \frac{-Q_1}{C_2}$$

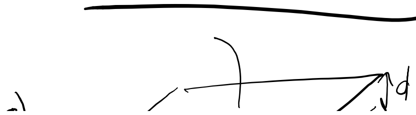
$$Q_1 + Q_2 = 0 \qquad Q_1 = 0$$

$$Q_1 = -Q_2$$

Capacitors

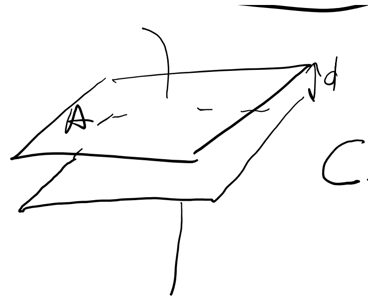


$$\boxed{\frac{Q}{\Delta V} = C} \qquad \boxed{C > 0}$$



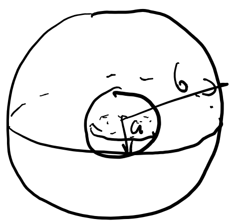
$[C] = \text{farad} = \frac{C}{V}$

1°)



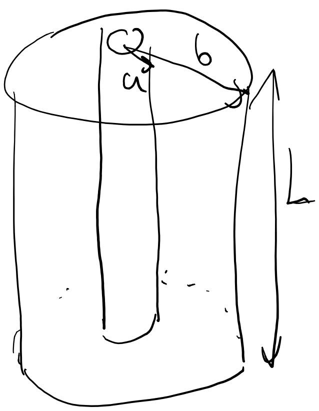
$C = \varepsilon_0 \frac{A}{d}$

2°)



$C_{sphere} = ?$

3°)



$C_{cylindrical} = ?$

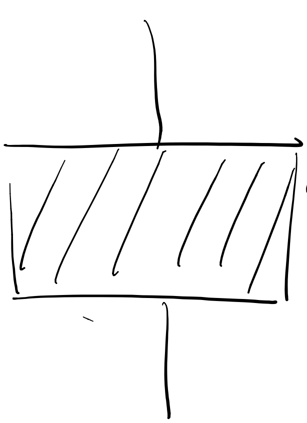
$E = \frac{1}{2} C V^2 = \frac{1}{2}\frac{Q^2}{C}$

$\boxed{F = -\frac{\partial E}{\partial x}}$

PHYS 101

$\vec{E} \Rightarrow$ stores energy

$u = \frac{1}{2}\varepsilon_0 |\vec{E}|^2$ per unit volume



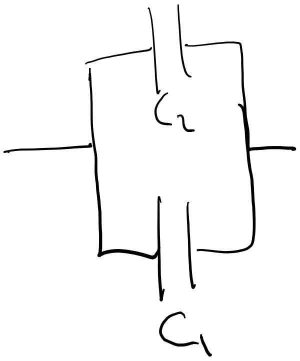
dielectrics

dielectric constant

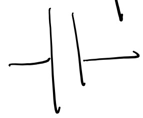
$C = K C_0$

material property

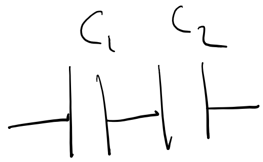
Parallel and Series Capacitors



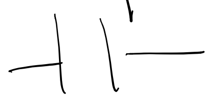
$\equiv$



$$C_{eq} = C_1 + C_2$$



$\equiv$

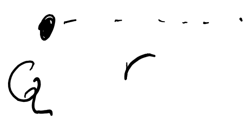


$$C_{eq}^{-1} = C_1^{-1} + C_2^{-1}$$

Electric Potential & Potential Energy

$$U(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$



$$U(r) = V(r)\, q$$

$V(r)$ ↓ electric potential

$$E_x = -\frac{\partial}{\partial x} V(x,y,z)$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$E_y = -\frac{\partial}{\partial y} V(x,y,z)$$

$$E_z = -\frac{\partial}{\partial z} V(x,y,z)$$

$$\Delta V = -\int_i^f \vec{E} \cdot \vec{dl}$$

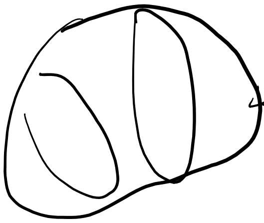
Generally $V(\infty) = 0$
unless there are
charges at ∞!

$V \rightarrow$ scalar $\quad [V] = \frac{J}{C} = $ Volt

Gauss' Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

charge inside



1°) Spherical Symmetry

2°) Cylindrical Symmetry

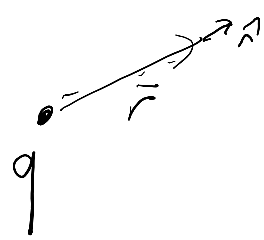
3°) Planar symmetry.

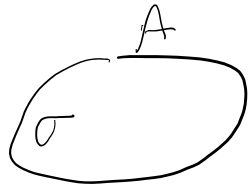
The charge distribution may be non-uniform !!

Conductor $\Longrightarrow$ $\vec{E}$ is 0 inside so charges are on the surface!

Coulomb's Law

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$





$$\lambda = \frac{Q}{L} \qquad \sigma = \frac{Q}{A} \qquad \rho = \frac{Q}{V}$$

Charge is conserved quantized