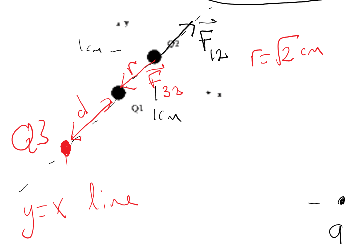
Week 2 Notes (Feb 1- 5)

Wednesday, January 27, 2016 3:13 PM

PHYSICS 102- Spring 2016- Instructor: M. Özgür OKTEL

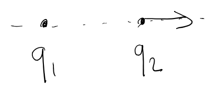
QUIZ-1

Charge Q1=+1 C is at the origin of the coordinates x=0,y=0. Charge Q2=+0.75 C is at x=1cm, y= 1cm. Find where a charge Q3=-4 C should be placed so that the second charge Q2 is in equilibrium. (To be in equilibrium the net force on Q2 must be zero. Find the x and y coordinates of Q3.)



$\Downarrow$

Net force on Q_2 should be zero!



$$|\vec{F}_{12}| = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

$$|\vec{F}_{32}| = \frac{1}{4\pi\varepsilon_0}\frac{|Q_3|\, Q_2}{(r+d)^2}$$

$$|\vec{F}_{12}| = |\vec{F}_{32}|$$

$$\cancel{\frac{1}{4\pi\varepsilon_0}}\frac{Q_1 \cancel{Q_2}}{r^2} = \cancel{\frac{1}{4\pi\varepsilon_0}}\frac{|Q_3|\,\cancel{Q_2}}{(r+d)^2}$$

$$\frac{1\,C}{r^2} = \frac{4\,C}{(r+d)^2}$$

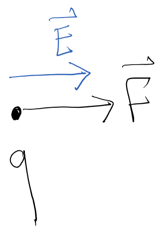
$$\frac{r+d}{r} = \sqrt{4} = 2$$

$$\boxed{d = r}$$

$$\boxed{\begin{aligned} x_{Q3} &= -1 \text{ cm} \\ y_{Q3} &= -1 \text{ cm} \end{aligned}}$$

What would change if $\boxed{Q_2 = -5\,C}$

Electric Field



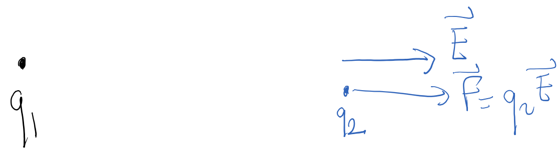
$$\boxed{\vec{E} = \frac{\vec{F}}{q}}$$

$$[\vec{E}] = N/C$$

Coulomb's Law

"Action at a distance"

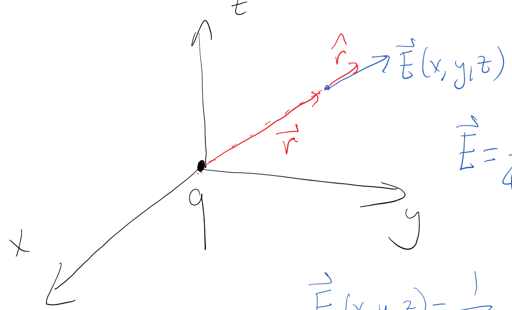
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$



$\vec{E} \to$ is the electric field
"real?"

can store and carry
→ energy
→ momentum
→ angular momentum

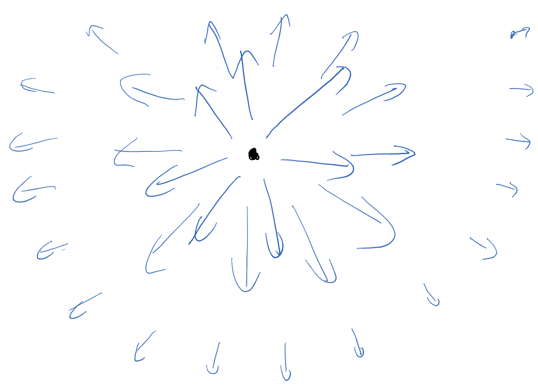
$\vec{E} \to$ is a vector



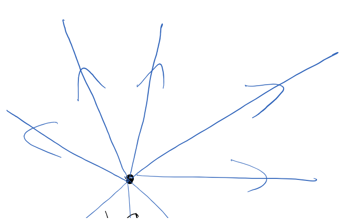
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$\underbrace{\vec{E}(x,y,z)}_{\text{vector}} = \underbrace{\underbrace{\frac{1}{4\pi\epsilon_0}\frac{q}{(x^2+y^2+z^2)}}_{\text{scalar}}\underbrace{\hat{r}}_{\text{direction}}}_{\text{vector}}$$

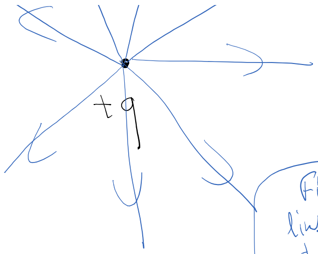
$\vec{E}(x,y,z) \to$ vector at each space point.



Electric field lines



1°) Electric field lines start on (+) charges or at ∞. They end up on (−) charges or at ∞.

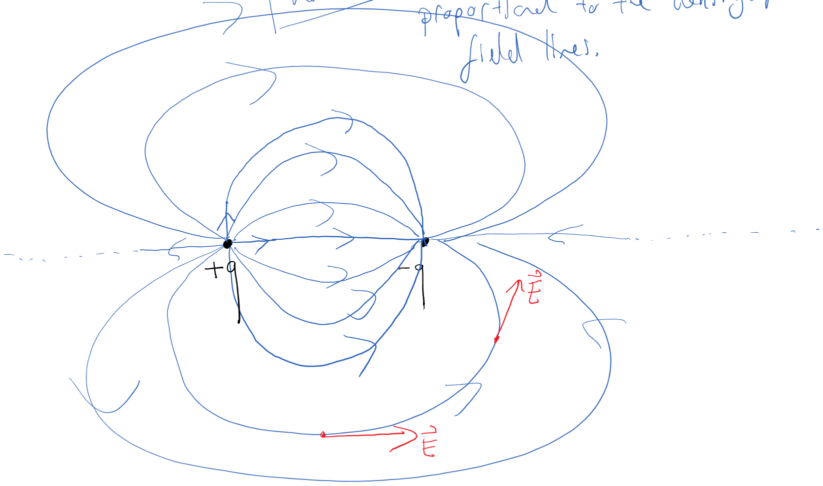


at ∞. They end up on (−) charges or at ∞.

2°) Electric field at any point is tangent to the lines.

Field lines do not cross

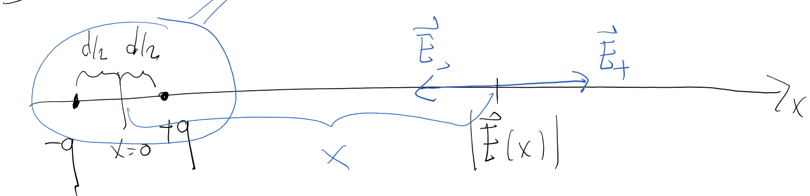
3°) Electric field magnitude is proportional to the density of field lines.



Calculating the electric field of a collection of charges.

Quiz

dipole



$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$|\vec{E}| = |\vec{E}_+| - |\vec{E}_-|$$

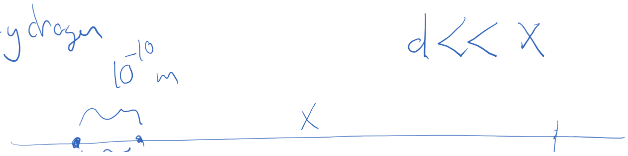
$$|\vec{E}_+| = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x-\frac{d}{2}\right)^2}$$

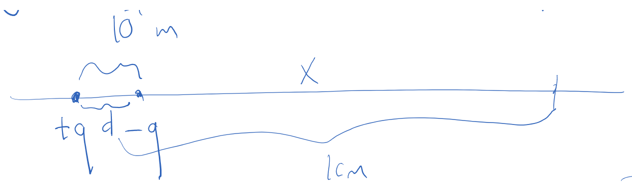
$$|\vec{E}_-| = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x+\frac{d}{2}\right)^2}$$

$$|\vec{E}(x)| = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(x-\frac{d}{2}\right)^2} - \frac{1}{\left(x+\frac{d}{2}\right)^2}\right]$$

Hydrogen

$d \ll x$





$$\vec{E}(x) = \frac{q}{4\pi\varepsilon_0}\frac{1}{x^2}\left[\frac{1}{\left(1-\frac{d}{2x}\right)^2} - \frac{1}{\left(1+\frac{d}{2x}\right)^2}\right]$$

$\varepsilon \ll 1$ $\qquad \varepsilon = 0.0001$

$$(1+\varepsilon)^n = 1 + n\varepsilon + \cancel{\binom{n}{2}\varepsilon^2 + \ldots}$$

$$(1+\varepsilon)^n \cong 1 + n\varepsilon$$

$$(1+0.0001)^5 \cong 1 + 0.0005$$

$$|\vec{E}(x)| \cong \frac{q}{4\pi\varepsilon_0 x^2}\left[\left(\cancel{1} - 2\left(-\frac{d}{2x}\right)\right) - \left(\cancel{1} - 2\frac{d}{2x}\right)\right]$$

$$\cong \frac{q}{4\pi\varepsilon_0 x^2}\left[\frac{d}{x} + \frac{d}{x}\right]$$

$qd = p \rightarrow$ the dipole moment

$$|\vec{E}(x)| \cong \frac{qd}{2\pi\varepsilon_0 x^3}$$

$$= \frac{p}{2\pi\varepsilon_0 x^3}$$

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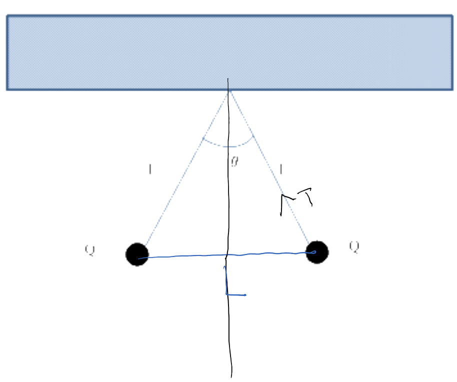
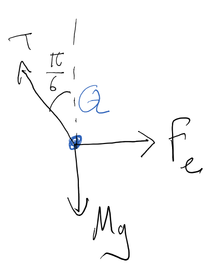
QUIZ-2

Two beads of mass M are hanging from the ceiling by non-conducting strings of length L. The beads carry equal charges Q and in equilibrium the angle between the ropes is $\theta = \frac{\pi}{3}$. Find Q in terms of L,M, gravitational acceleration g and universal constants.





$$Mg = T\cos\frac{\pi}{6}$$

$$F_e = T\sin\frac{\pi}{6}$$

$$\frac{F_e}{Mg} = \tan\frac{\pi}{6} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{L^2} = Mg\tan\frac{\pi}{6}$$

$$Q^2 = 4\pi\varepsilon_0 L^2 Mg\frac{1}{\sqrt{3}}$$

$$\boxed{Q = \sqrt{\frac{4\pi\varepsilon_0 Mg}{\sqrt{3}}}\, L}$$

1°) Goal ✓
2°) Units ✓
3°) Limits
$Q \to 0 \Rightarrow L \to 0$ ✓
$Mg \to \infty$ $Q \to \infty$ ✓

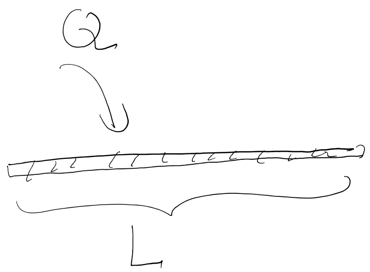
$$[\varepsilon_0] = \frac{C^2}{Nm^2} \qquad [Mg] = N$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}$$
$$[\varepsilon_0] = \frac{C^2}{Nm^2}$$

$$[\varepsilon_0 Mg] = \frac{C^2}{m^2}$$

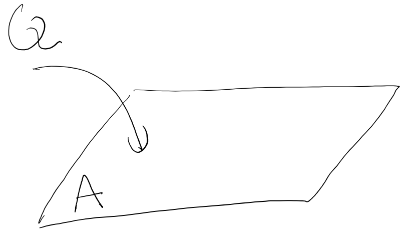
$$\Big[\underbrace{\sqrt{\varepsilon_0 Mg}}_{C/m}\ \underbrace{L}_{m}\Big] = C \ ✓$$

Charge Distributions



If Q is uniformly distributed on the rod of Length L

$$\lambda = \frac{Q}{L} \qquad [\lambda] \to C/m$$



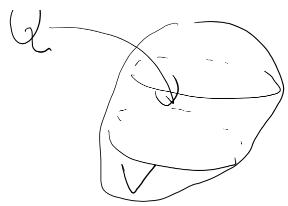
2D charge density

$$\sigma = \frac{Q}{A} \qquad [\sigma] \to C/m^2$$

3D charge density

$$Q \qquad [\rho] \to C/m^3$$

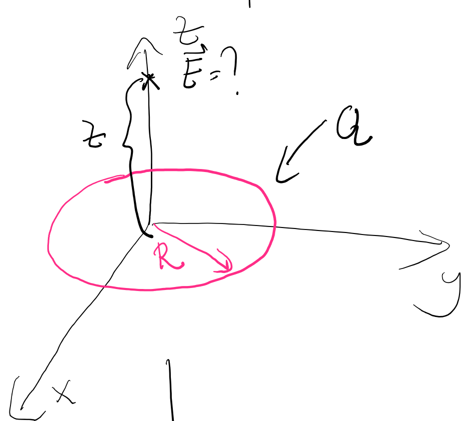
3D charge ...



$$\rho = \frac{Q}{V} \qquad [\rho] \to C/m^3$$

Ex

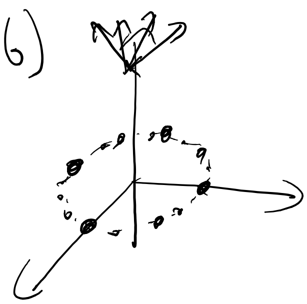
We have a ring of radius R charged with total charge Q. Find the electric field on the axis that is passing through the center of the ring.



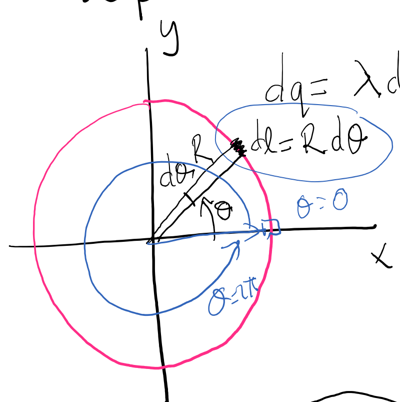
a) $\lambda = ?$

$$\lambda = \frac{Q}{2\pi R}$$

b)

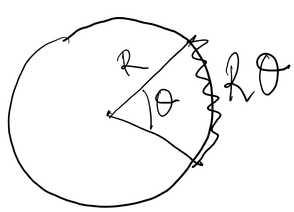


top view

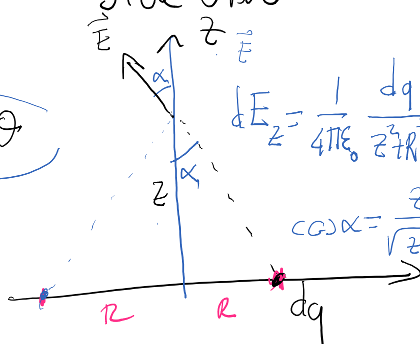


$dq = \lambda dl = \lambda R d\theta$

$dl = R d\theta$



side view



$$dE_z = \frac{1}{4\pi\epsilon_0} \frac{dq}{z^2+R^2} \cos\alpha$$

$$\cos\alpha = \frac{z}{\sqrt{z^2+R^2}}$$

$\vec{E}$ is // to $\hat{z}$

by symmetry!

$$dE_z = \frac{1}{4\pi\epsilon_0} \frac{dq}{z^2+R^2} \frac{z}{\sqrt{z^2+R^2}}$$

$$dE_z = \frac{1}{4\pi\varepsilon_0} \frac{z}{(z^2+R^2)^{3/2}} \lambda R \, d\theta$$

$$E_z = \int dE_z = \int_0^{2\pi} \frac{z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \lambda R \, d\theta$$

$$= \frac{z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \lambda R \underbrace{\int_0^{2\pi} d\theta}_{\theta\big|_0^{2\pi} = 2\pi}$$

$\int dx = x$

$$E_z = \frac{z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \underbrace{\lambda R \, 2\pi}_{Q}$$

$$\boxed{E_z = \frac{Q}{4\pi\varepsilon_0} \frac{z}{(z^2+R^2)^{3/2}}}$$

1°) Goal ✓

2°) Units ✓

$$[E] = \frac{[q]}{[\varepsilon_0][r^2]}$$

$$\left[\frac{z}{(z^2+R^2)^{3/2}}\right] = \frac{m}{m^{2\cdot\frac{3}{2}}} = \frac{1}{m^2}$$

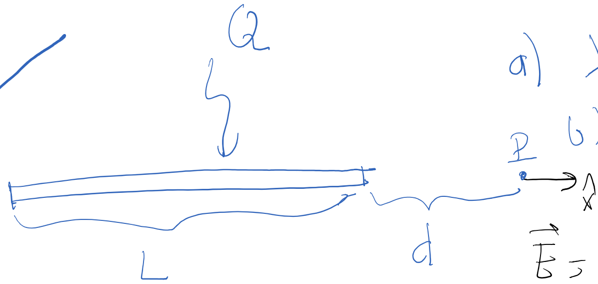
3°) Limits

$z = 0 \qquad E_z = 0$ ✓

$R \ll z \qquad E_z \simeq \frac{Q}{4\pi\varepsilon_0} \frac{z}{z^{2\cdot\frac{3}{2}}} = \frac{Q}{4\pi\varepsilon_0 z^2}$ ✓

point charge

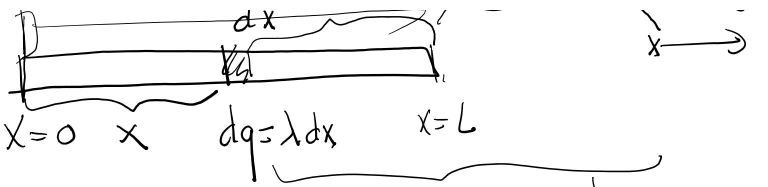
Ex



a) $\lambda = ?$ $\quad \lambda = \frac{Q}{L}$

b) What is the electric field at point P?

$\vec{E} = |\vec{E}| \, \hat{x}$ by geometry



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(d+(L-x))^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(d+L-x)^2}$$

$$E = \int dE = \int_0^L \frac{\lambda}{4\pi\varepsilon_0 (d+L-x)^2}\, dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{1}{(d+L-x)^2}\, dx$$

$y = d+L-x$

$x=L \Rightarrow y=d$

$x=0 \Rightarrow y=d+L$

$dy = -dx$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_{d+L}^{d} \frac{1}{y^2}(-dy)$$

$$\int y^n\, dy = \frac{y^{n+1}}{n+1}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_{d}^{d+L} \frac{1}{y^2}\, dy$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left.\frac{y^{-1}}{-1}\right|_d^{d+L} = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{d} - \frac{1}{L+d}\right]$$

$$\boxed{|\vec{E}| = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{d} - \frac{1}{L+d}\right]}$$

1°) Goal ✓ 2°) Units ✓ $\left[\frac{1}{L}\left[\frac{1}{d} - \frac{1}{L+d}\right]\right] = \frac{1}{m^2}$

3°) Limits $d \to \infty$ $d \gg L$

$$|\vec{E}| \cong \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{d} - \frac{1}{d}\frac{1}{(1+\frac{L}{d})}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{d} - \frac{1}{d}\left(1 - \frac{L}{d}\right)\right]$$

$$|\vec{E}| \simeq \frac{Q}{4\pi\varepsilon_0 d^2}$$ point charge

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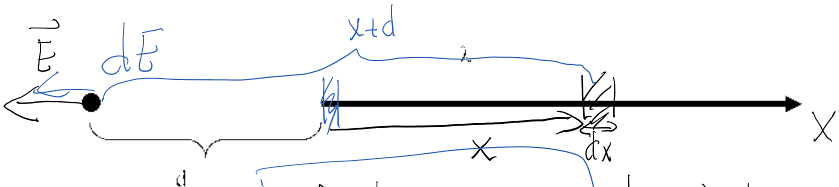
QUIZ-3

A rod of infinite length is placed on the x axis from x=0 to infinity. If the rod is charged uniformly with line charge density λ, find the electric field at point x -d. Give both the magnitude and the direction of the electric field.



Geometry

$$\boxed{\vec{E} = |\vec{E}|(-\hat{i})}$$

$$dq = \lambda\, dx$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(x+d)^2}$$

$$E = \int dE = \int_0^\infty \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x+d)^2}\,dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_0^\infty \frac{1}{(x+d)^2}\,dx$$

$u = x+d$ $\quad x=\infty \rightarrow u=\infty$ $\quad x=0 \rightarrow u=d$

$du = dx$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_d^\infty \frac{1}{u^2}\,du$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{-1}{u}\right]_d^\infty = \boxed{\frac{\lambda}{4\pi\varepsilon_0 d}}$$

1°) Goal

2°) $[\lambda] = C/m$

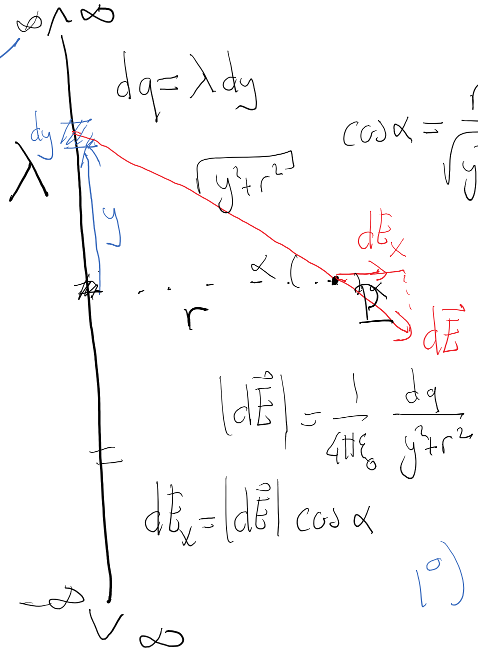
$$\left[\frac{\lambda}{d}\right] = \frac{C}{m^2}$$

$$\left[\frac{1}{\varepsilon_0}\right]\frac{C}{m^2} = \frac{N}{C}$$

3°) Limits $d \to \infty$ $\quad E \to 0$

3°) Limits $d \to \infty$ $E \to 0$ ✓ $\left[\frac{1}{\varepsilon_0}\right] \frac{C}{m^2} = \frac{N}{C}$ ✓

Ex



$dq = \lambda\, dy$

$\cos\alpha = \frac{r}{\sqrt{y^2+r^2}}$

Find the electric field a distance r away from an ∞ uniformly charged line.

1°) Direction?

2°) Magnitude?

$|d\vec{E}| = \frac{1}{4\pi\varepsilon_0} \frac{dq}{y^2+r^2}$

$dE_x = |d\vec{E}| \cos\alpha$

1°) By symmetry $\vec{E}$ field is perpendicular to the line $\vec{E} = |\vec{E}|\,\hat{i}$

$$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dy}{y^2+r^2} \frac{r}{(y^2+r^2)^{1/2}}$$

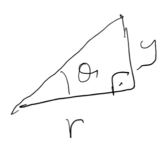
$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda r}{(y^2+r^2)^{3/2}}\, dy$$

$$E_x = \int dE_x = \int_{-\infty}^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{\lambda r}{(y^2+r^2)^{3/2}}\, dy$$

$$= \frac{\lambda r}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{1}{(y^2+r^2)^{3/2}}\, dy$$

$y = +\infty \Rightarrow \theta = \frac{\pi}{2}$

$y = -\infty \Rightarrow \theta = -\frac{\pi}{2}$



$\tan\theta = \frac{y}{r}$

$\frac{1}{\cos^2\theta}\, d\theta = \frac{1}{r}\, dy$

$$= \frac{\lambda r}{4\pi\varepsilon_0} \int^{\pi/2} \frac{1}{[\;\;]^3} \frac{r}{\cos^2\theta}\, d\theta$$

$\cos\theta = \frac{r}{\sqrt{r^2+y^2}}$

$dy = \frac{r}{\cos^2\theta}\, d\theta$

$$= \frac{\lambda r}{4\pi\varepsilon_0}\int_{-\frac{\pi}{2}} \frac{1}{\left[\frac{r}{\cos\theta}\right]^3}\frac{1}{\cos^2\theta}\,d\theta$$

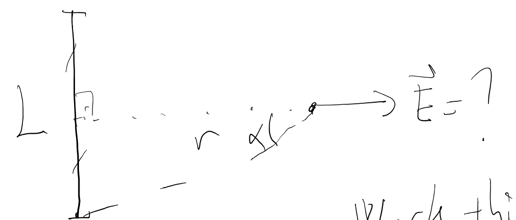
$\sqrt{r^2+y^2}$ $\qquad (r^2+y^2)^{1/2} = \frac{r}{\cos\theta}$ $\qquad dy = \frac{1}{\cos^2\theta}\,d\theta$

$$= \frac{\lambda \cancel{r}}{4\pi\varepsilon_0}\frac{1}{r^{\cancel{2}}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\overset{\cos\theta}{\cancel{\cos^3\theta}}}{\cancel{\cos^2\theta}}\,d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\underbrace{\sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}_{\underbrace{1-(-1)}_{2}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 r}}$$

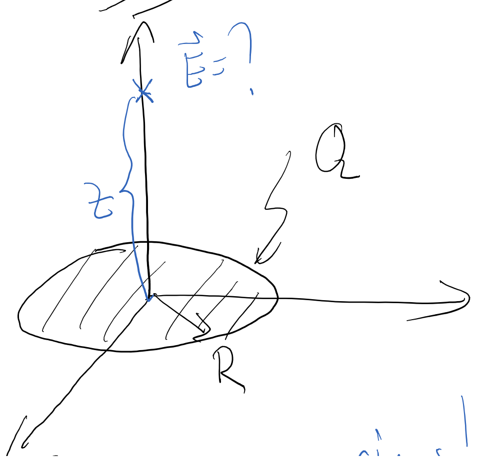
1°) Goal ✓
2°) Units ✓
3°) Limit ✓
$r \to \infty$
$E \to 0$

Question



Work this out yourself.

Find the electric field on the axis of a uniformly charged disc of radius R and total charge Q.



1°) Direction?

By symmetry $\vec{E} = |\vec{E}|\,\hat{k}$

2°) $\sigma = \frac{Q}{\pi R^2}$

Break up the disc into

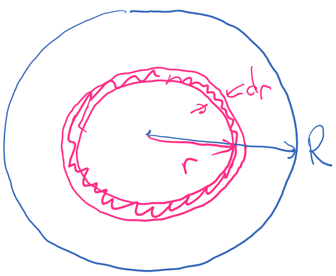
R Break up the disc ...

rings!

$$E_z = \frac{Q}{4\pi\varepsilon_0} \frac{z}{(z^2+R^2)^{3/2}}$$

Electric field of a ring of radius R and charge Q

Top view



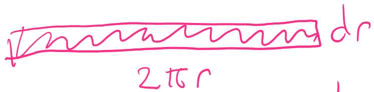
small charge in the ring

$$dq = \sigma\, dA$$

$$dA = \pi(r+dr)^2 - \pi r^2$$

$$= \cancel{\pi r^2} + \underbrace{2\pi r\, dr} + \cancel{\pi (dr)^2} - \cancel{\pi r^2}$$

$(dr)^2$ is ~~negligible~~, really zero!



$$dq = \sigma\, 2\pi r\, dr$$

$$dE_z = \frac{dq}{4\pi\varepsilon_0} \frac{z}{(z^2+r^2)^{3/2}} = \frac{\sigma\, 2\pi r\, z}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}}\, dr$$

$$E_z = \int dE_z = \int_0^R \frac{\sigma\, \cancel{2\pi} r\, z}{\cancel{4}\pi\varepsilon_0 (z^2+r^2)^{3/2}}\, dr$$

$$= \frac{\sigma z}{2\varepsilon_0} \int_0^R \frac{r}{(z^2+r^2)^{3/2}}\, dr$$

$$\int^{z^2+R^2}$$

$$u = z^2 + r^2 \qquad du = 2r\, dr$$

$$E_z = \frac{\sigma z}{2\varepsilon_0}\int_{z^2}^{z^2+R^2} \frac{\frac{1}{2}}{u^{3/2}}\,du$$

$u = z^2 + r^2 \qquad du = 2r\,dr$

$\int u^n\,du = \frac{u^{n+1}}{n+1}$

$$= \frac{\sigma z}{4\varepsilon_0}\left.\frac{u^{-1/2}}{-1/2}\right|_{z^2}^{z^2+R^2}$$

$$\boxed{E_z = \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2+R^2}}\right]}$$

$$= \frac{Q\,z}{2\pi R^2\,\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

1°) Goal ✓

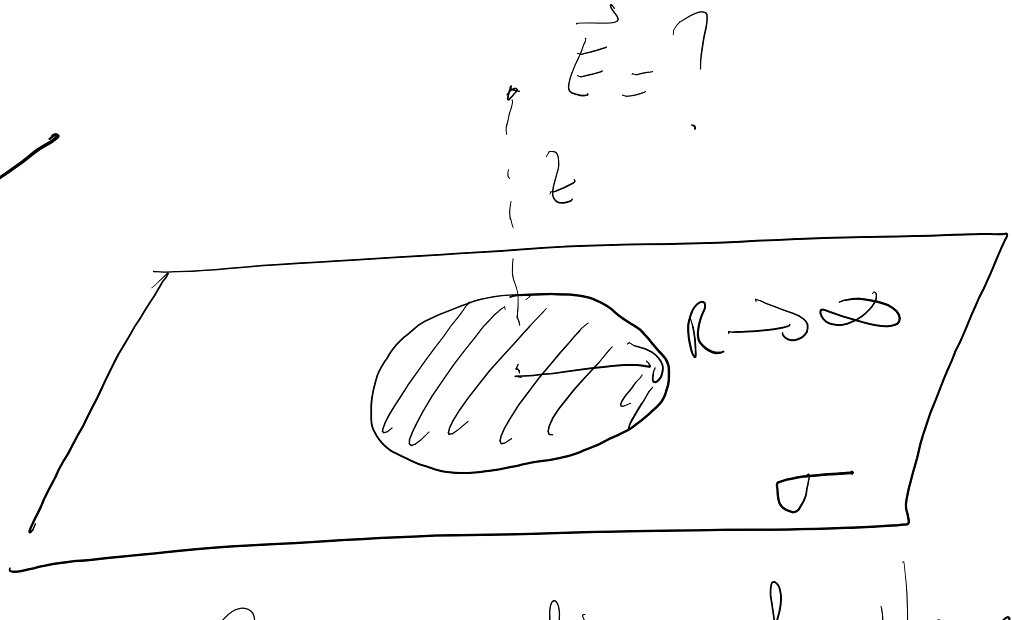
2°) Units ✓

3°) Limits.

$z \gg R$

point charge ✓

Ex



Infinite plane

σ

... $R \to \infty$ limit of the previous problem!

Just $R \to \infty$ limit of the previous problem!

$$E_z = \lim_{R\to\infty} \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

$$= \frac{\sigma \cancel{z}}{2\varepsilon_0}\frac{1}{\cancel{z}} = \boxed{\frac{\sigma}{2\varepsilon_0}}$$