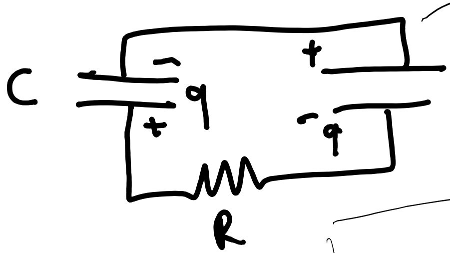
Week 9 Notes (Mar 21-25)

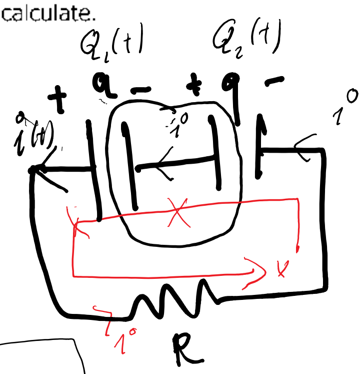
Wednesday, January 27, 2016 4:47 PM

QUIZ-14

Two capacitors $C_1 = C$ and $C_2 = C$ initially have equal charges q, but have ~~opposite polarity (see figure).~~ There is also a resistance R between the capacitors. After the switch between them is closed at t=0, calculate.

a) The current in the circuit as a function of time i(t).
b) Plot the charge on C1 as a function of time.
c) What is the time constant of this circuit?





$$\boxed{i(t) = -\frac{dQ_1(t)}{dt}} = -\frac{dQ_2}{dt} \Rightarrow Q_1 = Q_2$$

$$\boxed{+\frac{Q_2(t)}{C} + \frac{Q_1(t)}{C} - i(t)R = 0}$$

$$\frac{2Q_1(t)}{RC} = i(t)$$

$$\frac{2}{RC} Q_1(t) = -\frac{dQ_1(t)}{dt}$$

$$Q_1(t) = A e^{\alpha t} \Rightarrow \frac{dQ_1}{dt} = \alpha A e^{\alpha t}$$

$$\frac{2}{RC} A \cancel{e^{\alpha t}} = -\alpha A \cancel{e^{\alpha t}} \Rightarrow \boxed{\alpha = -\frac{2}{RC}}$$

$$Q(t=0) = q = A e^{0} \Rightarrow A = q$$

$$\boxed{Q_1(t) = q\, e^{-\frac{2t}{RC}}}$$

$$i(t) = -\frac{dQ_1}{dt}$$

$$\boxed{i(t) = \frac{2q}{RC} e^{-\frac{2t}{RC}}}$$

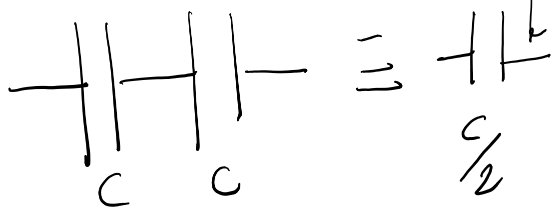
$$i(t) = \frac{2q}{RC} e^{\;RC}$$

current is counterclockwise

c) time constant

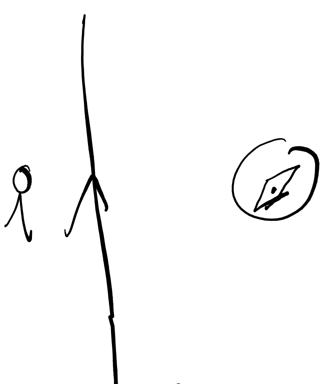
$\tau = ?$ $\qquad e^{-t/\tau}$

$$\boxed{\tau = \frac{RC}{2}}$$



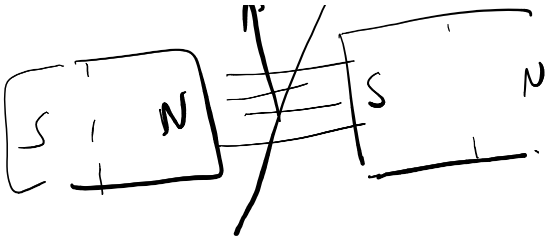
Manisa → Magnesia ⟶ Magnets

$\boxed{\text{Magnetism}}$

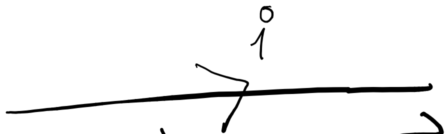


Currents create magnetic fields

Magnetic field applies a force on



applies a force on currents!



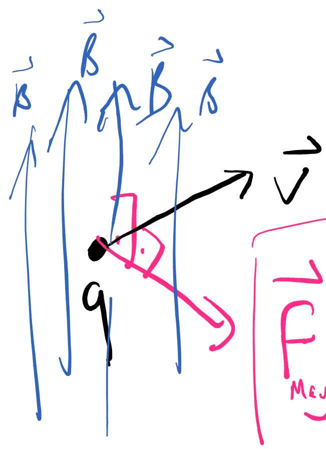
$\vec{E} \longleftrightarrow \vec{B}$

Uniform

$\boxed{\text{Electromagnetic Field}}$

Forces on a moving charge

Observation

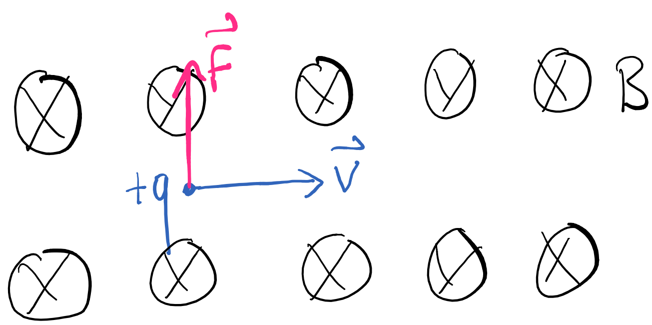


$$[B] = \frac{[F]}{[q][v]} = \frac{N}{C\,m/s} = \frac{Ns}{Cm} = \boxed{\text{Tesla}}$$

$B_{earth} = 1\,\text{Gauss}$

$\sim 10^{-4}$ tesla

$$\boxed{\vec{F}_{Mag} = q\,\vec{v} \times \vec{B}}$$



⊗ into

⊙ out of.

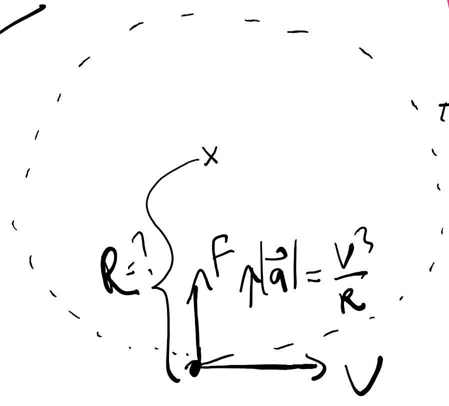
$$\vec{F} = q\,\vec{v} \times \vec{B}$$

$$P = \vec{F}\cdot\vec{v} = q\underbrace{(\vec{v}\times\vec{B})\cdot\vec{v}}_{0\ \ \vec{F}\perp\vec{v}} \Rightarrow$$ Magnetic force does no work!

EX

Ex

$① \ \vec{F} \perp \vec{v}$ does no work!

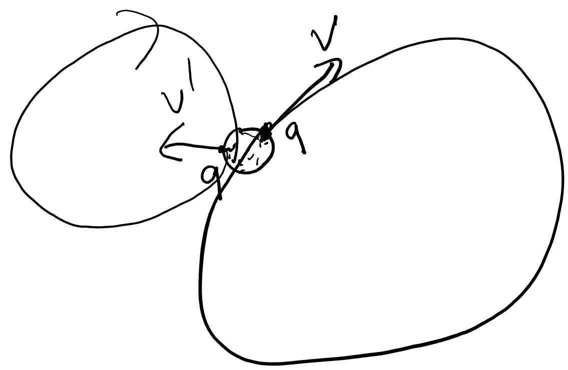


$T = ?$ $\quad m|\vec{a}| = |\vec{F}|$

$$m \frac{v^{\cancel{2}}}{R} = q \cancel{v} B$$

$$\boxed{R = \frac{mv}{qB}}$$

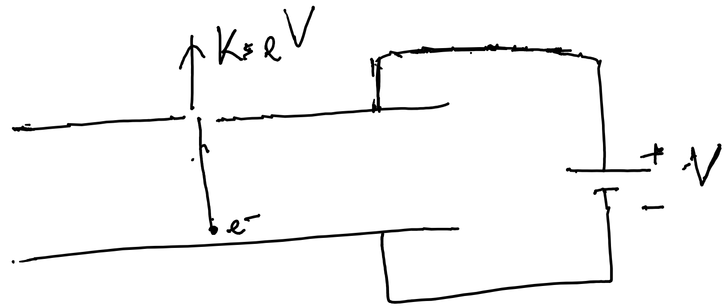
$$T = \frac{2\pi R}{v} = \frac{2\pi m \cancel{v}}{\cancel{v} qB} = \frac{2\pi}{\left(\frac{qB}{m}\right)} = \frac{2\pi}{\omega_c}$$

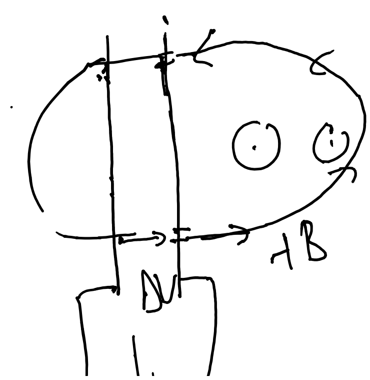


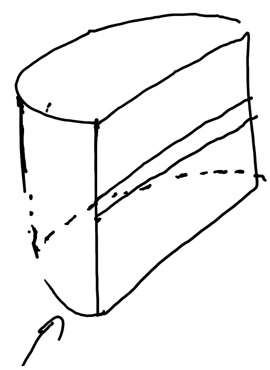
$$\omega_c = \frac{qB}{m}$$

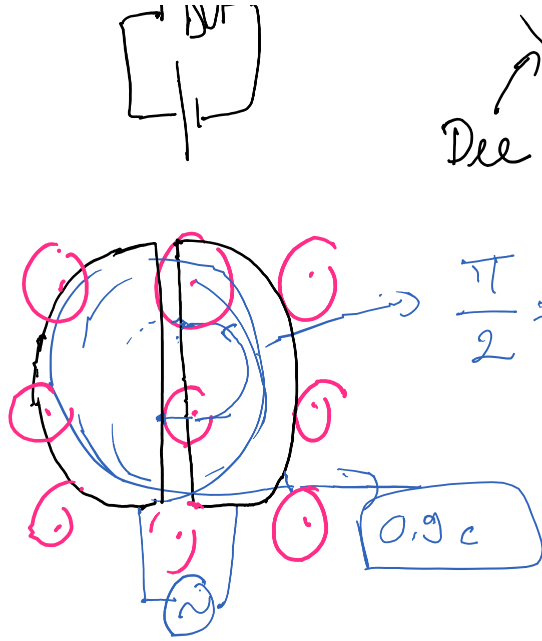
$\underbrace{\text{Cyclotron}}$ frequency

First particle accelerator.









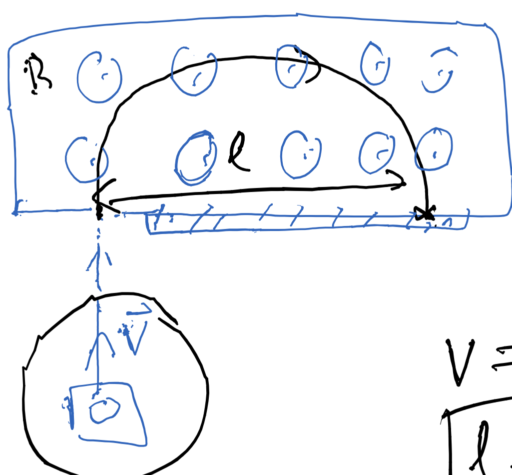
$$\frac{T}{2} = \frac{\pi}{qB/m}$$

$$F = q \cdot \vec{v} \times \vec{B}$$

Lorentz force

Ex

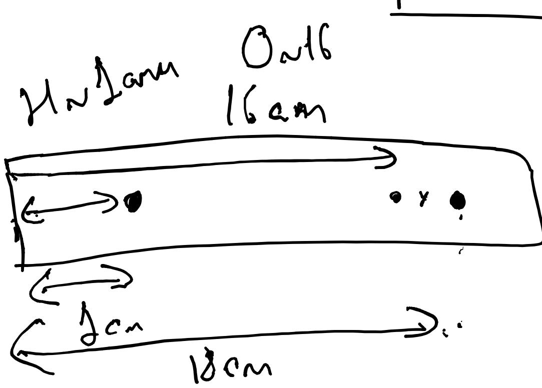
Mass spectrometer



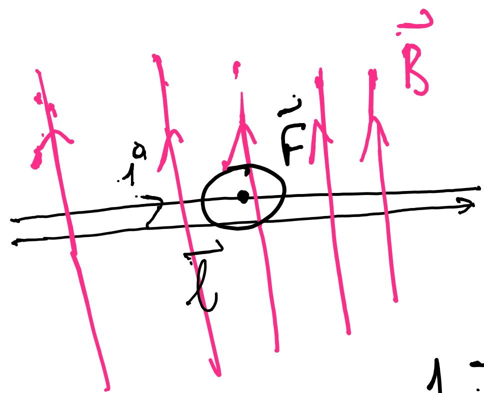
$$l = \frac{2mv}{qB}$$

$V \Rightarrow$ constant

$$\boxed{l \propto m}$$



Force on a current carrying wire

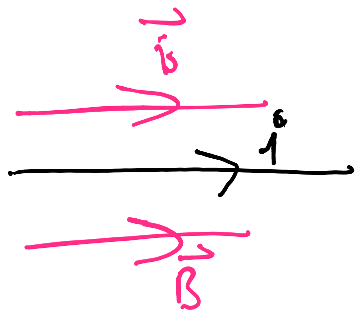


straight wire

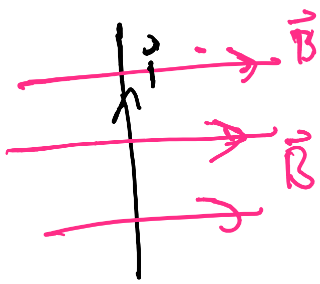
$$\vec F = i\,\vec \ell \times \vec B$$

SI of $|\vec B|$

1 Tesla of magnetic field creates 1 N of force on 1 m of wire carrying 1 A current.



$\vec F = 0$



$\vec F$ ⊗

$$\boxed{d\vec F = i\, d\vec \ell \times \vec B}$$

Force on the diff. segment $d\vec \ell$

$$\vec F = \int d\vec F$$

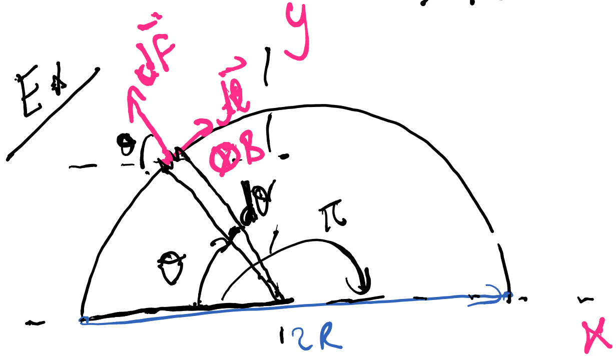
⊗ $\vec B$ uniform

Ex

$\otimes \vec{B}$ uniform

What is the direction and magnitude of the total force?

Ex



By symmetry total force will be along $\hat{y}$

$$dl = R\,d\theta \qquad \vec{dl} \times \vec{B} = dl\,B$$
$$= R\,B\,d\theta$$

$$|d\vec{F}| = i\,|\vec{dl} \times \vec{B}| = i\,R\,B\,d\theta$$

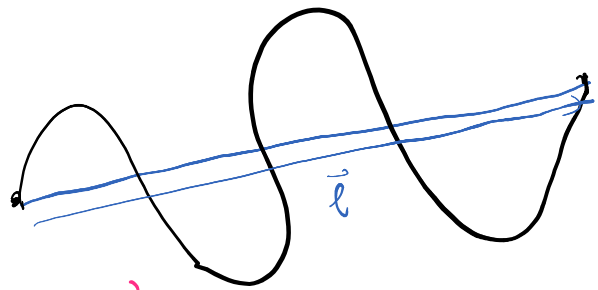
$$dF_y = |d\vec{F}|\sin\theta = i\,R\,B\sin\theta\,d\theta$$

$$F_y = \int dF_y = \int_0^{\pi} i\,R\,B\sin\theta\,d\theta$$

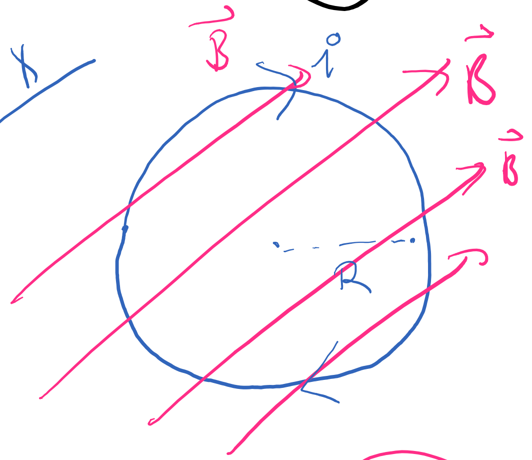
$$= i\,R\,B\int_0^{\pi}\sin\theta\,d\theta$$

$$= i\,R\,B\,\underbrace{\Big[-\cos\theta\Big]_0^{\pi}}_{1+1} = \boxed{i\,2RB}$$

in the $\hat{j}$ direction



Ex



$$\vec{F}_{net} = \int d\vec{F}$$

$$= \int i\, d\vec{l} \times \vec{B}$$

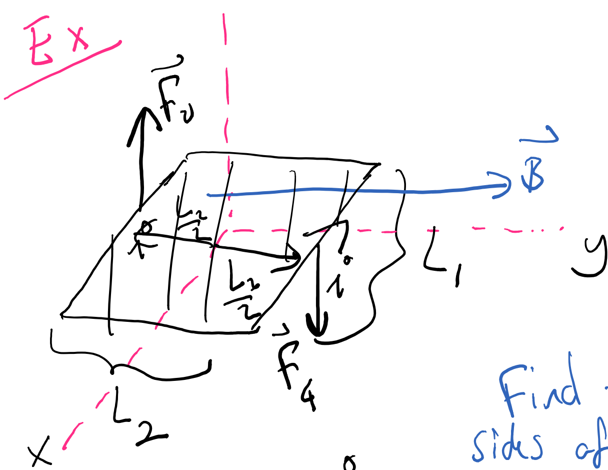
uniform $\vec{B}$

$$= i\underbrace{\left(\oint d\vec{l}\right)}_{=0} \times \vec{B}$$

$= 0$ for any closed loop

any shape loop will have net force $= 0$

Ex

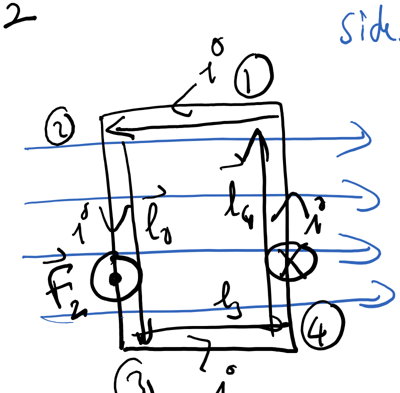


A rectangular current loop is carrying current i as shown.

$$\vec{B} = B_0\hat{j}$$

Find the forces on 4 sides of the rectangle.

Top view



$$\vec{F}_1 = 0$$

$$\vec{F}_2 = i\,\vec{l}_2 \times \vec{B}$$

$$= i\,L_1\hat{i} \times B_0\hat{j}$$

③ $\hat{i}$

$$\vec{F}_3 = 0 \qquad \vec{F}_4 = i\,\vec{l}_4 \times \vec{B} = i L_1 \hat{i} \times B_0 \hat{j} = i L_1 B_0 \underbrace{\hat{i} \times \hat{j}}_{\hat{k}}$$

$$= (i L_1 B_0)(-\hat{k})$$

No net force but net torque!

$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = \frac{L_2}{2}|\vec{F}_2| + \frac{L_2}{2}|F_4|$$

$$= i\,\frac{L_1 B_0 L_2}{\cancel{2}}\,\cancel{2} = \boxed{i(L_1 L_2) B_0}$$

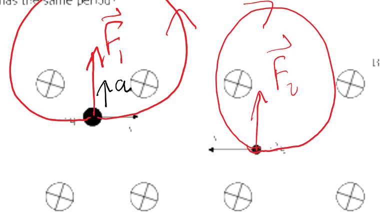
$$\vec{\mu} = i\vec{A}$$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

QUIZ-15

Two particles one positively the other negatively charged, are moving under a constant magnetic field B as shown in the figure. Their velocities are also shown in the figure

a) Draw the force (vector) that is acting on each particle.
b) Draw the approximate trajectories of both particles, indicating the direction of motion on the trajectory.
c) If positive particle and the negative particles have charges +q and −2q respectively, what should be the ratio of their masses so that their motion has the same period?



$$\vec{F}_L = q\,\vec{v} \times \vec{B}$$

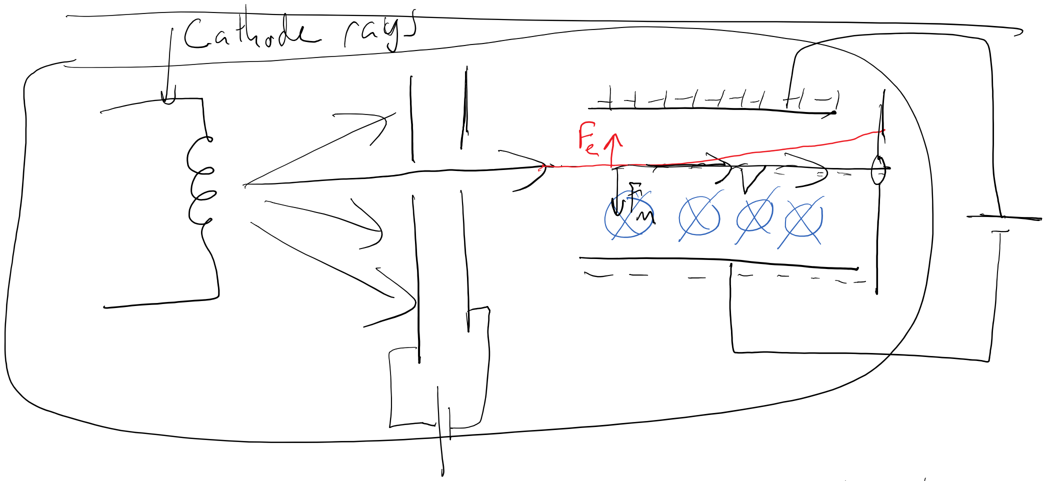
$$\omega_c = \frac{|q|B}{m}$$

$+q \qquad -2q$

$m \qquad 2m$

$$|\vec{a}| = \frac{v^{\cancel{2}}}{R} = \frac{1}{m} q \cancel{v} B$$

$$v = \frac{1}{m} q B R \qquad T = \frac{2\pi \cancel{R}}{\frac{qB\cancel{R}}{m}} = \frac{2\pi m}{qB}$$



How can magnetic force cancel the electric force

$$\vec F_e = e\vec E$$

$$\vec F_m = e\vec v \times \vec B$$

$$|\vec F_e| = |\vec F_m|$$

$$\cancel{e}|\vec E| = \cancel{e}|\vec v|\,|\vec B|$$

$$|\vec v| = \frac{|\vec E|}{|\vec B|}$$

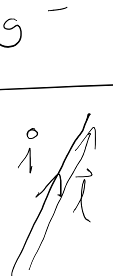
1 N/C Electric field

$$|\vec F_e| = q|\vec E| \sim 1.6\ 10^{-19}\ N$$

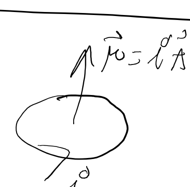
g 10 m/s²

$$\vec F_g = mg = 9.1\ 10^{-30}\ N$$

$$\vec F_L = q\vec v \times \vec B$$

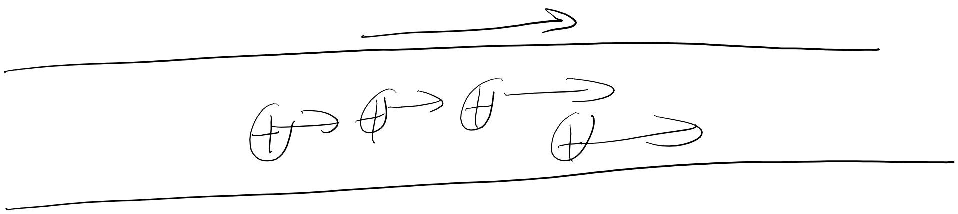


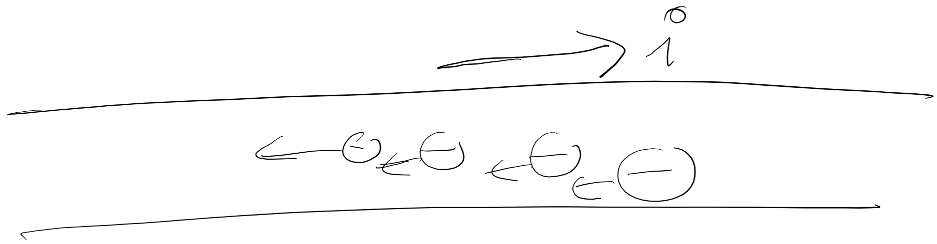
$$\vec F = i\vec l \times \vec B$$



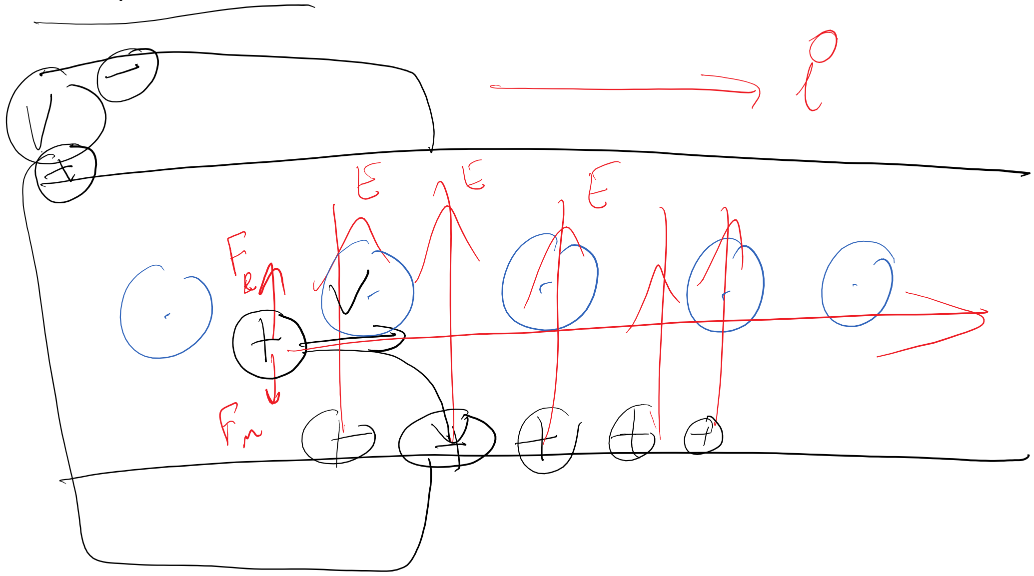
$$\vec\tau = \vec\mu \times \vec B$$

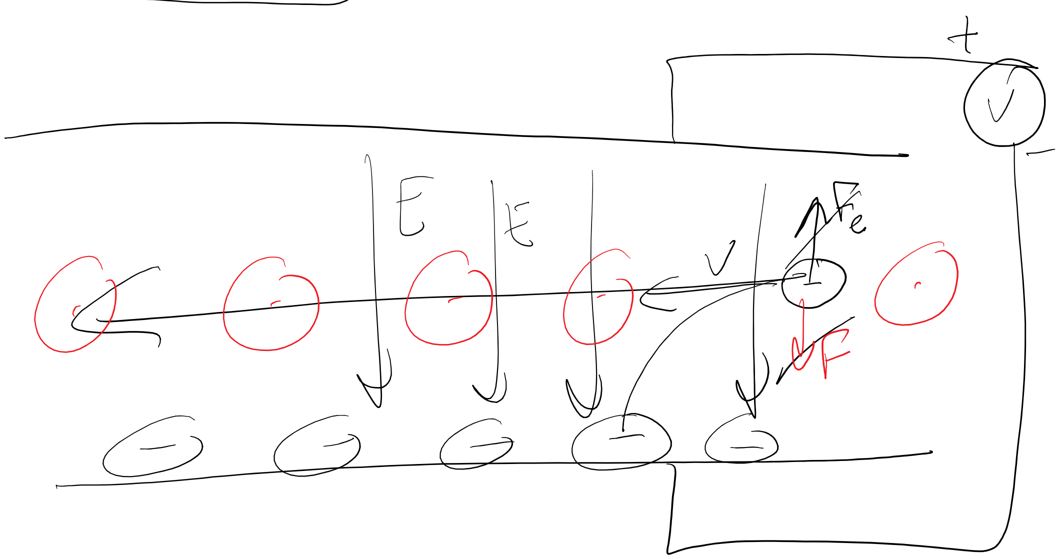
i





Hall





In metals e⁻ which have (−)

charge carry the current!

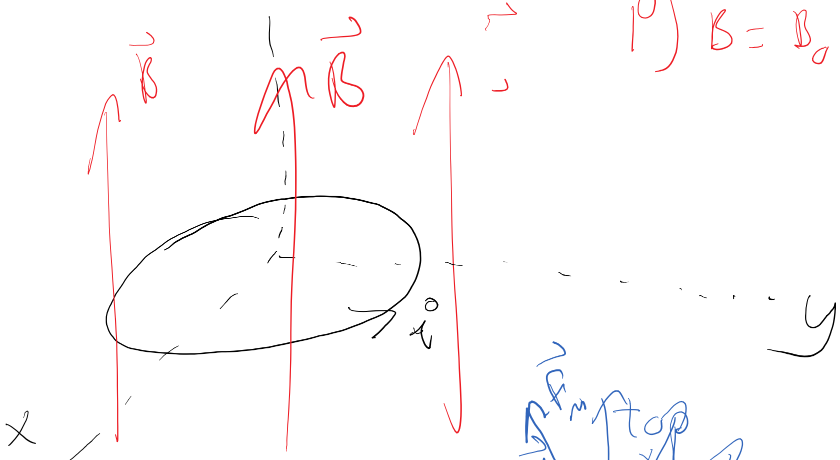
Hall effect

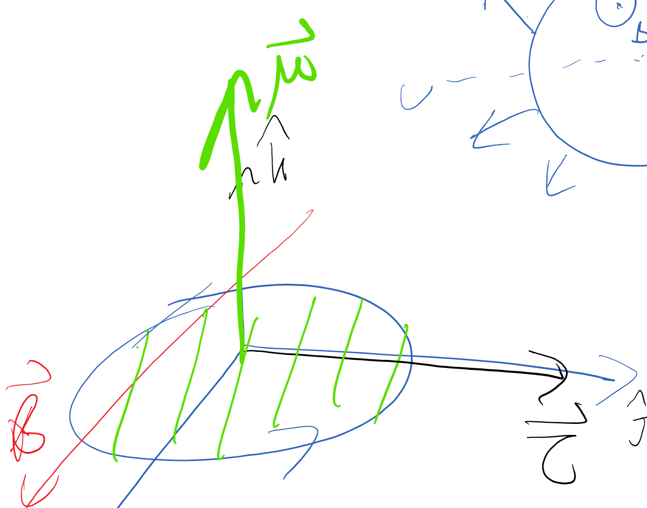
Quantum Hall Effect

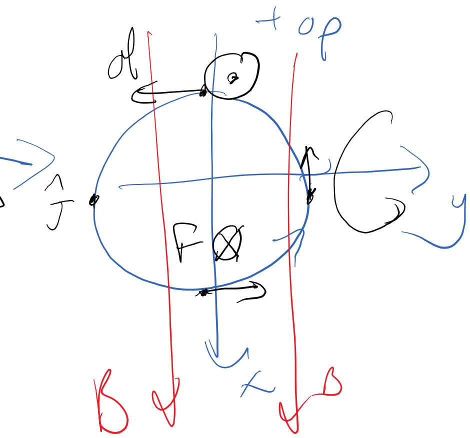
Fractional Quan Hall Effect

Torque on a current loop

1) $\vec{B} = B_0 \hat{z}$







$|\vec{\mu}| = i A$

$\vec{\mu} \Rightarrow$ Magnetic dipole moment

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$