Week 10 Notes (Mar 28 - Apr 1)

Wednesday, January 27, 2016 4:47 PM

Today 2 Lecture hours

Saturday April 2^{nd} 3 Lecture hours

(2 Quizzes)

12:00 → 12:50

13:00 → 13:50

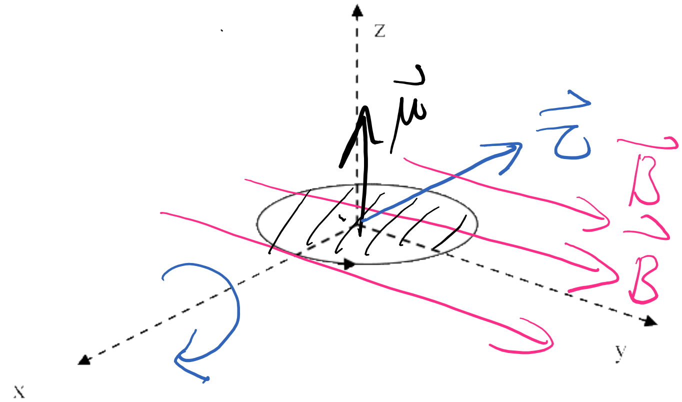
14:00 → 14:50

QUIZ-17

A ring of radius R is made from N turns of wire. The ring is placed on the x-y plane, and the wire carries a current i as shown in the figure.

a) What is the magnetic dipole moment $\vec{\mu}$ of the ring? Give both magnitude and direction.

b) If a magnetic field $\vec{B} = B_0 \hat{j}$ is applied on to the ring, calculate the torque on the ring. Describe which way the ring will turn if the only forces on it are due to the magnetic field.



a) $\vec{\mu} = i\vec{A}N$

$\vec{\mu} = |\vec{\mu}|\hat{k}$

$|\vec{\mu}| = iN\pi R^2$

b) $\vec{\tau} = \vec{\mu} \times \vec{B}$

$= |\vec{\mu}|\hat{k} \times B_0\hat{j} = iNB_0\pi R^2 \underbrace{\hat{k} \times \hat{j}}_{-\hat{i}}$

$$\boxed{|\vec{\tau}| = iNB_0\pi R^2}$$

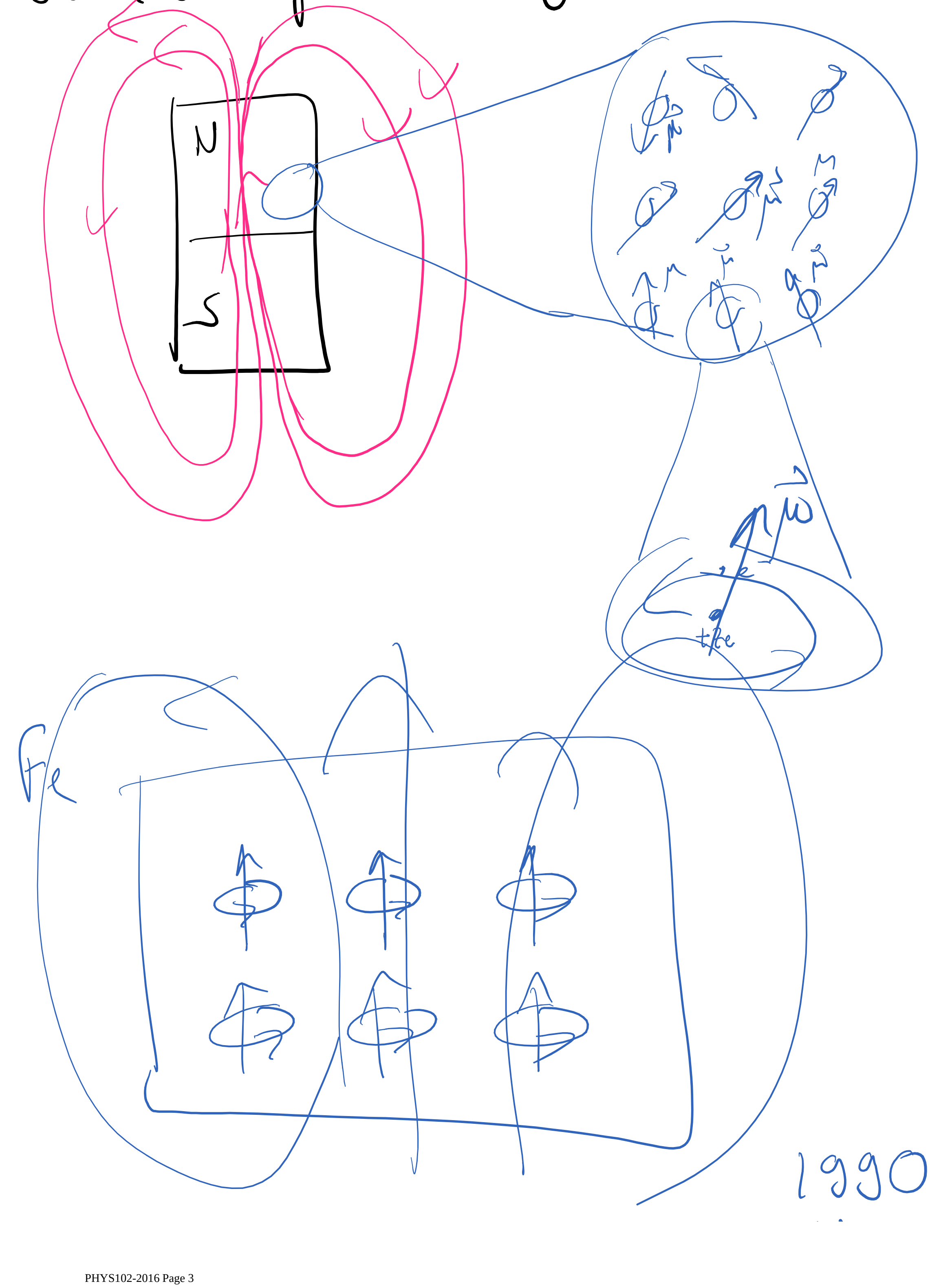
Sources of magnetic field

* Magnetic fields are created by

moving charges!

• currents

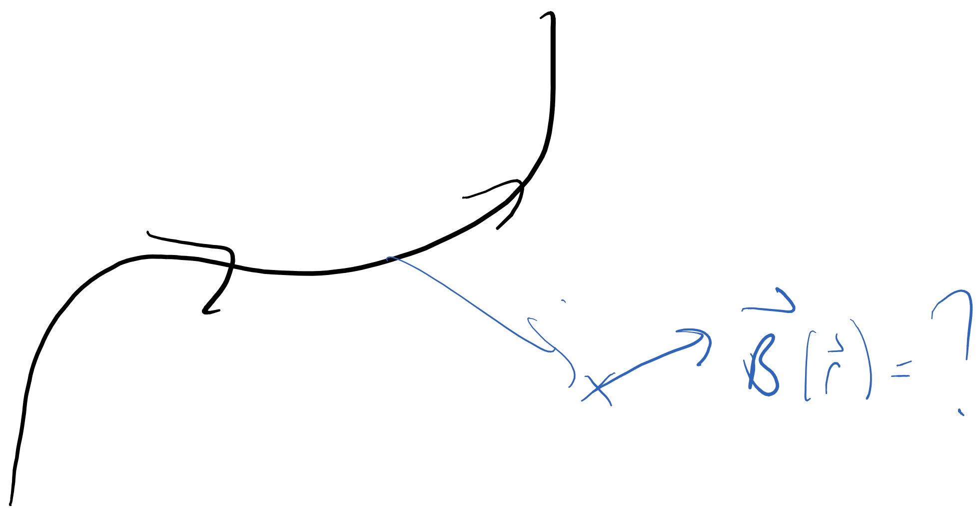
→ How about permanent magnets



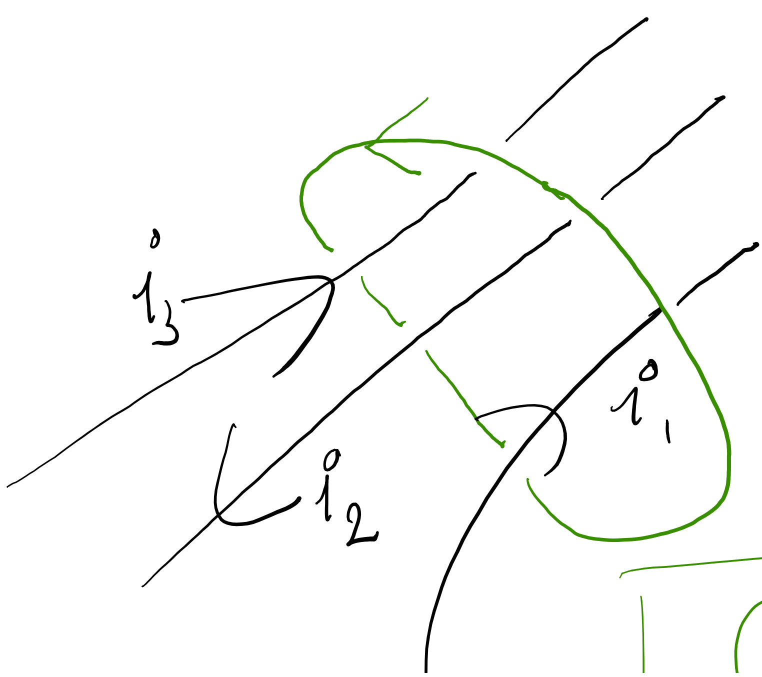
Neodymyum

This is hard!

Let's just find the magnetic field created by current carrying wires.



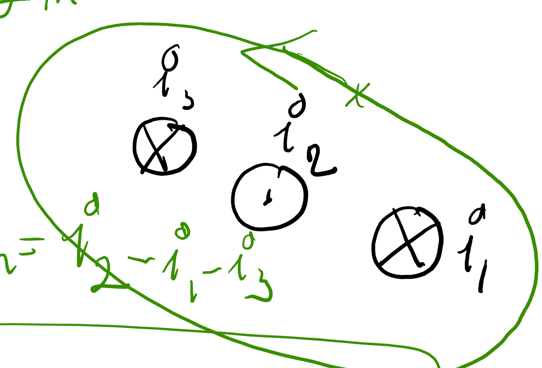
Ampere's law



Sense of rotation!

Cross-section

I_{in} ⊙

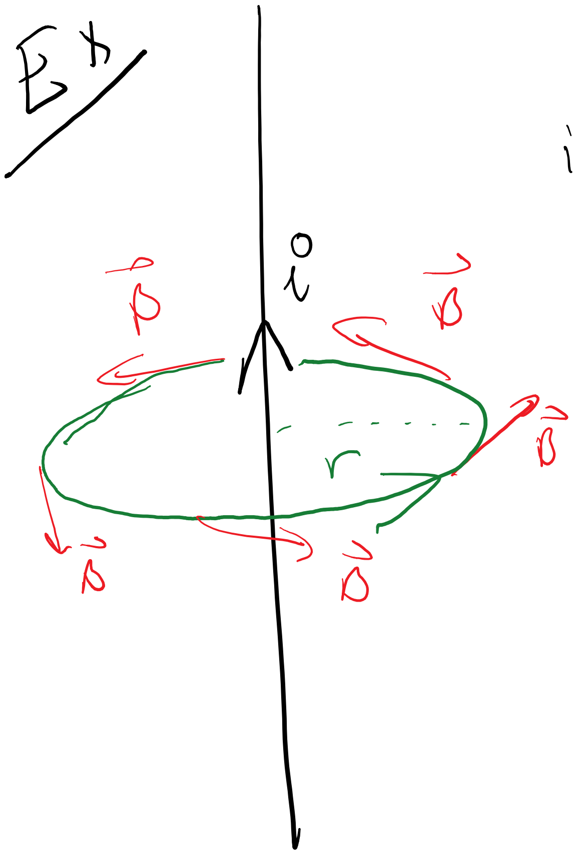


$I_{in} = i_2 - i_1 - i_3$

$$\boxed{\oint_{loop} \vec{B}\cdot d\vec{l} = \mu_0 I_{in}}$$

Ex

An infinite wire which is infinitesimally thin is carrying a current i. Find the direction and the magnitude of the magnetic field created by this wire.



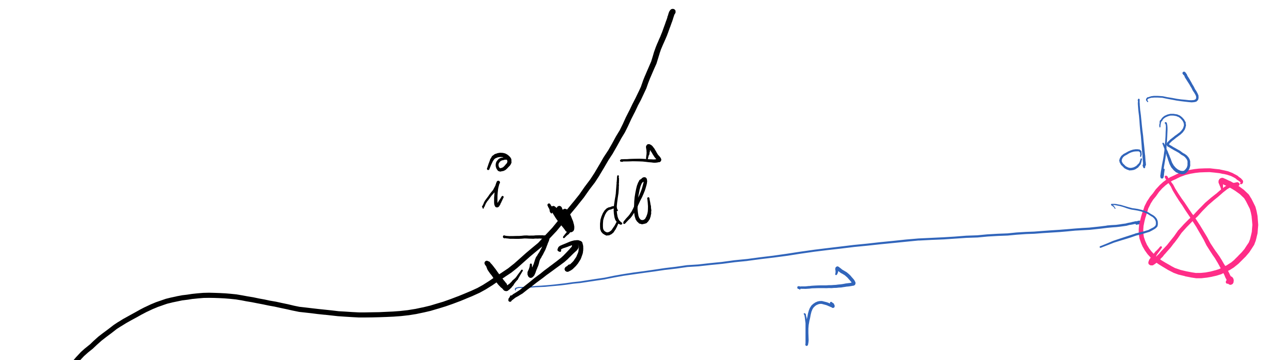
$$\oint_{loop} \underbrace{\vec{B}\cdot d\vec{l}}_{\vec{B} // d\vec{l}} = \mu_0 \underbrace{I_{in}}_{i}$$

$$|\vec{B}| \underbrace{\oint dl}_{2\pi r} = \mu_0 i$$

$$\left|\vec{B}(\vec{r})\right| = \frac{\mu_0}{2\pi}\frac{i}{r}$$

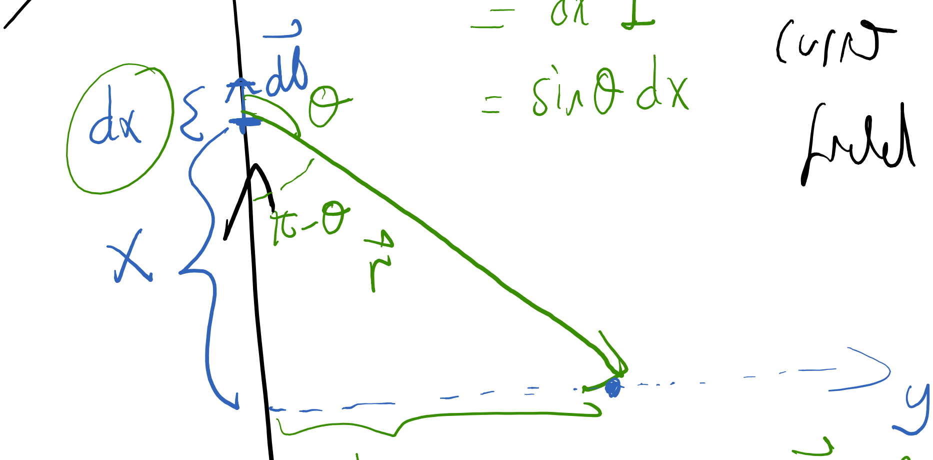
top view

Biot-Savart Law



$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{d\vec{l}\times\hat{r}}{r^2} \quad \hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

Ex



$$|d\vec{l}\times\hat{r}| = |d\vec{l}||\hat{r}|\sin\theta$$
$$= dx \cdot 1$$
$$= \sin\theta \, dx$$

Infinite, thin wire carrying current i, find the magn. field at a distance h away from the wire.

$$\sin(\pi-\theta) = \sin\theta$$
$$= \frac{h}{\sqrt{h^2+x^2}}$$

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

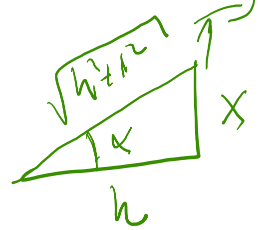
$d\vec{l}\times\hat{r}$ ⊗

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi} \frac{dx}{h^2+x^2} \frac{h}{\sqrt{h^2+x^2}}$$

$d\vec{l} \times \hat{r}$ $\otimes$

$$|\vec{B}| = \int |d\vec{B}| = \int_{-\infty}^{\infty} \frac{\mu_0 i}{4\pi} \frac{h}{(h^2+x^2)^{3/2}} dx$$

$$= \frac{\mu_0 i}{4\pi} h \int_{-\infty}^{\infty} \frac{1}{(h^2+x^2)^{3/2}} dx \quad ✓$$



$$\frac{h}{(h^2+x^2)^{1/2}} = \cos\alpha$$

$$(h^2+x^2)^{1/2} = \frac{h}{\cos\alpha}$$

$$(h^2+x^2)^{3/2} = \frac{h^3}{\cos^3\alpha}$$

$$= \frac{\mu_0 i}{4\pi} h \int_{-\pi/2}^{\pi/2} \frac{1}{\frac{h^3}{\cos^3\alpha}} h \left(\frac{1}{\cancel{\cos^2\alpha}}\right) d\alpha$$

$$\tan\alpha = \frac{x}{h}$$

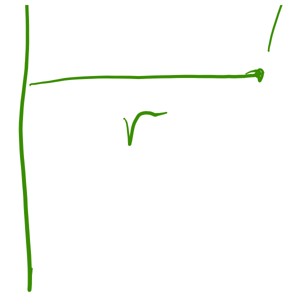
$$(1+\tan^2\alpha)\, d\alpha = \frac{1}{h} dx$$

$$dx = h \left(\frac{1}{\cos^2\alpha}\right) d\alpha$$

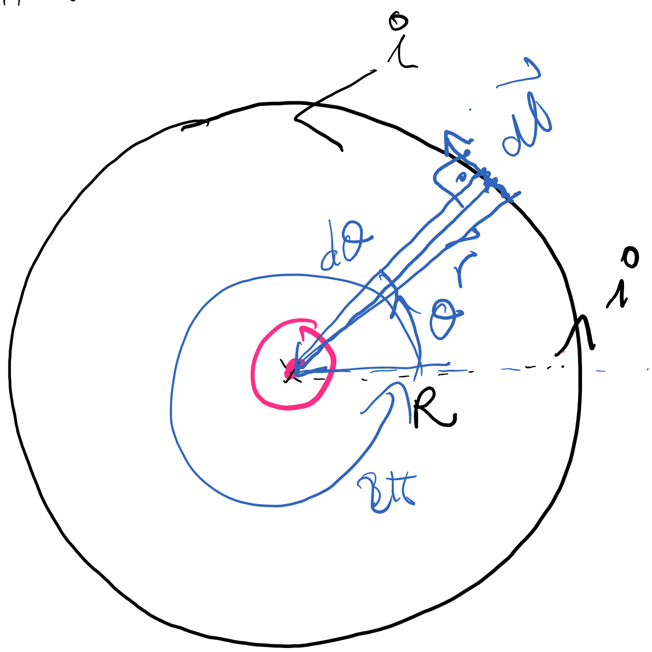
$$= \frac{\mu_0 i}{4\pi} \frac{\cancel{h^2}}{\cancel{h^3}} \underbrace{\int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha}_{\underbrace{\sin\alpha \Big|_{-\pi/2}^{\pi/2}}_{1-(-1) = 2}}$$

$$= \frac{\mu_0 i}{4\pi h} 2$$

$$|\vec{B}(r)| = \boxed{\frac{\mu_0 i}{2\pi r}}$$

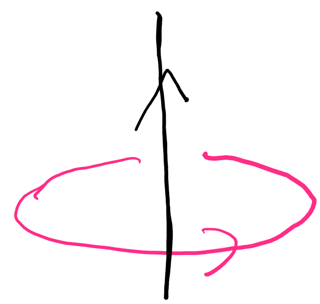


Ex Find the magnetic field created by a circular loop of radius R, carrying current i at its center.



$\vec{B}$ at center?

Direction



$$|\vec{dl}\times\hat{r}| = \underbrace{|\vec{dl}|}_{R\,d\theta}\,\underbrace{|\hat{r}|}_{1} \qquad |\vec{B}| = \int |d\vec{B}|$$

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi}\,\frac{|\vec{dl}\times\hat{r}|}{R^2} = \frac{\mu_0 i}{4\pi}\,\frac{\cancel{R}\,d\theta}{R^{\cancel{2}}}$$

2π

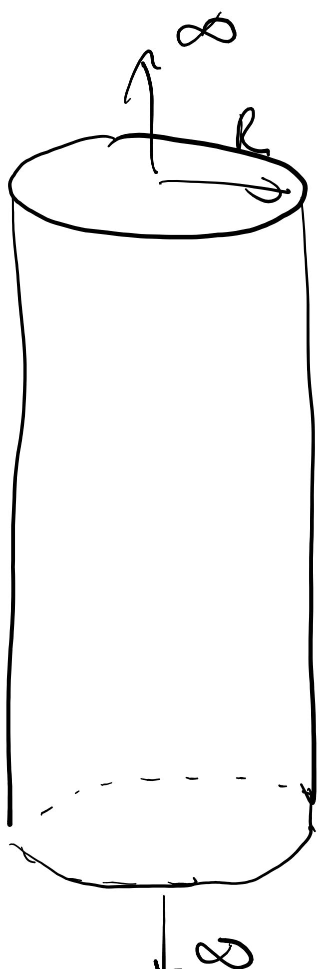
$4\pi \quad R^2 \quad 4\pi \quad R$

$$|\vec{B}| = \int dB = \int_0^{2\pi} \frac{\mu_0 i}{4\pi} \frac{1}{R}\, d\theta$$

$$= \frac{\mu_0 i}{4\pi R} \underbrace{\theta \Big|_0^{2\pi}}_{2\pi} = \boxed{\frac{\mu_0 i}{2R}}$$

out of the page.

Ex



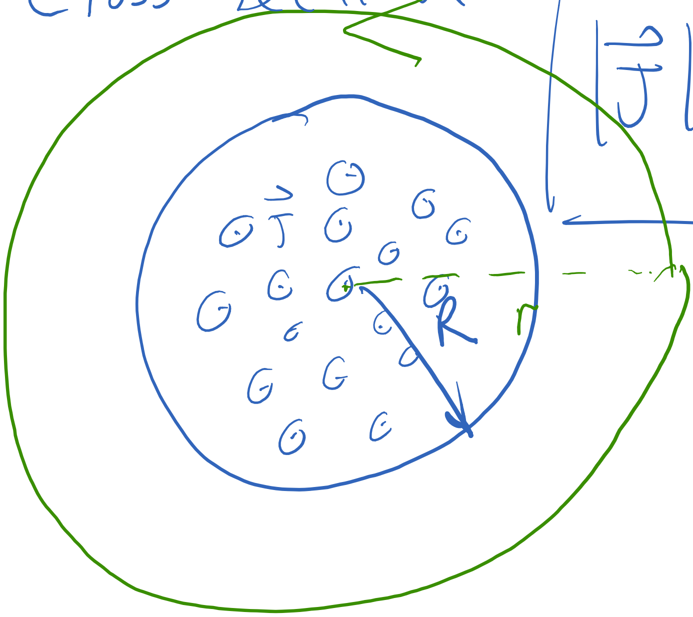
A thick wire of radius R carries a total current i uniformly through its cross section

a) Calculate $|\vec{J}|$.

b) Find the magnetic field outside the wire

c) Find the magnetic field inside the wire!

Cross section



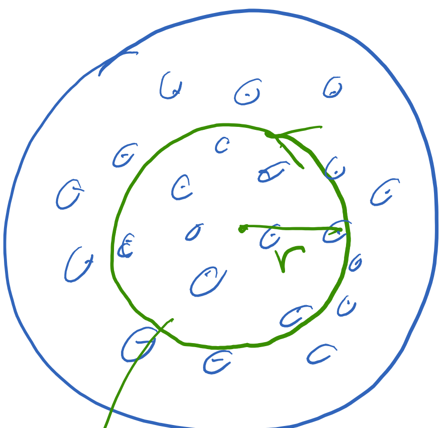
a)

$$\boxed{|\vec{J}| = \frac{i}{\pi R^2}}$$

$$\underbrace{\oint \vec{B}\cdot d\vec{l}}_{\vec{B} // d\vec{l}} = \mu_0 I_{in}$$

$$|\vec{B}| \underbrace{\oint dl}_{2\pi r} = \mu_0 i$$

$$|\vec{B}| = \frac{\mu_0 i}{2\pi r} \qquad r > R$$



$$\underbrace{\oint \vec{B}\cdot d\vec{l}}_{\vec{B} // d\vec{l}} = \mu_0 I_{in}$$

$I_{in} = ?$

$$|\vec{B}| \underbrace{\oint |d\vec{l}|}_{2\pi r} = \mu_0 i \frac{r^2}{R^2}$$

$$I_{in} = |\vec{J}|\pi r^2 = \frac{i \cancel{\pi} r^2}{\cancel{\pi} R^2}$$

$$\boxed{|\vec{B}| = \frac{\mu_0 i}{2\pi} \frac{r}{R^2}} \qquad r < R$$

$|\vec{B}|$


$|B|$
$\frac{\mu_0 i}{2\pi R}$
R
r
$\sim \frac{1}{r}$
i