Week 3 Notes (Feb 8- 12)

Wednesday, January 27, 2016 3:14 PM

PHYSICS 102- Instructor: M. Özgür OKTEL- Spring 2016

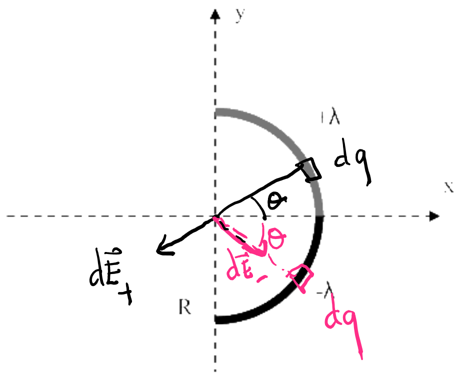
QUIZ-4

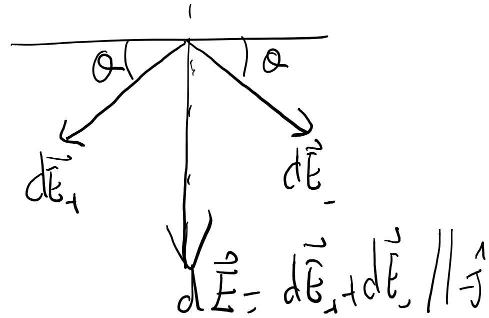
A non-conducting rod is bent into a semicircle of radius R as shown in the figure. Half of the semicircle(quarter of the circle) is charged with $+\lambda$ line density while the other half is charged with $-\lambda$.

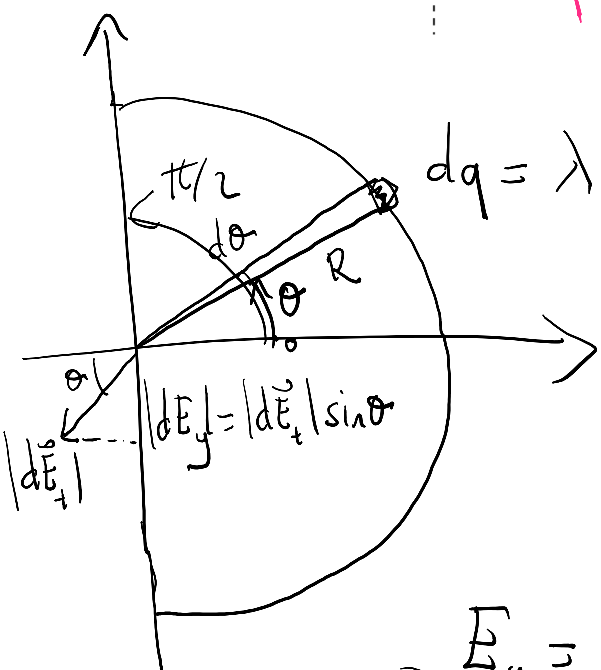
At the center of the circle:

a) Which way does the electric field vector point? (Give the unit vector in terms of $\hat{i}, \hat{j}$.) → $\vec{E}$ points in the $(-\hat{j})$ direction.

b) What is the magnitude of the electric field?







$$dq = \lambda R\, d\theta$$

$$|d\vec{E}_+| = \frac{1}{4\pi\varepsilon_0} \frac{dq}{R^2}$$

$$|dE_y| = \frac{1}{4\pi\varepsilon_0} \frac{\sin\theta}{R^{\cancel{2}}} \lambda \cancel{R}\, d\theta$$

$$E_y = \int dE_y = \int_0^{\pi/2} \frac{\lambda}{4\pi\varepsilon_0} \frac{\sin\theta}{R}\, d\theta$$

created only by the upper part

$$= \frac{\lambda}{4\pi\varepsilon_0 R} \underbrace{(-\cos\theta)\Big|_0^{\pi/2}}_{1}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 R}$$

$$\text{Total } |\vec{E}| = 2E_y = \boxed{\frac{\lambda}{2\pi\varepsilon_0 R}}$$

1°) Goal ✓

2°) Units ✓

Total $|\vec{E}| = 2E_y = \boxed{\dfrac{\lambda}{2\pi\varepsilon_0 R}}$

2°) units ✓

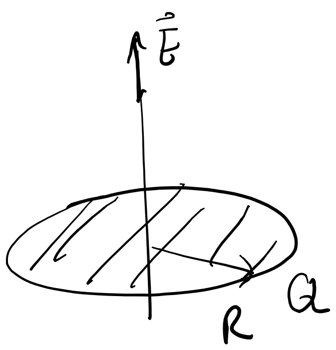
$[\vec{E}] = \dfrac{C}{[\varepsilon_0]\, m^2}$

$[\lambda] = C/m$

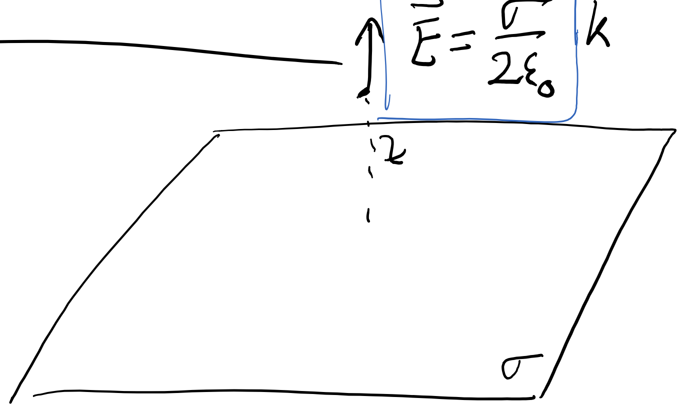
$\left[\dfrac{\lambda}{\varepsilon_0 R}\right] = N/C$ ✓

3°) limit $R \to \infty$ $E \to 0$ ✓

$\lambda \to 0$ $E \to 0$ ✓

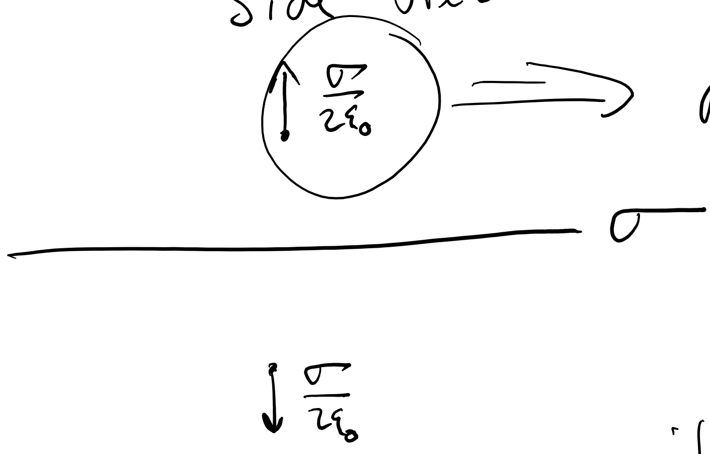


$\xRightarrow[\text{keep } \sigma \text{ constant}]{\lim R\to\infty}$



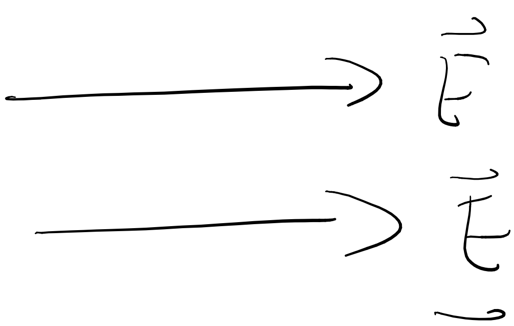
$\boxed{\vec{E} = \dfrac{\sigma}{2\varepsilon_0}\hat{k}}$

Side view

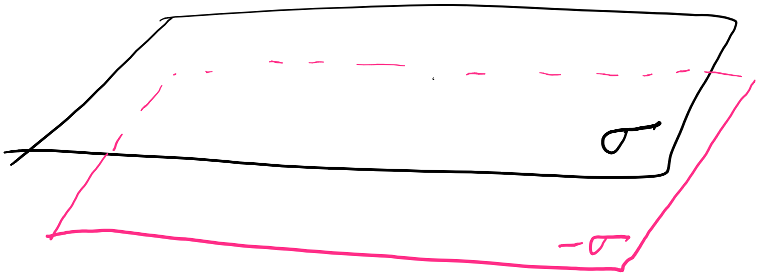


$\Longrightarrow$ formula for ∞ plane

but it works nicely for finite planes if you are close.



uniform electric field



Side

$\frac{\sigma}{2\varepsilon_0}\uparrow \quad \cancel{\downarrow \frac{\sigma}{2\varepsilon_0}} \qquad \vec{E} = 0$

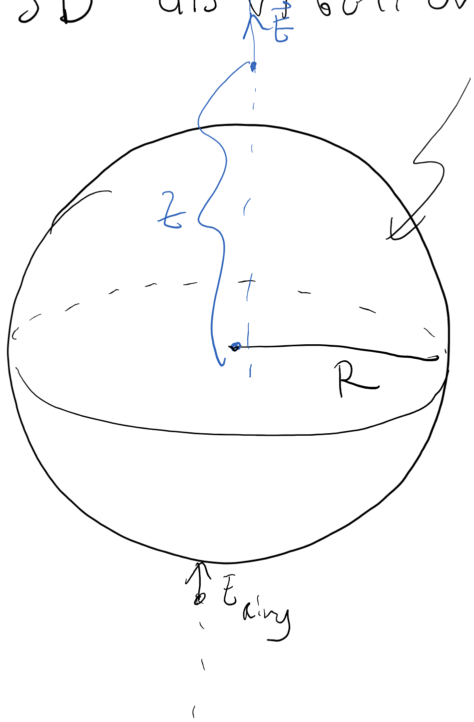
σ

$\frac{\sigma}{2\varepsilon_0}\downarrow + \downarrow \frac{\sigma}{2\varepsilon_0} = \downarrow \frac{\sigma}{\varepsilon_0}$ uniform $\vec{E}$ field.

$-\sigma$

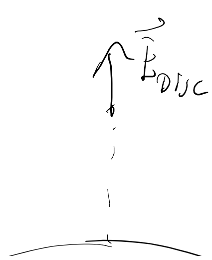
$\frac{\sigma}{2\varepsilon_0}\downarrow \quad \cancel{\uparrow \frac{\sigma}{2\varepsilon_0}} \qquad \vec{E} = 0$

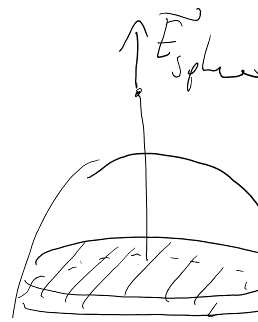
3D distributions

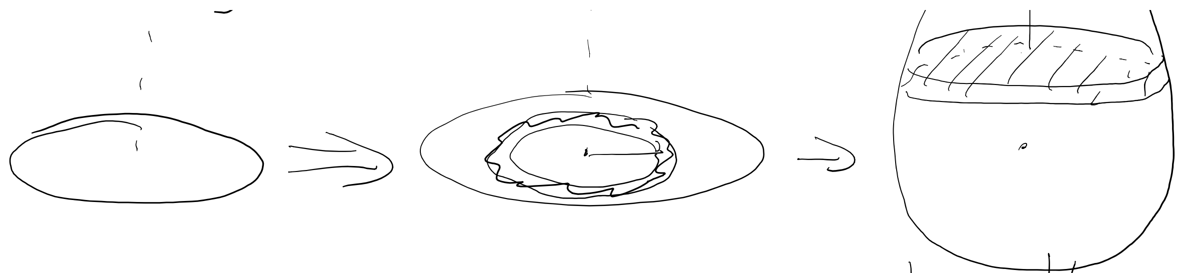


uniformly charged inside with total charge Q

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4\pi}{3}R^3}$$

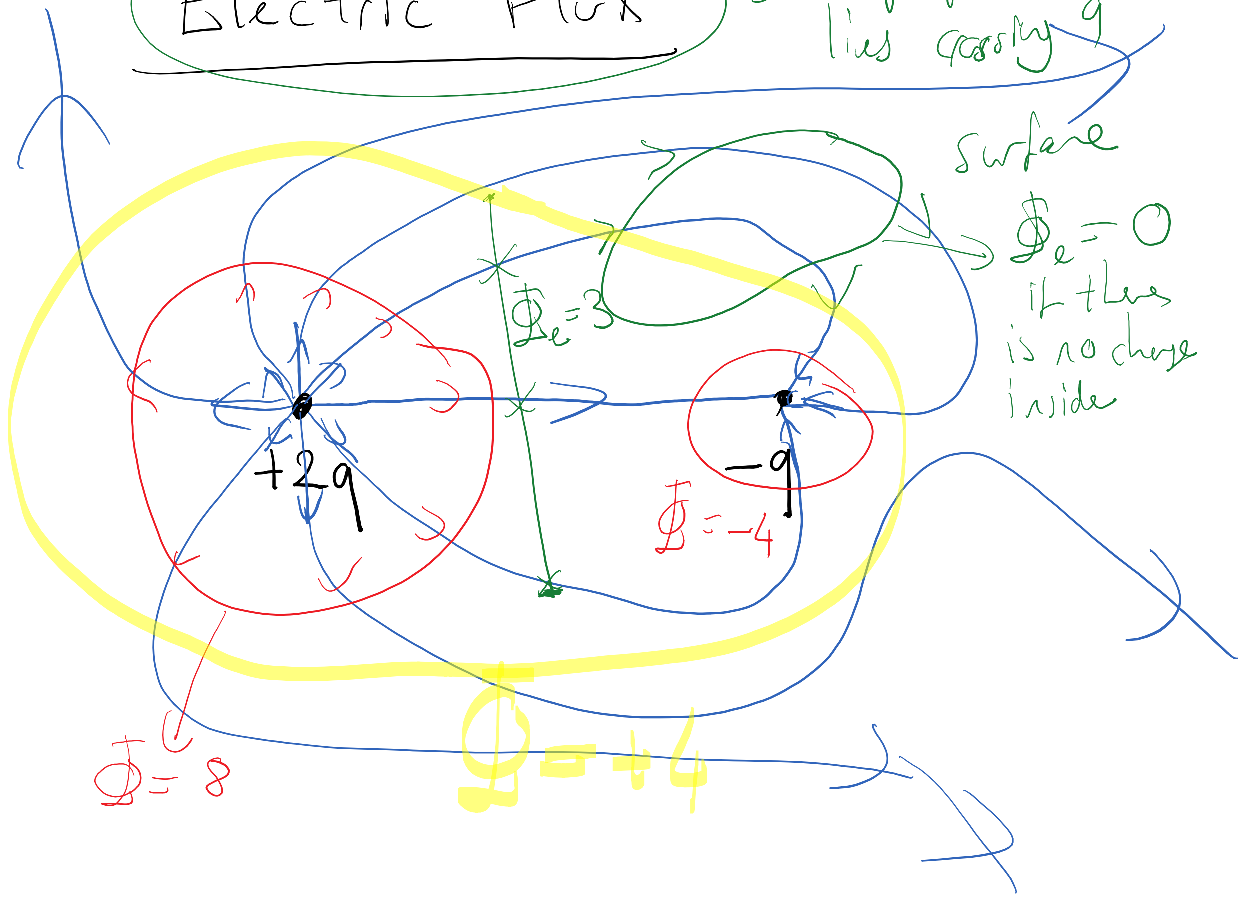






Do Try this at home!!

Electric Flux → # of field lines crossing a surface



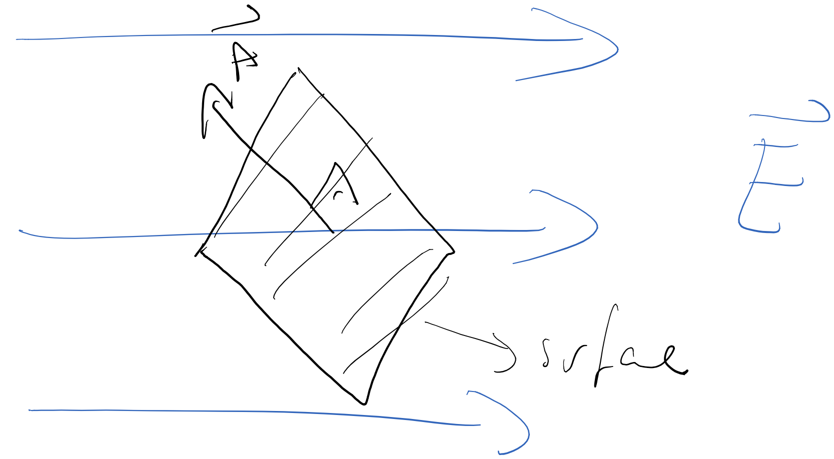
# of lines going out of a closed surface is proportional to the total charge inside.

A better definition of Flux

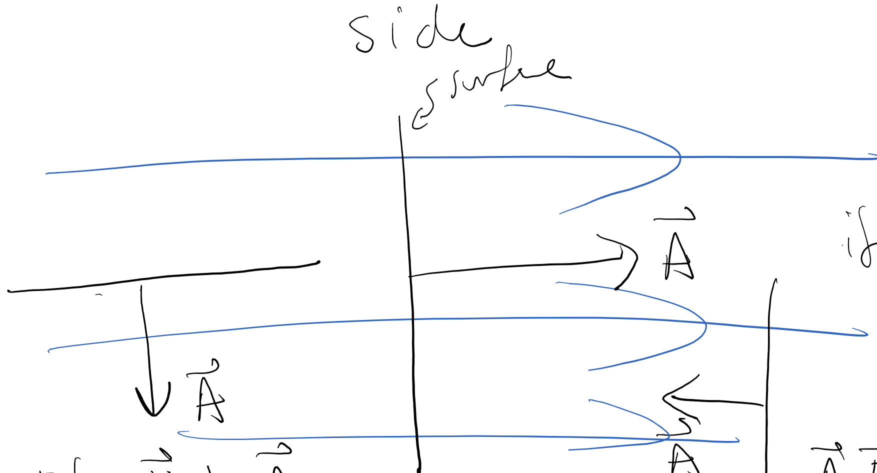
A better definition of .

$$\Phi_e = \vec{E} \cdot \vec{A}$$

area vector.



$$\Phi = \vec{E} \cdot \vec{A}$$



If $\vec{E} \perp \vec{A}$

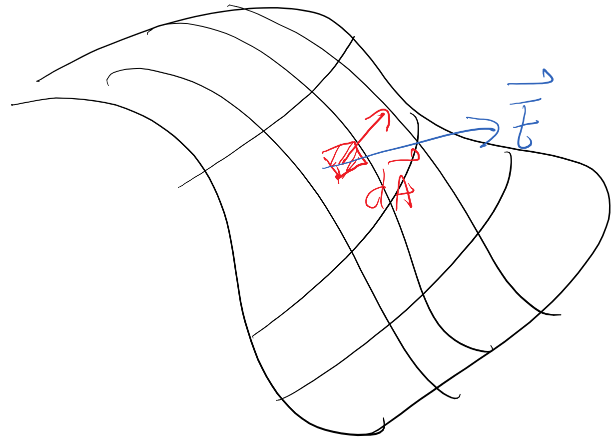
$\Phi_e = 0$

if $\vec{A} \parallel \vec{E}$

Φ_e is max.

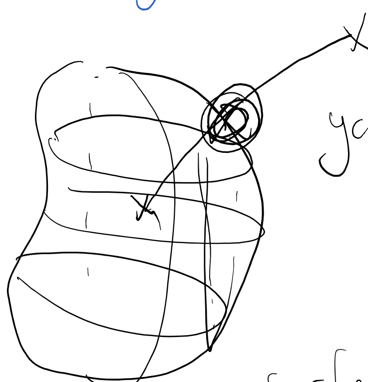
$\underbrace{\vec{A} \cdot \vec{E} < 0}$

$\vec{A}$ definition is generally ambiguous.

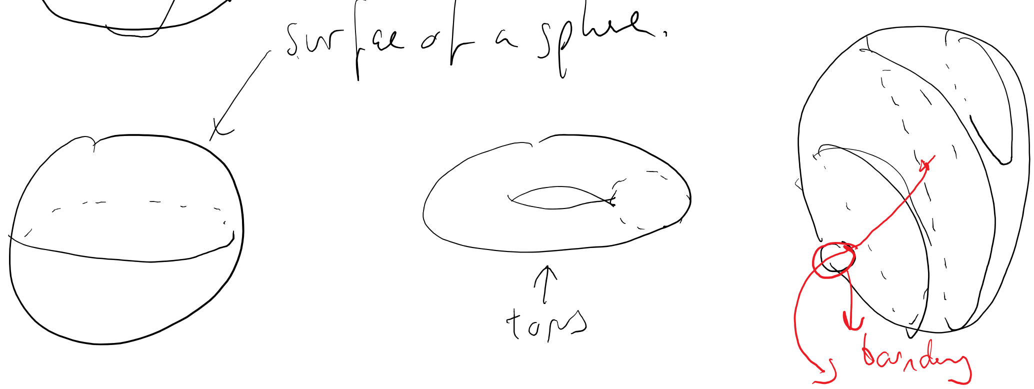


$$\Phi_e = \int_{\underline{\underline{Surface}}} \vec{E} \cdot d\vec{A}$$

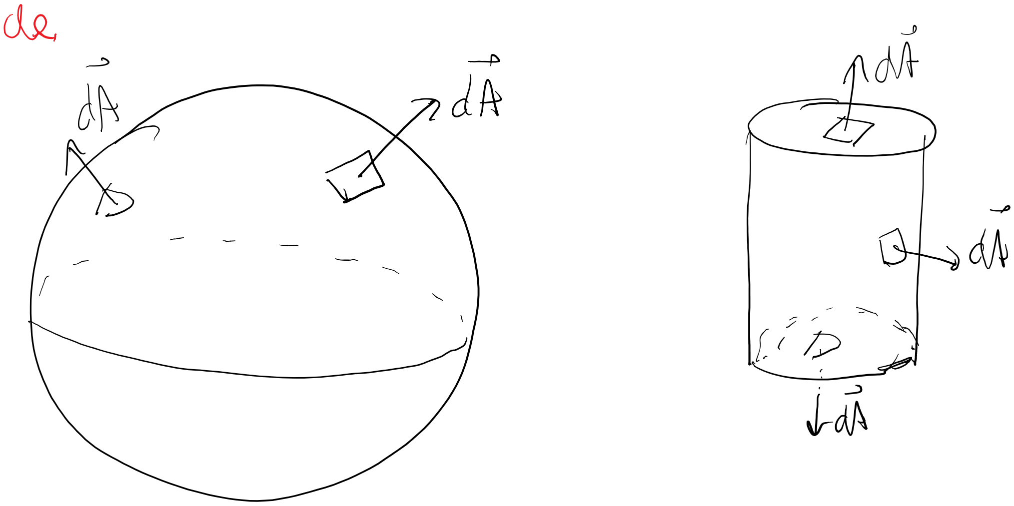
<u>Closed Surface:</u> is a surface which divides the space into an inside and outside region.



you can not go from inside to outside without crossing the surface.



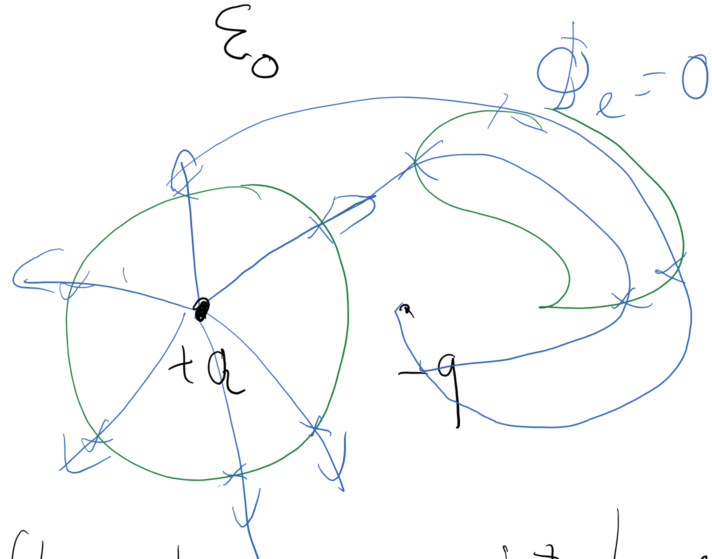
For closed surfaces $d\vec{A}$ points from inside to outside



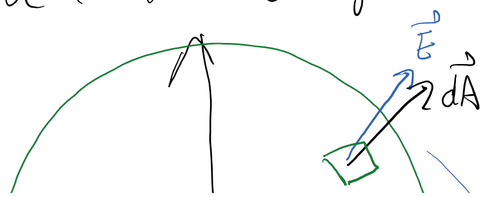
Gauss' Law $\Longrightarrow$ Equivalent to Coulomb's Law!!

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

means closed surface

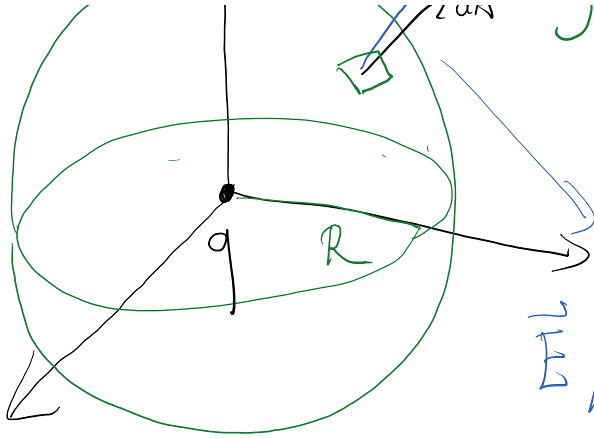


Calculate the flux due to a point charge q at the origin



$\oint \vec{E} \cdot d\vec{A}$

$|\vec{E}|$ | $\frac{q}{\ }$



$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$

By symmetry $\vec{E} \parallel d\vec{A}$ everywhere on the sphere.

$$\oint \underbrace{\vec{E}\cdot d\vec{A}}_{\vec{E}\parallel d\vec{A}} = \oint \underbrace{|\vec{E}|}_{\text{same on all points of the surface}} dA = |\vec{E}| \underbrace{\oint dA}_{\text{total surface area}} = |\vec{E}|\, \underbrace{A}_{4\pi R^2}$$

$$= \frac{1}{\cancel{4\pi}\epsilon_0}\frac{q}{\cancel{R^2}}\cancel{4\pi R^2} = \frac{q}{\epsilon_0}$$

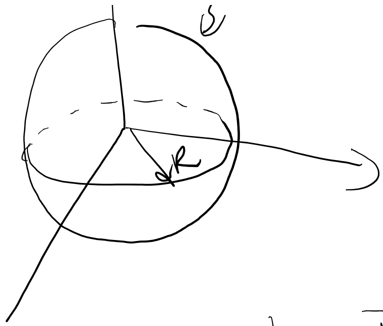
$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$$

checks out for point charges.

How will this help us?

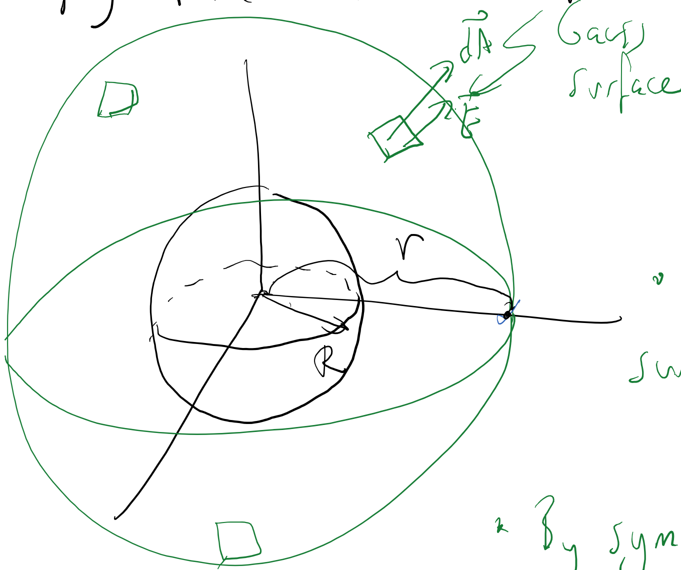
Find the electric field of a uniformly filled sphere

Find $\vec{E}$ both inside and outside the sphere.



and outside the sphere.

1°) Find the $\vec{E}$ field outside $r > R$



$$\oint \vec{E} \cdot \vec{dA} = \frac{Q_{in}}{\varepsilon_0}$$

$Q_{in} \to Q$

• By symmetry $\vec{E} \parallel \vec{dA}$ on the surface

$$\vec{E} \cdot \vec{dA} = |\vec{E}|\, dA$$

• By symmetry $|\vec{E}|$ is constant on the surface.

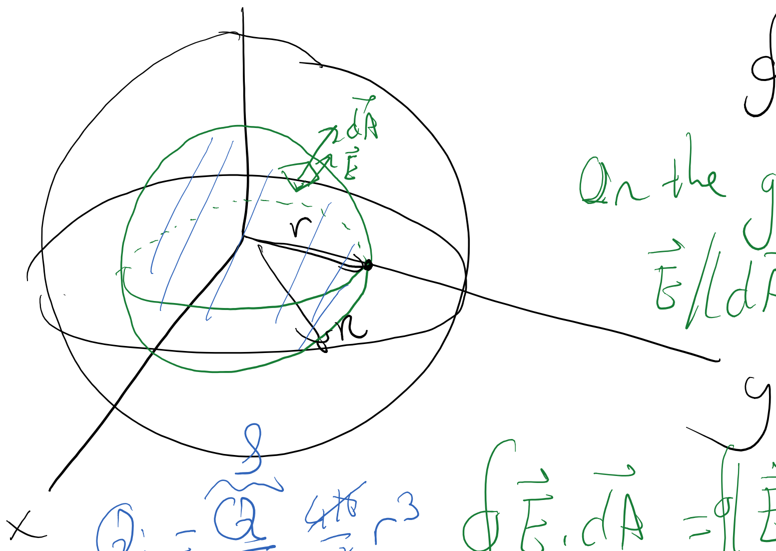
$$\oint \vec{E} \cdot \vec{dA} = \oint |\vec{E}|\, dA = |\vec{E}| \underbrace{\oint dA}_{A = 4\pi r^2} = |\vec{E}|\, 4\pi r^2$$

$$\oint \vec{E} \cdot \vec{dA} = \frac{Q_{in}}{\varepsilon_0}$$

$$4\pi r^2 |\vec{E}| = \frac{Q}{\varepsilon_0} \Rightarrow |\vec{E}(\vec{r})| = \frac{Q}{4\pi \varepsilon_0 r^2} \quad \checkmark$$

$r > R$

How about $r < R$?



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

On the gauss surface

$\vec{E} \parallel d\vec{A}$

$$Q_{in} = \overbrace{\frac{Q}{\frac{\cancel{4\pi}}{\cancel{3}}R^3}}^{\delta}\,\frac{\cancel{4\pi}}{\cancel{3}}r^3$$

$$\oint \vec{E}\cdot d\vec{A} = \oint |\vec{E}|\,dA = |\vec{E}|\underbrace{\oint dA}_{4\pi r^2}$$

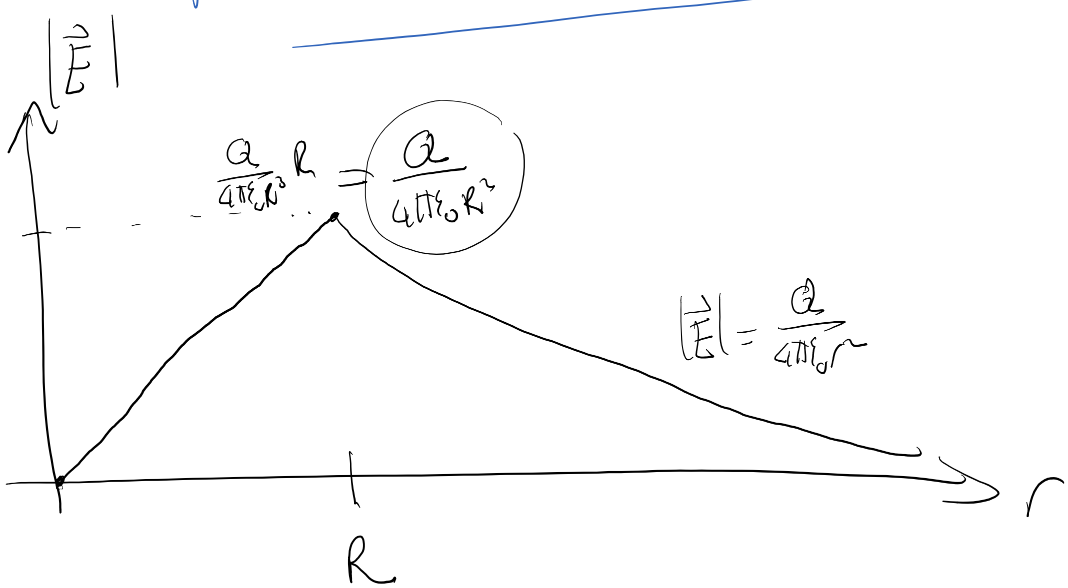
$$\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 |\vec{E}|$$

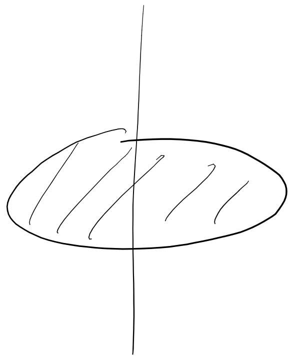
$$Q_{in} = \frac{Q r^3}{R^3}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$4\pi \cancel{r^2}|\vec{E}| = \frac{Q}{\varepsilon_0}\,\frac{\cancel{r^3}\;r}{R^3}$$

$$\boxed{|\vec{E}| = \frac{Q}{4\pi\varepsilon_0}\,\frac{r}{R^3} \qquad r < R}$$





Disc problem is ~~not easy~~ impossible! to solve by Gauss

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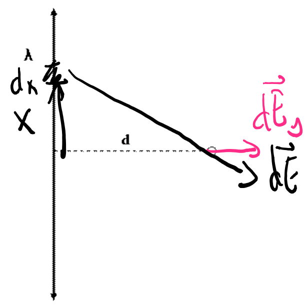
QUIZ-5
An infinite line is charged with +λ line charge density.

What is the magnitude of the electric field at a distance d from the line?

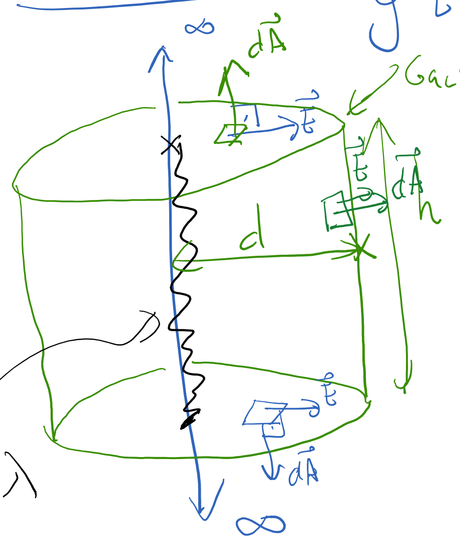


$E_y = \int dE_y \Rightarrow$ ok!

$$\boxed{E_y = \frac{\lambda}{2\pi\varepsilon_0 d}}$$

Gauss law

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \;?$$



$Q_{in} = h\lambda$

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{top} \vec{E}\cdot d\vec{A}}_{0} + \underbrace{\int_{bottom} \vec{E}\cdot d\vec{A}}_{0} + \int_{side} \vec{E}\cdot d\vec{A}$$

$\vec{E} \perp d\vec{A}$, $\vec{E}\cdot d\vec{A} = 0$ (top)

$\vec{E} \perp d\vec{A}$, $\vec{E}\cdot d\vec{A} = 0$ (bottom)

$\vec{E} \parallel d\vec{A}$, $\vec{E}\cdot d\vec{A} = |\vec{E}|\,dA$ (side)

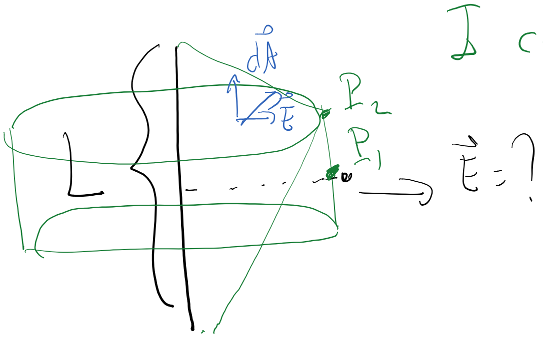
$$= \int_{side} |\vec{E}|\,dA = |\vec{E}| \int_{side} dA = \boxed{2\pi d\,h\,|\vec{E}|}$$

$|\vec{E}|$ is constant on the surface by symmetry!

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$2\pi d h|\vec{E}| = h\lambda \Rightarrow \boxed{|\vec{E}| = \frac{\lambda}{\ldots}}$$

$$2\pi d \cancel{h} |\vec{E}| = \frac{\cancel{h}\lambda}{\varepsilon_0} \Rightarrow \boxed{|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 d}}$$

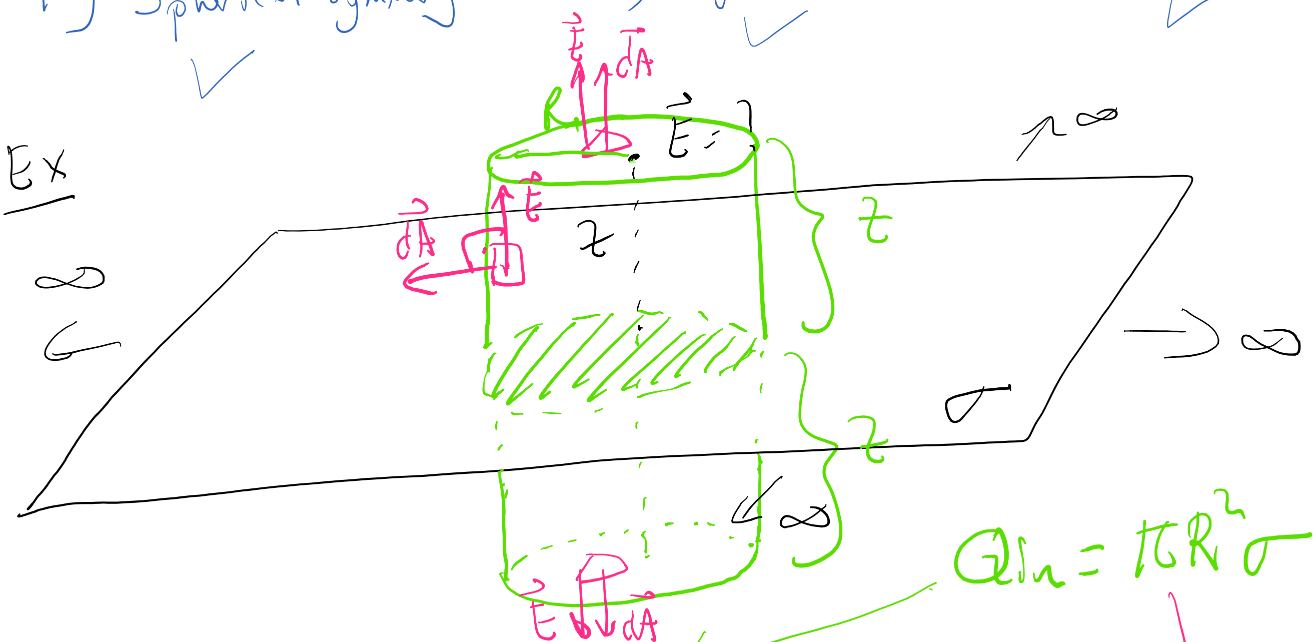


I can not use Gauss' Law to calculate $|\vec{E}|$ when it is not a constant on this surface!

Are you sure that you can use Gauss' Law!

1°) Spherical Symmetry ✓ 2°) Cylindrical Symmetry ✓ 3°) Planar Symmetry ✓

Ex



$Q_{in} = \pi R^2 \sigma$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \Longrightarrow 2\cancel{\pi}\cancel{R^2}|\vec{E}| = \frac{\cancel{\pi}\cancel{R^2}\sigma}{\varepsilon_0}$$

$$\int \cancel{\vec{E}\cdot d\vec{A}}^{\,0} + \int \vec{E}\cdot d\vec{A} + \int \vec{E}\, d\vec{A} \qquad \boxed{|\vec{E}| = \frac{\sigma}{\dots}}$$

$$\cancel{\int_{side} \vec{E}\cdot d\vec{A}} + \int_{top} \vec{E}\cdot d + \int_{bottom} \vec{E}\cdot d\vec{A}$$

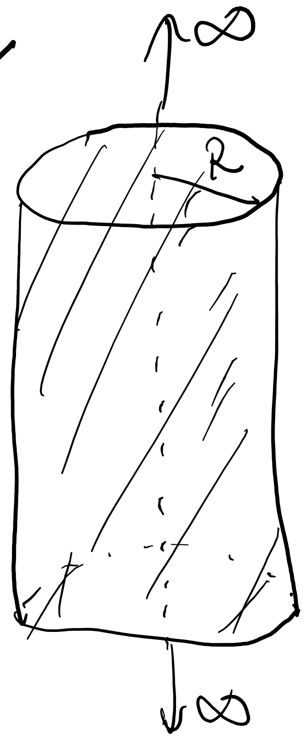
$\vec{E} \perp d\vec{A}$ $\quad$ $\vec{E} \parallel d\vec{A}$ $\quad$ $\vec{E} \parallel d\vec{A}$

$\vec{E}\cdot d\vec{A} = 0$ $\quad$ $\vec{E}\cdot d\vec{A} = |\vec{E}|\,dA$

$$\Rightarrow |\vec{E}|\int_{top} dA + |\vec{E}|\int_{bottom} dA = 2|\vec{E}|\pi R^2$$

$$\boxed{|\vec{E}| = \frac{\sigma}{2\epsilon_0}}$$

EX



An ∞ cylinder of radius R is filled uniformly with charge density ρ.

a) Find λ, the charge density per unit length along the cylinder.

b) Find $|\vec{E}|$ outside the cylinder

c) Find $|\vec{E}|$ inside the cylinder.

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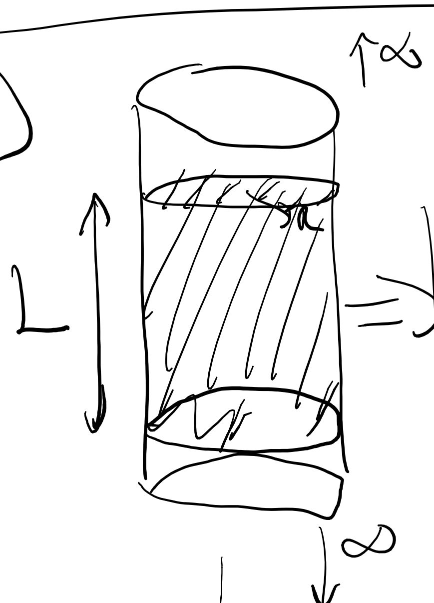
a)



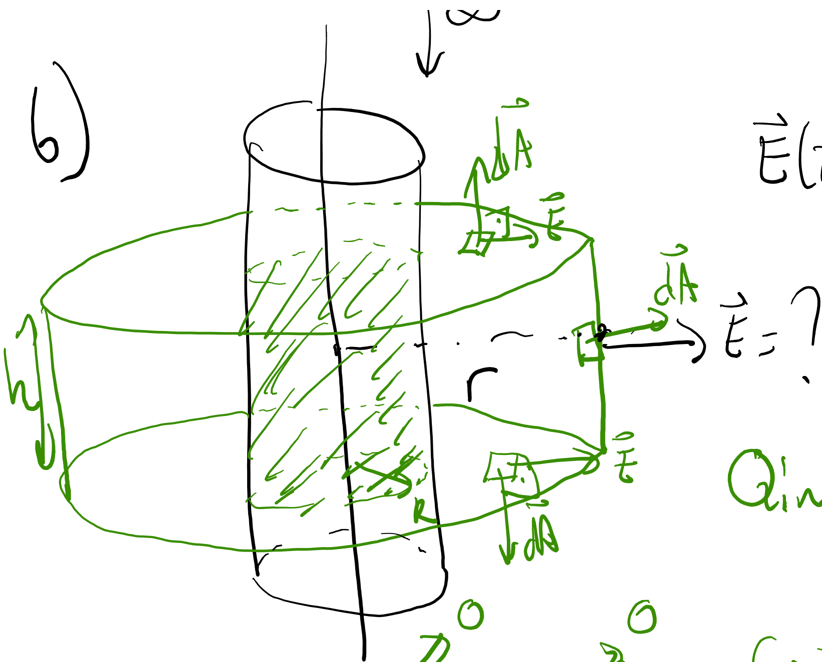
$$\Rightarrow Q = \rho\, \pi R^2 L$$

$[\lambda] = C/m$

$[\rho] = C/m^3$

$$\frac{Q}{L} = \boxed{\lambda = \rho \pi R^2}$$

b)



$\vec{E}(\vec{r})$ for $r > R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

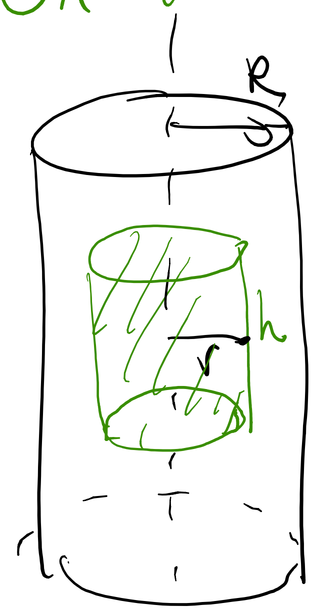
$$Q_{in} = \pi R^2 h \rho = \lambda h$$

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\cancel{\int_{top} \vec{E}\cdot d\vec{A}}}_{\vec{E}\perp d\vec{A}}^{\,0} + \underbrace{\cancel{\int_{bottom} \vec{E}\cdot d\vec{A}}}_{\vec{E}\cdot d\vec{A}}^{\,0} + \underbrace{\int_{side} \vec{E}\cdot d\vec{A}}_{\vec{E}\,/\!/\,d\vec{A}} = |\vec{E}| \int_{side} dA = \boxed{|\vec{E}|\, h\, 2\pi r}$$

$$|\vec{E}|\,\cancel{h}\,2\pi r = \frac{\lambda \cancel{h}}{\varepsilon_0}$$

$$|\vec{E}| = \frac{\lambda}{2\pi r \varepsilon_0} = \frac{\cancel{\pi} R^2 \rho}{2\cancel{\pi} r \varepsilon_0} = \boxed{\frac{\rho}{2\varepsilon_0}\frac{R^2}{r}}$$

c) On the inside



$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\cancel{\int_{top} \vec{E}\cdot d\vec{A}}}_{\vec{E}\perp d\vec{A}}^{\,0} + \cancel{\int_{bottom} \vec{E}\cdot d\vec{A}}^{\,0} + \underbrace{\int_{side} \vec{E}\cdot d\vec{A}}_{\vec{E}\,/\!/\,d\vec{A}}$$

$$= 2\pi h r |\vec{E}|$$

$$Q_{in} = \pi r^2 h \rho$$

$$2\cancel{\pi} \cancel{h} r |\vec{E}| = \frac{\cancel{\pi} r^{\cancel{2}} \cancel{h} \rho}{\varepsilon_0}$$ ✓

$$\boxed{|\vec{E}| = \frac{\rho}{2\varepsilon_0} r}$$

$[\rho] = C/m^3$

$[r] = m$