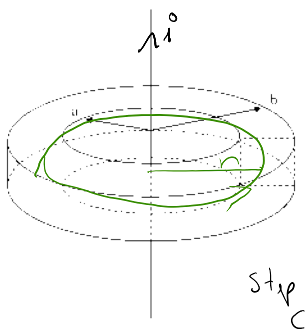
Week 14 Notes (Apr 25-29)

Wednesday, January 27, 2016 4:49 PM

A toroid is made by winding N turns of wire around a torus of inner radius **a** and outer radius **b**. The torus has a square cross section, so its height is **(b-a)**.

a) Find the self inductance **L** of the toroid.

b) If a thin infinite wire is passing through the center of the toroid, as shown in the figure, find the mutual inductance **M** between the wire and the toroid.



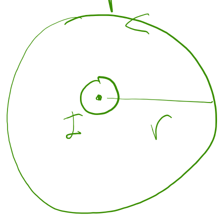
$$\Phi_{toroid} = M\, i_{wire}$$

?

Step 1. Calculate $\vec{B}$ created by the wire

Step 2. Calculate flux through the toroid.

top

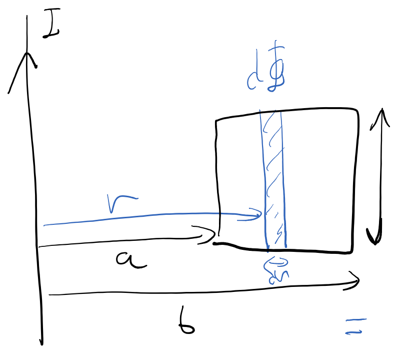


$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{in}$$

$$2\pi r |\vec{B}| = \mu_0 I$$

$$|\vec{B}| = \mu_0 \frac{I}{2\pi r}$$

Side



$$\Phi_{sq} = ?$$

$$d\Phi = \frac{\mu_0 I}{2\pi r}(b-a)\,dr$$

$$\Phi_{sq} = \int d\Phi$$

$$= \int_a^b \frac{\mu_0 I}{2\pi r}(b-a)\,dr$$

$$\Phi_{sq} = \frac{\mu_0 I}{2\pi}(b-a)\int_a^b \frac{1}{r}\,dr = \boxed{\frac{\mu_0 I}{2\pi}(b-a)\ln\left(\frac{b}{a}\right)}$$

$$\Phi_{toroid} = N\,\Phi_{sq} = \underbrace{\left[\frac{\mu_0 N (b-a)}{2\pi}\ln\left(\frac{b}{a}\right)\right]}_{M} I$$

(a) Self inductance?

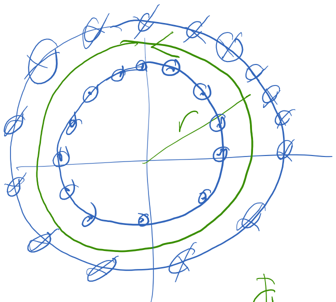
1°) Think a current I_{tor} is running through toroid → Find it's magnetic field

2°) Calculate Φ through the toroid

c) Calculate Φ_{tor} ...

$$\boxed{\Phi_{tor} = L \, I_{tor}}$$

Magnetic field inside the toroid



$$\underbrace{\oint \vec{B} \cdot d\vec{l}}_{2\pi r |\vec{B}|} = \mu_0 I_{in}$$

$$2\pi r |\vec{B}| = \mu_0 N I_{tor}$$

$$|\vec{B}| = \frac{\mu_0 N I_{tor}}{2\pi r}$$

$$\Phi_{tor} = \underbrace{\left[\frac{\mu_0 N^2 (b-a)}{2\pi} \ln\left(\frac{b}{a}\right) \right]}_{L} I_{tor}$$

Exactly the same integral!

Maxwell's equations and Electromagnetic waves.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \qquad \text{(Gauss' Law)}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss' Law for magnetic fields)}$$

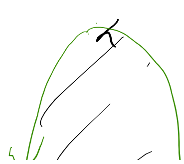
<u>No magnetic monopoles!</u>

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \text{(Ampere's Law)}$$

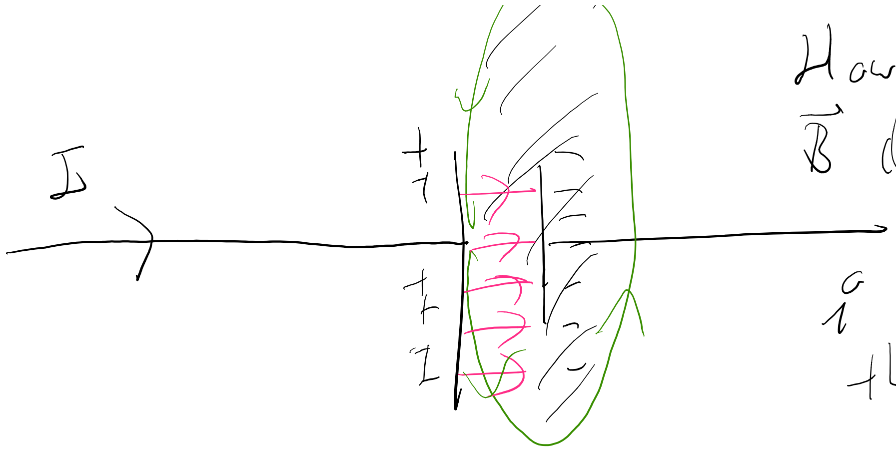
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

Maxwell

$$\int \vec{B} \cdot d\vec{l} = \mu_0 i^0$$



How come



How come $\vec{B}$ depends only on i crossing the surface!

$$I = \frac{dQ}{dt} \propto \frac{d\Phi_E}{dt}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{in} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$= \mu_0 \left(i_{in} + \underbrace{\varepsilon_0 \frac{d\Phi_E}{dt}}_{\text{the displacement current.}} \right)$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \qquad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

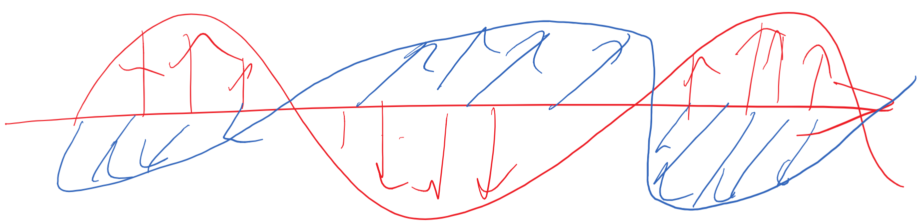
$$\oint \vec{B}\cdot d\vec{A} = 0 \qquad \oint \vec{B}\cdot d\vec{l} = \mu_0 i_{in} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Source terms

induction terms!

$$\vec{B}(t) \xRightarrow{\text{Ampere's law}} \vec{E}(t) \Rightarrow \vec{B}(t) \Rightarrow \ldots$$

$\vec{E}$ & $\vec{B}$ can support each other without the need for charges or currents.



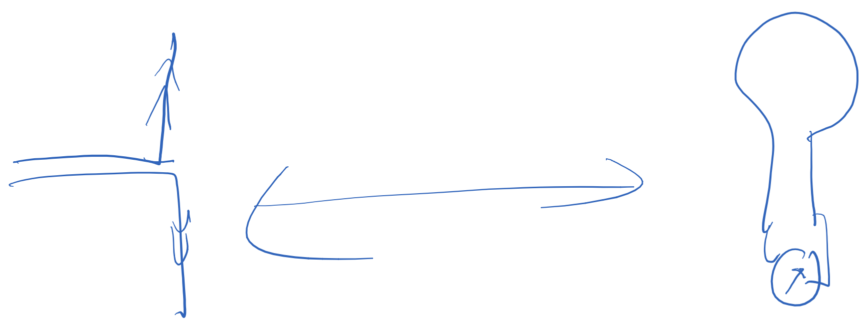
$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$c \cong 3\ 10^8 \text{ m/s}$$

speed of light!

EM waves $\Longrightarrow$ light

Hertz



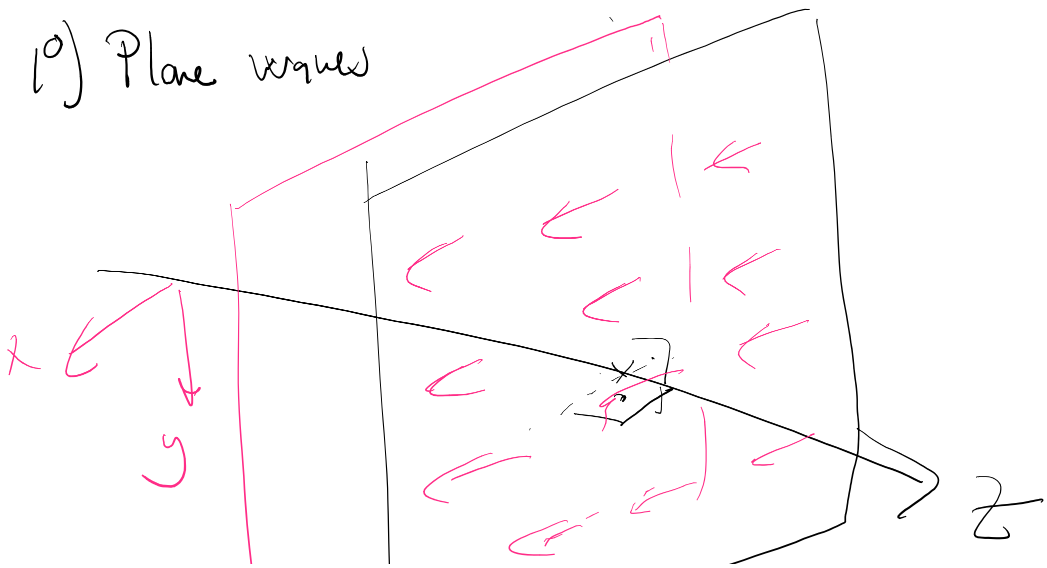
What do electromagnetic waves look like?

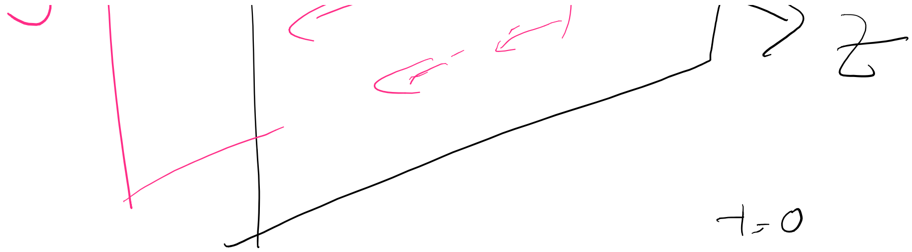
Propagation

propagation direction

$$\vec{E}(x,y,z,t) = E_0 \,\hat{\imath}\, \cos(kz-\omega t)$$

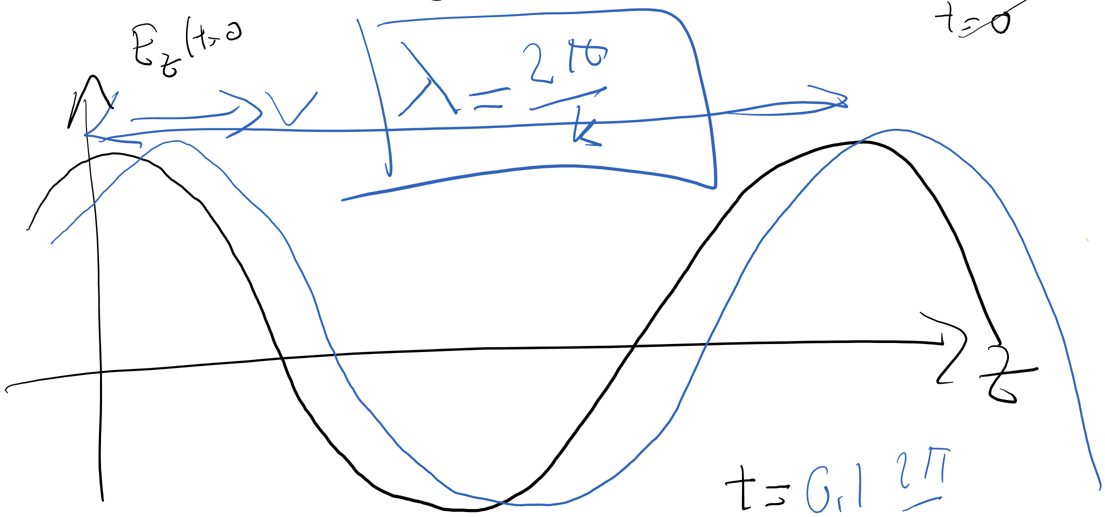
1°) Plane waves





2°) Behavior along z $\qquad E_x = E_0\cos(kz - \cancel{\omega t})$

$t=0$



$t = 0.1\ \frac{2\pi}{\omega}$

$$E_x = E_0\cos\left(kz - \cancel{\omega}\frac{2\pi}{\cancel{\omega}}\,0.1\right)$$

$t = 0.2\ \frac{2\pi}{\omega}$

$$E_x = E_0\cos(kz - \omega t)$$
$$= E_0\cos\left(k\left(z - \frac{\omega}{k}t\right)\right)$$

$$c = v = \frac{\omega}{k} \qquad \boxed{\omega = ck}$$

$$\lambda = \frac{2\pi}{k} \Rightarrow \omega = c\frac{2\pi}{\lambda}$$

$\omega \to$ ang. freq $\to$ frequency

$\omega = 2\pi \boxed{\nu}$

$\cancel{2\pi}\nu = \cancel{2\pi}\frac{c}{\lambda}$

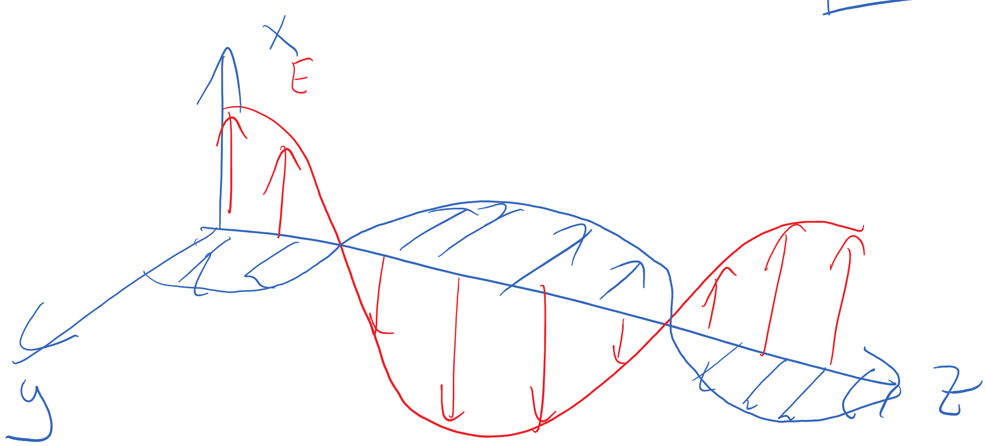
$$\boxed{c = \lambda \nu}$$

3°) $\vec{E}$ and $\vec{B}$ are $\perp$.

they are also $\perp$ to the direction of propagation

$$\hat{E} \times \hat{B} = \hat{k}$$

Transverse waves

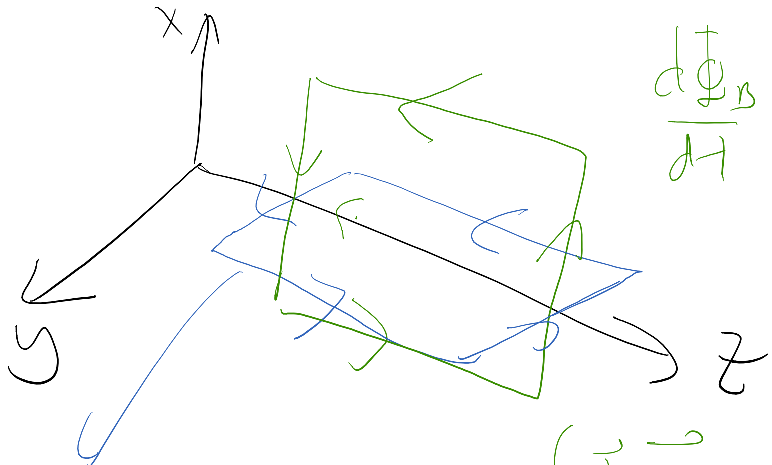


4°) $\vec{E}$ and $\vec{B}$ are in phase

An EM wave propagating along z would be

$$\vec{E} = E_0 \, \hat{i} \cos(kz - \omega t)$$

$$\vec{B} = B_0 \, \hat{j} \cos(kz - \omega t)$$



$$\frac{d\Phi_B}{dt} \propto B_0$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt} \propto B_0$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \underbrace{\frac{d\Phi_E}{dt}}_{\propto E_0}$$

$$\left(\frac{E_0}{B_0}\right)^2 = \frac{1}{\mu_0 \varepsilon_0} \qquad E_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} B_0$$

$$\boxed{E_0 = c B_0}$$

$$u_E = \frac{1}{2} \varepsilon_0 E_0^2 \qquad u_B = \frac{1}{2\mu_0} B_0^2$$

$$= \frac{1}{2\mu_0}\frac{E_0^2}{c^2}$$

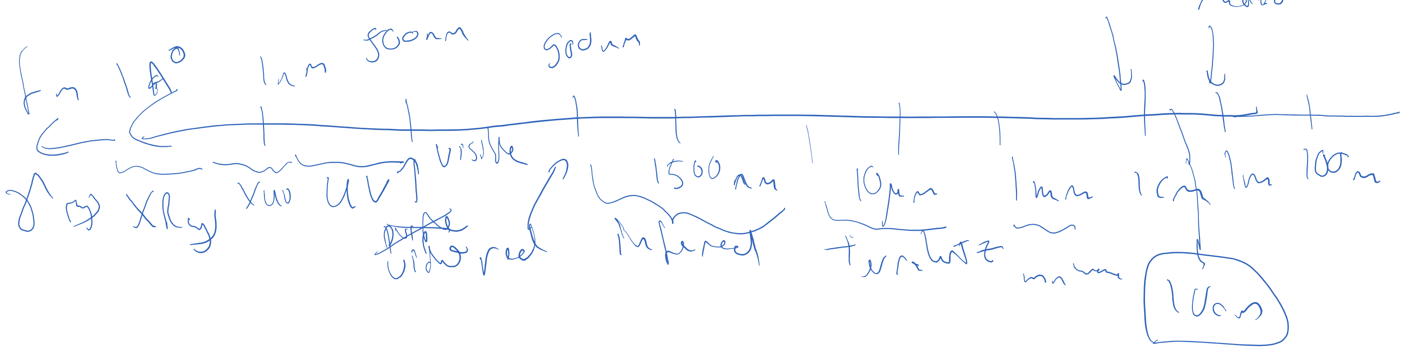
$$= \frac{1}{2\cancel{\mu_0}}\frac{E_0^2}{\frac{1}{\cancel{\mu_0}\varepsilon_0}} = \frac{1}{2}\varepsilon_0 E_0^2$$

$$\boxed{U_E = U_B}$$

Energy can be carried by EM waves!

* Momentum can also be carried by EM waves.

λ



Quiz 27

Bilkent Radio 96.6 MHz

Megahertz

$\nu = 96.6\ 10^6$ Hz

$\lambda = ?$

$\omega = c\,k$

$2\pi\nu = c\,\frac{\cancel{2\pi}}{}$

$$\lambda = \frac{c}{\nu}$$

$$\lambda = \frac{3\,10^{8}\ m/\cancel{sec}}{96.6\ 10^{6}\ 1/\cancel{sec}} = \boxed{3.1\ m}$$

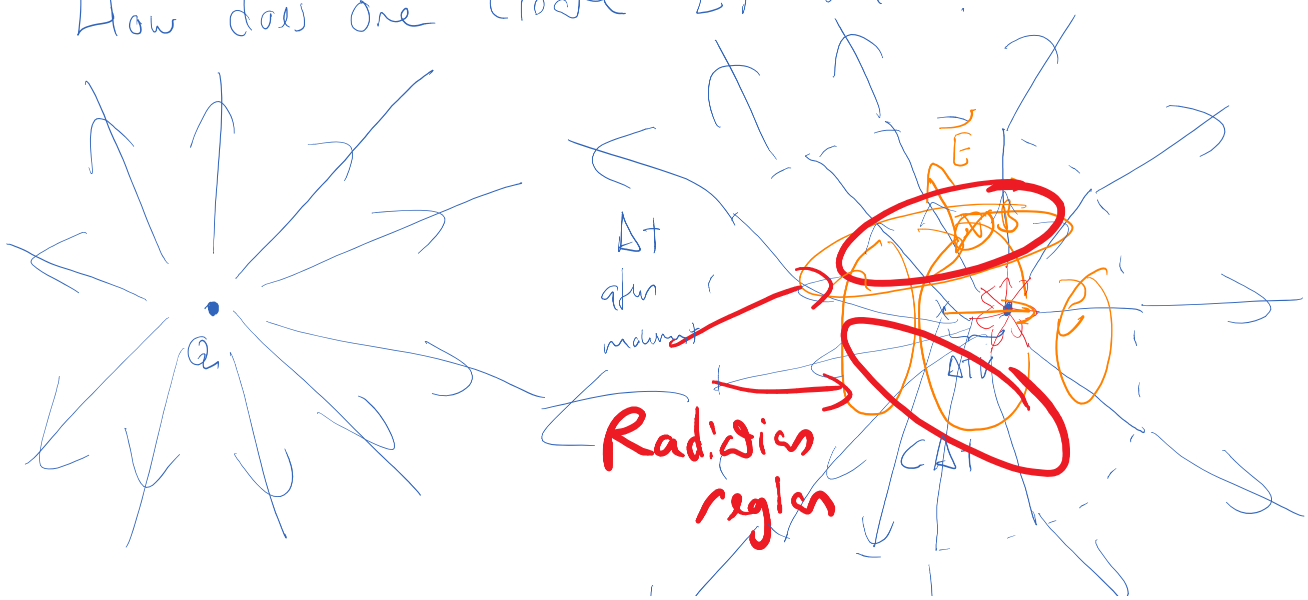
B) $$T = \frac{1}{\nu} = \frac{1}{96.6\ 10^{6}} \simeq 10^{-8}\ sec$$
$$= 10\ nsec$$

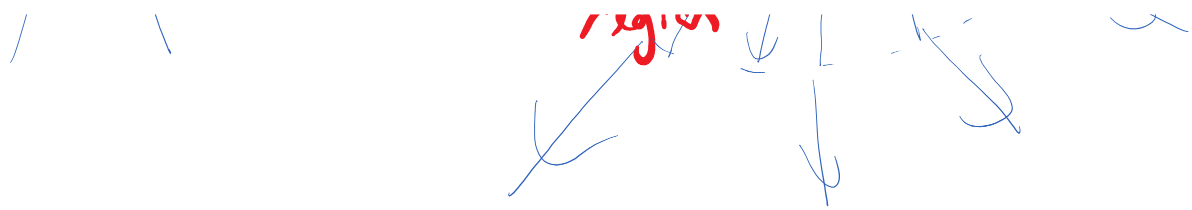
C) 10cm ~ λ ~ 10cm

$$\nu = \frac{c}{\lambda} = \frac{3\,10^{8}\ m/s}{0.1\ m} = 30\,10^{8}\ Hz$$
$$\simeq 3\ GHz$$

Energy in EM waves

How does one create EM waves?





Accelerated Charges create EM fields

EM wave

$\hat{k}$

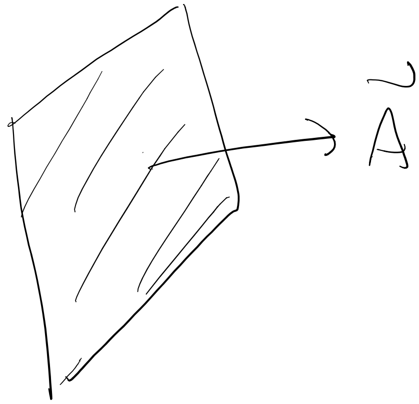
$$\vec{E} = \vec{E}_0 \cos(\vec{r}\cdot\vec{k} - \omega t)$$

$$\vec{B} = \vec{B}_0 \cos(\vec{r}\cdot\vec{k} - \omega t)$$

$$\omega = c|\vec{k}| \qquad |\vec{E}_0| = c|\vec{B}_0|$$

$$\hat{E} \times \hat{B} = \hat{k}$$

What is the energy transmitted per unit area per unit time by an EM wave?



$$P = \underbrace{\vec{S}}\cdot\vec{A}$$

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$$

Poynting

$$[\vec{S}] = \frac{W}{m^2} = \frac{J}{s\,m^2}$$

$\vec{S}$ also tells us about

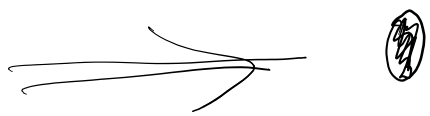
How much momentum is carried by EM

wave.

$$\vec{P} = \frac{1}{c}\vec{S}$$

$$\boxed{E = cP}$$

If we have a 10 Watt laser absorbed by a black body with mass $m = 1\,g$ for 1 sec. what is its final velocity?



$\to V = ?$

$$\frac{P}{c}\Delta t = Mv$$

~ 3

$$\frac{\cancel{10}\,\frac{J}{s}\,1\,sec}{\cancel{3}\,10^{8}\,m/s} = 10^{-3}\,kg\;V$$

$$\boxed{V = 3.3\,10^{-5}\,m/s}$$