Week 12 Notes (Apr 11-15)

Wednesday, January 27, 2016 4:48 PM

QUIZ-22

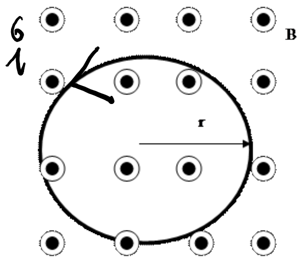
A ring of radius r is under a magnetic field that is out of the page which has a magnitude

$B(t) = B_0 e^{-5t}$

.

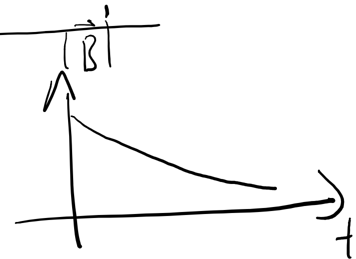
a) What is the direction of the induced current?

b) Find the induced current as a function of time if the total resistance of the ring is R.



Lenz's law!

$$B(t) = B_0 e^{-5t}$$



$\Phi(t) \downarrow$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

B_{loop} should be in the same direction as $\vec{B}$ ⊙

$$\Phi_B(t) = \int \vec{B}\cdot d\vec{A} = |\vec{B}|A = \pi r^2 B_0 e^{-5t}$$

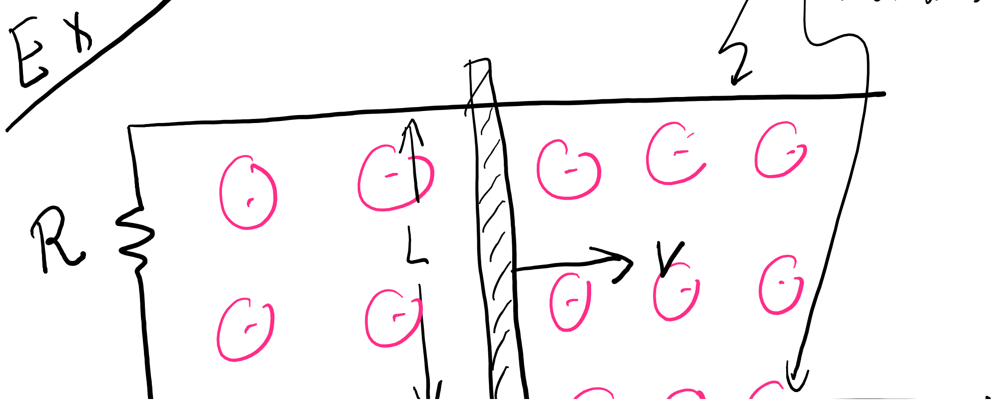
i_{loop} Counterclockwise

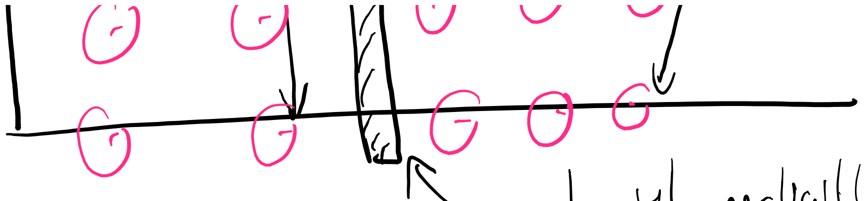
$$\mathcal{E} = -\frac{d}{dt}\Phi_B(t) = -\pi r^2 B_0 \frac{d}{dt} e^{-5t} = \boxed{5\pi r^2 B_0 e^{-5t}}$$

$$i_{loop} = \frac{\mathcal{E}}{R} = \boxed{\frac{5\pi r^2 B_0}{R} e^{-5t}}$$

Ex

frictionless rails

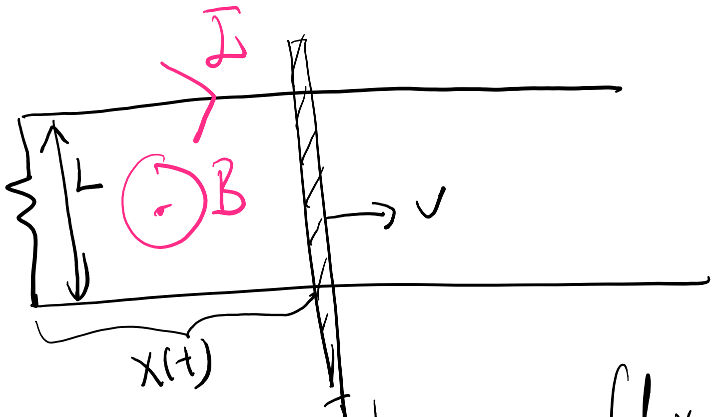




The rod is moving with velocity v as shown.

a) Calculate the current in the circuit.

b) Find the force that is needed to keep the rod going with velocity v.



B_{ext} $\odot$

$\Phi(t) \uparrow$

B_{loop} must oppose B_{ext}

B_{loop} $\otimes$ I_{loop} $\circlearrowright$ clockwise

Area is changing $\Rightarrow$ flux is changing!

I_{loop} is clockwise

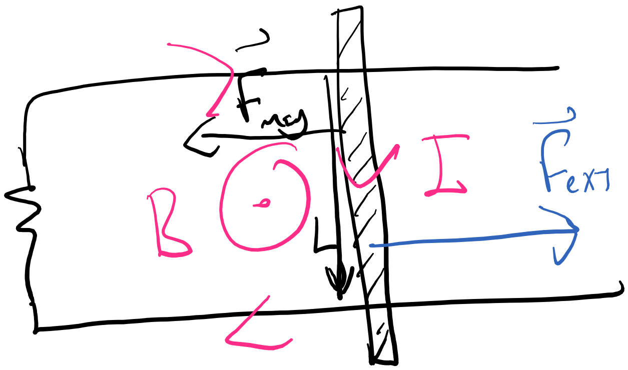
$$\Phi(t) = L\, x(t)\, B$$

$$\mathcal{E} = -\frac{d\Phi(t)}{dt} = -LB \underbrace{\frac{dx(t)}{dt}}_{v} = \boxed{-BvL}$$

$$i = \frac{|\mathcal{E}|}{R} = \boxed{\frac{BvL}{R}}$$

$\tilde{v}$

$$i = \frac{|\varepsilon|}{R} = \boxed{\frac{BvL}{R}}$$



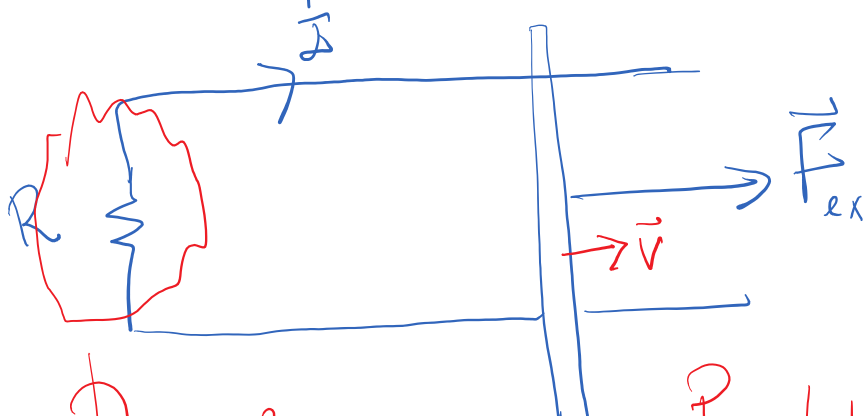
$$\vec F_{mag} = i \vec L \times \vec B$$

$$|\vec F_{ext}| = |\vec F_{mag}|$$

$$= i L B$$

$$|F_{ext}| = \frac{BvL}{R} L B$$

$$\boxed{F_{ext} = \frac{B^2L^2v}{R}}$$



$$P = I^2 R$$

$$= \frac{B^2v^2L^2}{R^{\cancel{2}}}\cancel{R}$$

$$P_{applied} = \vec F \cdot \vec v$$

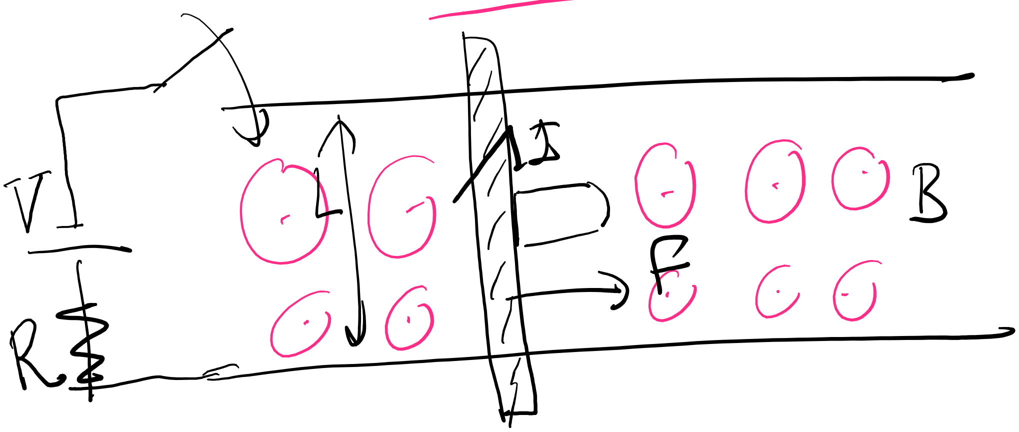
$$= \frac{B^2L^2v}{R} v$$

$$P = \frac{B^2v^2L^2}{}$$

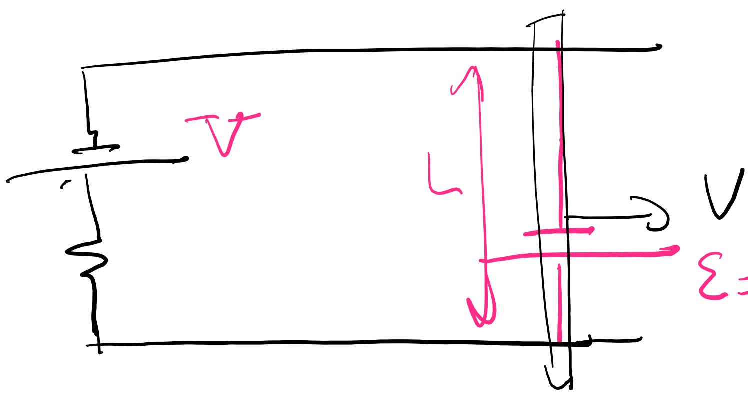
$$P = B^2L^2v^2$$

$$\boxed{P = \frac{B^2 v^2 L^3}{R}} \Longleftarrow \boxed{P_{apply} = \frac{B^2 L^2 v^2}{R}}$$

EM $\boxed{\text{rail gun}}$



What is the maximum velocity in this rail gun?



$\mathcal{E} = BvL$

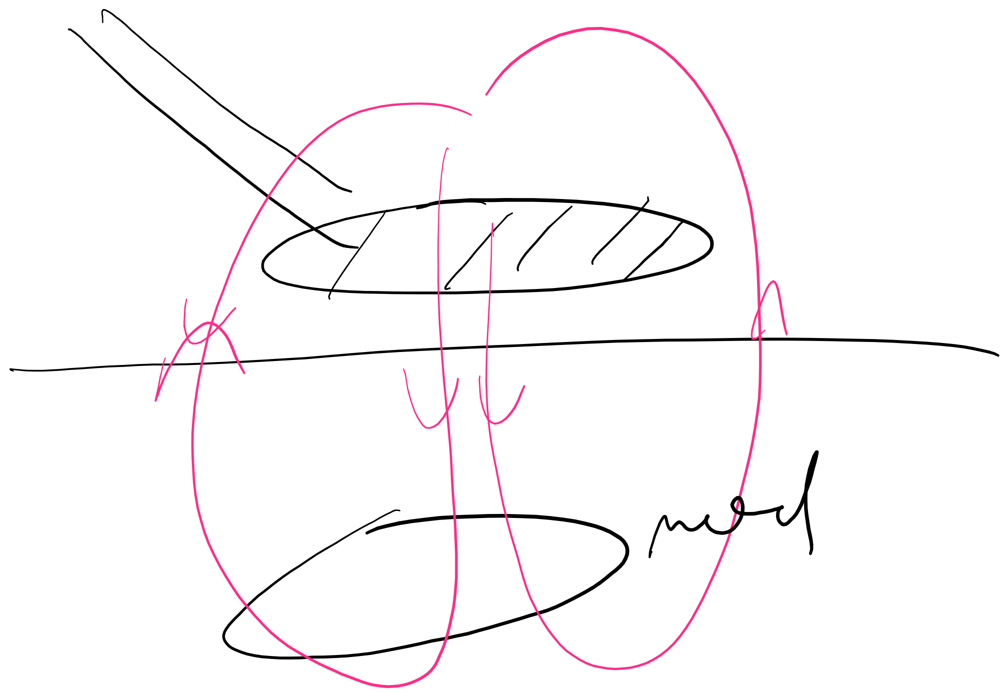
Δ If $V = BvL$

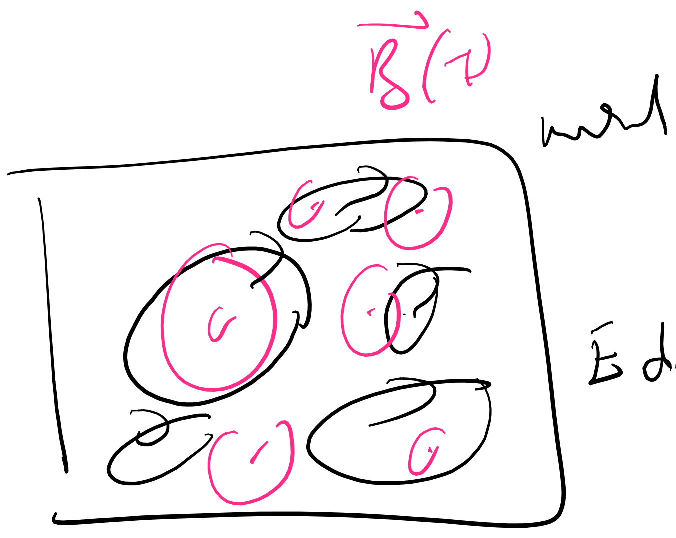
no current flows in the circuit!

$$\boxed{v_{max} = \frac{V}{BL}}$$

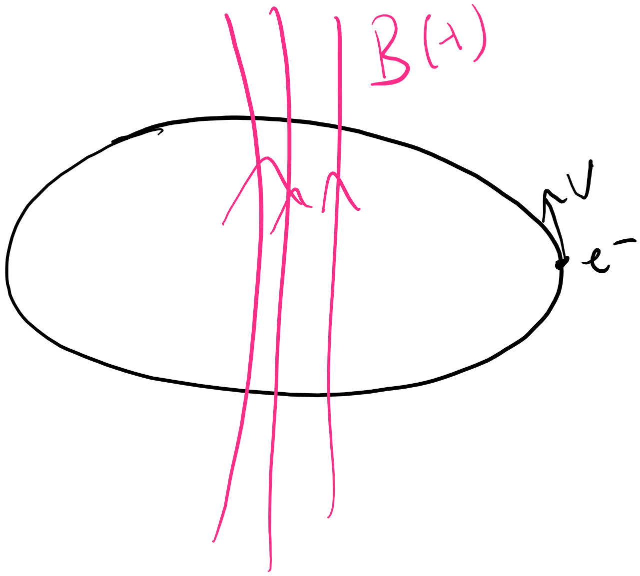
$$\boxed{V_{max} = \frac{V}{BL}}$$

follows in the circuit!





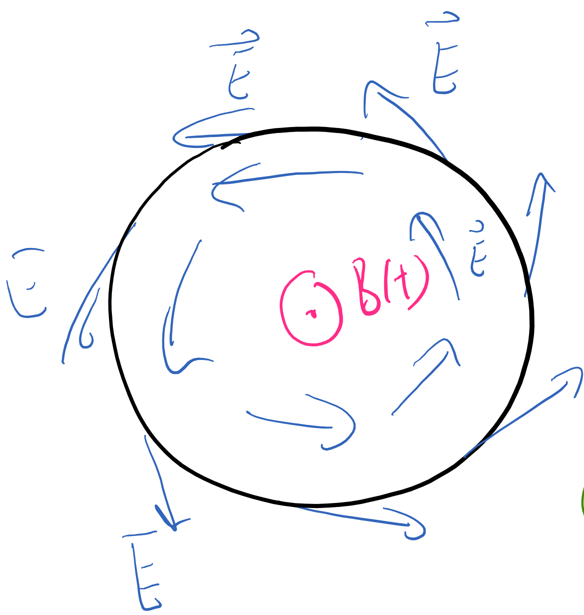
Eddy currents



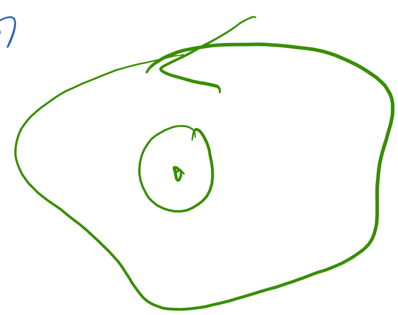
1°) $\vec{F} = q\vec{v} \times \vec{B}$

2°) $\vec{B}$ may be far away!

Even in free space $\boxed{\dfrac{d\vec{B}}{dt} \Rightarrow \vec{E}}$!!!



$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B(t)}{dt}$$

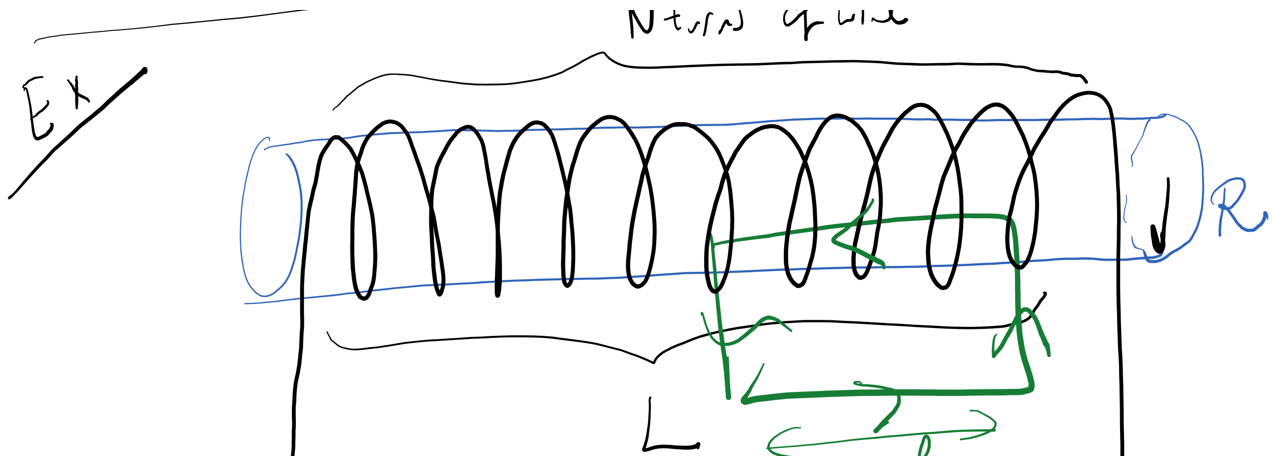


Electric field is induced even when there are no wire loops to carry a current.

Ex.

N turns of wire

Ex



$i(t) = I_0 \frac{t}{\tau}$ $\quad$ I_0 and τ are constants

Find the magnetic field and the electric field everywhere in space as a function of time.

Assuming a long & tightly wound solenoid.

$$\underbrace{\oint \vec{B} \cdot d\vec{l}}_{|\vec{B}|\, l} = \mu_0 \underbrace{i_{in}}_{i \frac{N}{L} l} \Longrightarrow |\vec{B}|\, l = i \frac{N}{L}\, l\, \mu_0$$

inside $\vec{B}$ is constant, outside it is zero

$$\vec{B}(t) = \mu_0 \frac{N}{L} i(t)$$

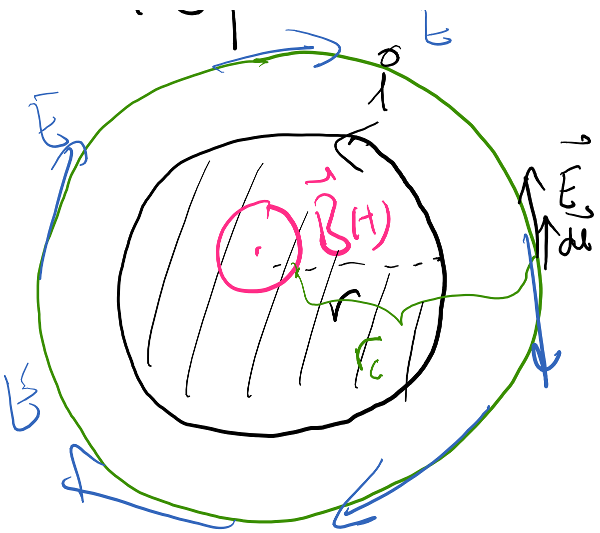
$\vec{B}$ inside $\Rightarrow$ $\boxed{\vec{B}(t) = \mu_0 \frac{N}{L} I_0 \frac{t}{\tau}}$

$\vec{B}_{outside} = 0$

Top view

Find $\vec{E}$ field outside!

Find E field outside.



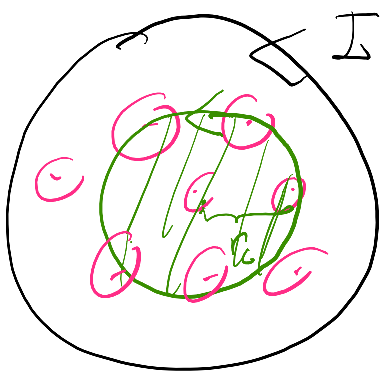
$$\oint \underbrace{\vec{E}\cdot d\vec{l}}_{\vec{E} \parallel d\vec{l}} = -\frac{d\Phi_B}{dt}$$

$$2\pi r_c |\vec{E}| = -\frac{d}{dt}\left(\pi r^2 B(t)\right)$$

$$|\vec{E}| = \frac{+1}{2\pi r_c}\pi r^2 \mu_0 \frac{N}{L} I_0 \frac{d}{dt}\left(\frac{t}{\tau}\right)$$

$$\boxed{|\vec{E}| = \frac{1}{2\pi r_c}\pi r^2 \mu_0 \frac{N I_0}{L\tau}}$$

$\vec{E}$ inside the solenoid?



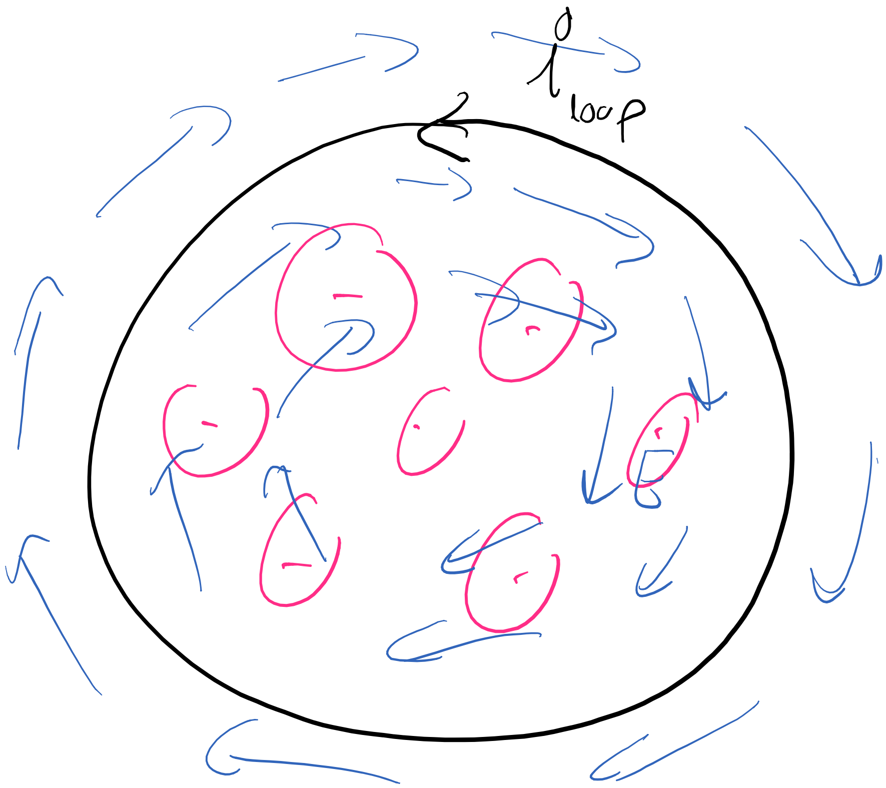
$$\underbrace{\oint \vec{E}\cdot d\vec{l}}= -\frac{d}{dt}\Phi_B$$

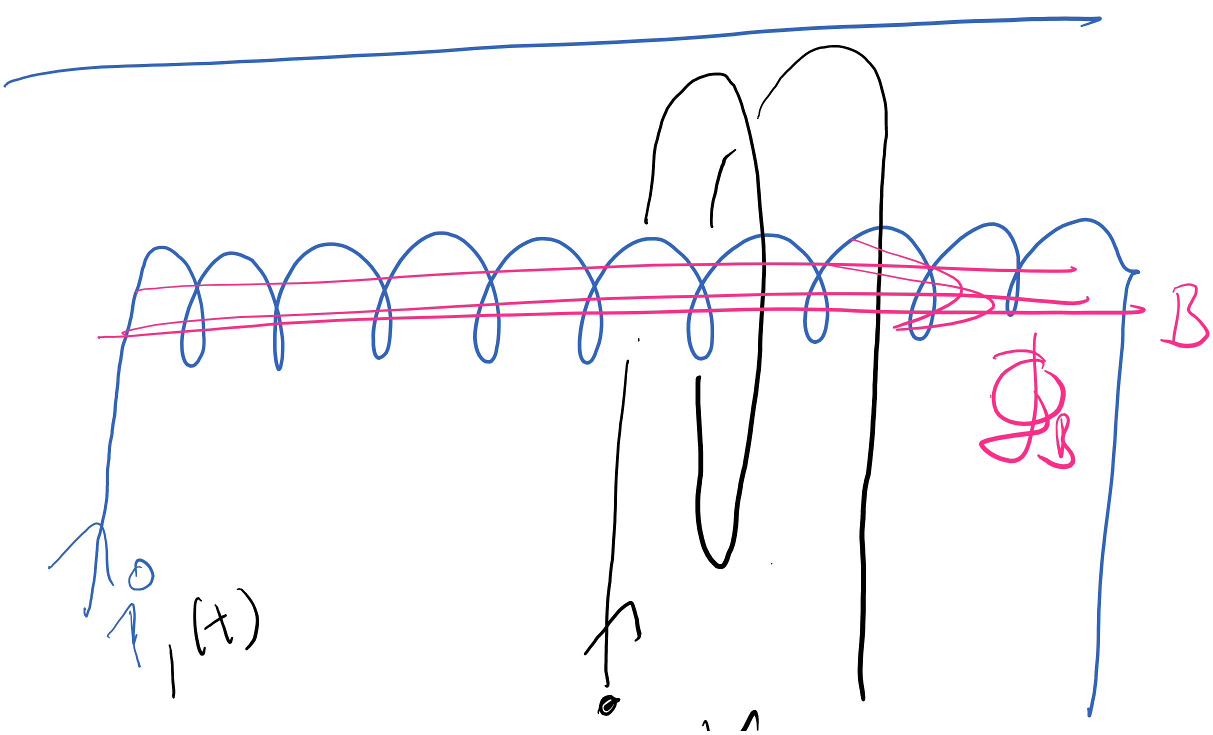
$$|\vec{E}|\,2\pi r_c = +\frac{d}{dt}\left(\pi r_c^2 B(t)\right)$$

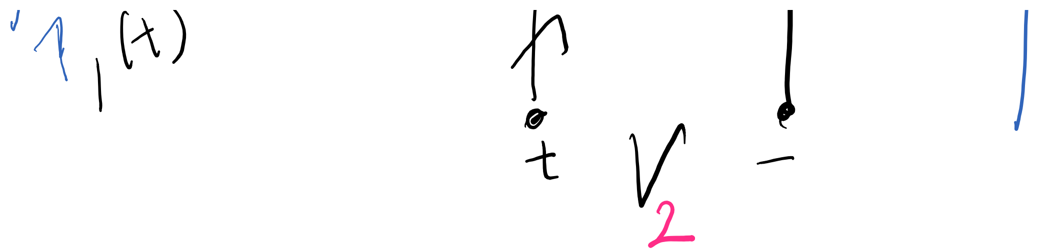
$$|\vec{E}|\,2\cancel{\pi} r_c = +\frac{d}{dt}\left(\cancel{\pi} r_c^2 \mu_0 \frac{N}{L} I_0 \frac{t}{\tau}\right)$$

$$\boxed{|\vec{E}| = \frac{r_c}{2}\mu_0 \frac{N}{L}\frac{i_0}{\tau}}$$

$r_c < r$

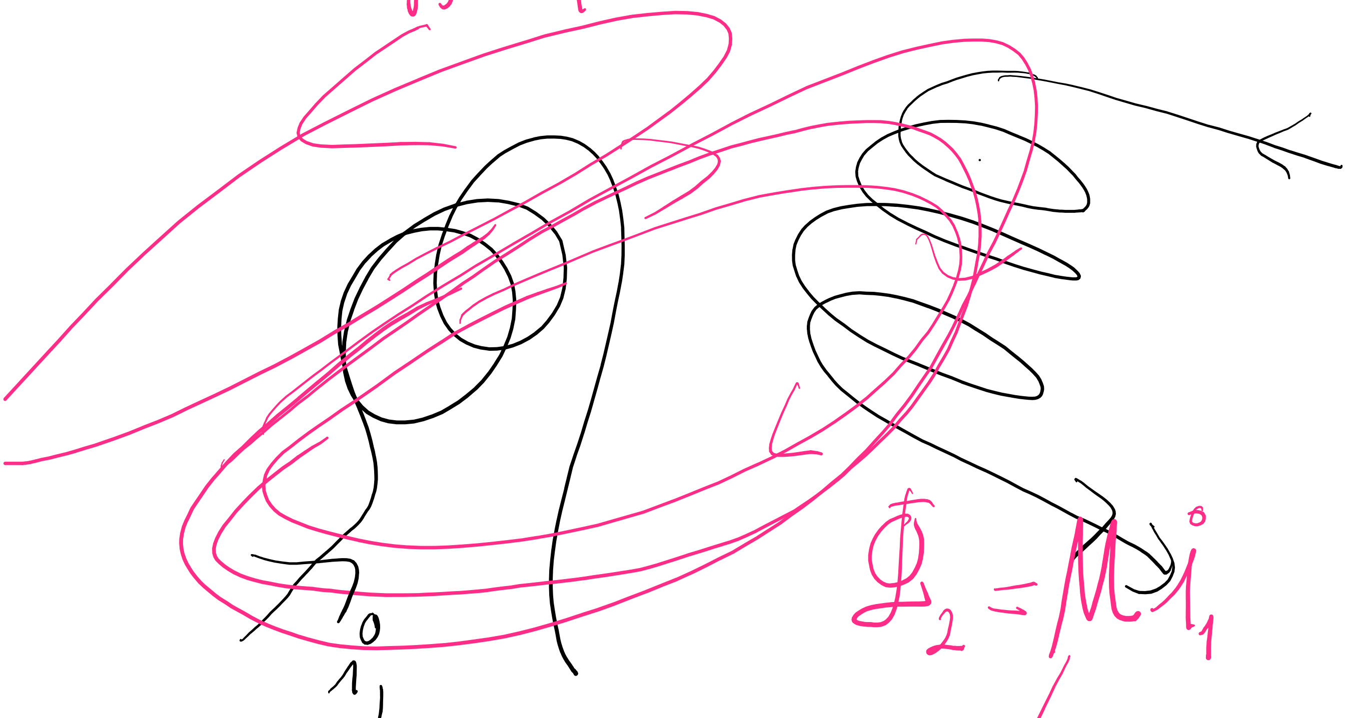






$$|V_2| = \frac{d\Phi_B(t)}{dt}$$ ← controlled by i_1

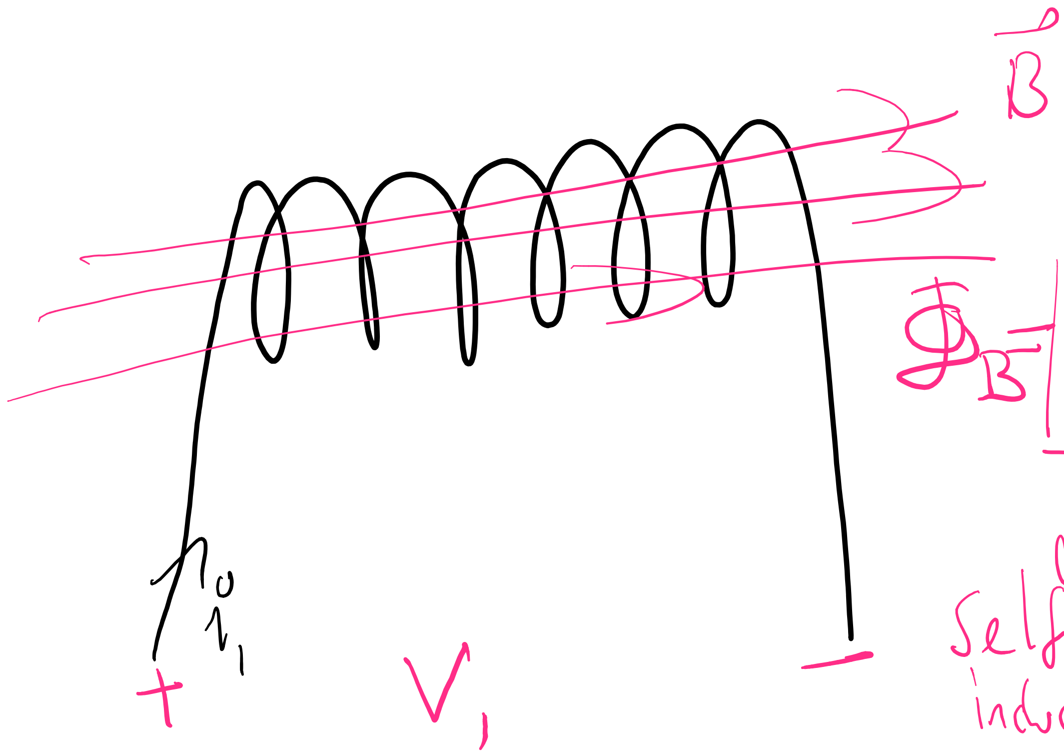
Changing i_1 with time creates a V_2!



$$\Phi_2 = M i_1$$

mutual inductance of the loops

$$|V_2| = \frac{d}{dt}\Phi_2 = M\frac{di_1}{dt}$$



$\Phi_B = \boxed{L}\, i_1$

Self inductance

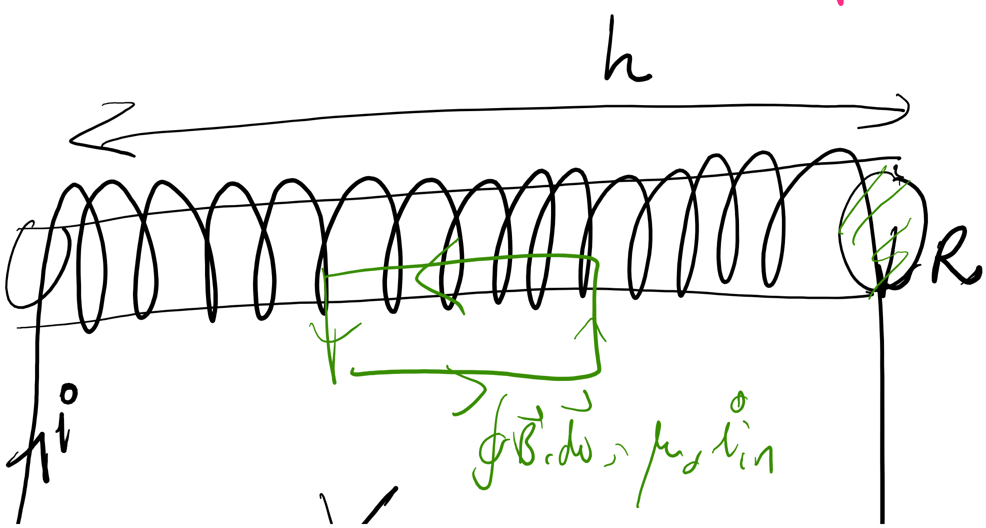
$$|v_1| = \frac{d\Phi_B}{dt} = \frac{d}{dt}(L i_1)$$

$$\boxed{V_1 = L \frac{di}{dt}}$$

$$i_c = C \frac{dV}{dt}$$

inductance ~ capacitance

Ex



$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{in}$

who is the inductance of the solenoid.

$$V = L \frac{di}{dt}$$

$$B = \mu_0 \, i(t) \frac{N}{h}$$

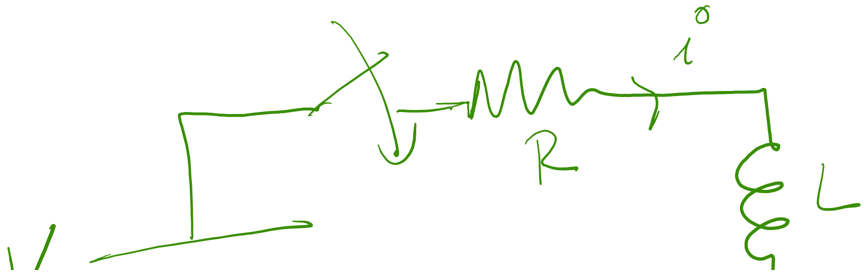
$$\Phi_B = \pi R^2 B$$

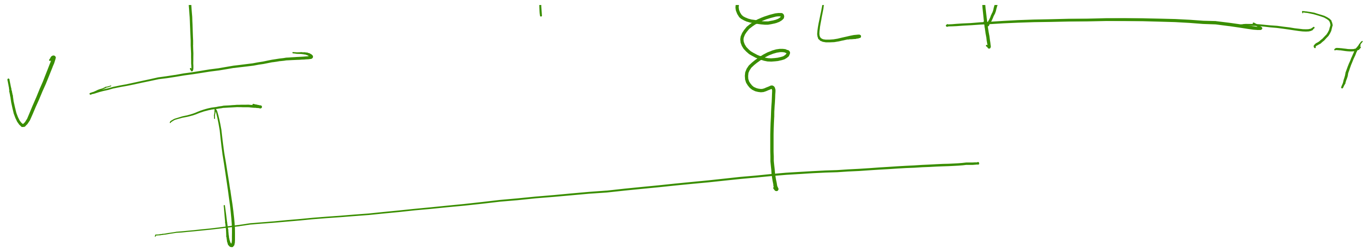
$$\Phi_{total} = N \Phi_B = N \pi R^2 \mu_0 \frac{N}{h} i(t)$$

$$\Phi_{total} = \pi R^2 \mu_0 \frac{N^2}{h} i(t)$$

$$V = \frac{d\Phi_{tot}}{dt} = \underbrace{\pi R^2 \mu_0 \frac{N^2}{h}}_{L} \frac{di}{dt}$$

$$\boxed{L = \pi R^2 \mu_0 \frac{N^2}{h}}$$





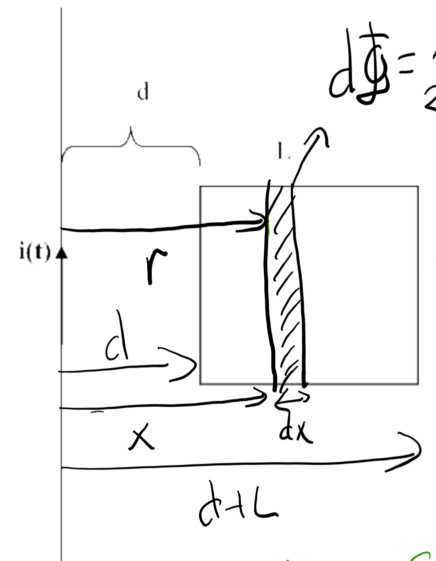
QUIZ-23

An infinite wire carries a time dependent current $i(t) = I_0 e^{-t/\tau}$. At a distance **d** away from the wire is a square loop of wire, which has side **L** and total resistance **R**.

a) Find the mutual inductance between the wire and the loop.
b) Find the current that runs through the loop as a function of time. Clearly indicate whether the current is clockwise or counterclockwise.
c) Find the total force the wire applies to the loop. Is the wire pulling or pushing the loop?

?

$$d\Phi = \frac{\mu_0 i}{2\pi x} L\,dx \qquad \Phi_{loop} = M\, i_{wire}$$



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{wire}$$

$$2\pi r |\vec{B}| = \mu_0 i_{wire}$$

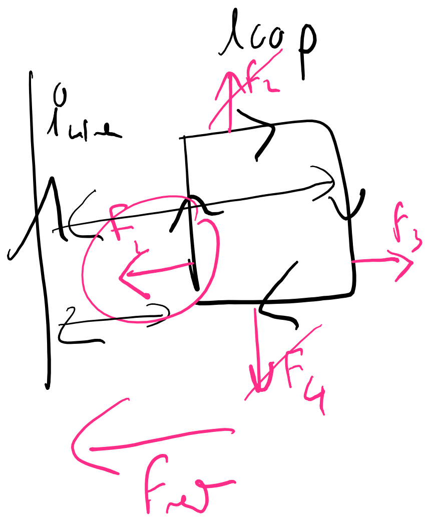
$$\boxed{|\vec{B}(r)| = \frac{\mu_0 i_{wire}}{2\pi r}}$$

$$\Phi_{loop} = \int \vec{B}\cdot d\vec{A} = \int d\Phi = \int_d^{d+L} \frac{\mu_0 i}{2\pi x} L\,dx$$

$$\Phi_{loop} = \frac{\mu_0 i L}{2\pi}\int_d^{d+L} \frac{1}{x}\,dx = \frac{\mu_0 i L}{2\pi} \ln\left(\frac{L+d}{d}\right)$$

$$M = \frac{\mu_0 L}{2\pi} \ln\left(\frac{L+d}{d}\right)$$

loop



$i_{wire} \downarrow$

$\vec{B}_{wire}$ is $\otimes$ within the loop

$\Phi_{loop} \downarrow$

$\vec{B}_{loop}$ is in the same direction as $\vec{B}_{wire}$ $\otimes$

i_{loop} clockwise

$$i_{wire}(t) = I_0 e^{-t/\tau}$$

$$\Phi_{loop}(t) = M\, i_{wire}(t) = M I_0 e^{-t/\tau}$$

$$|\mathcal{E}| = \left|\frac{d\Phi_{loop}}{dt}\right| = \left| M I_0 \frac{d}{dt} e^{-t/\tau} \right|$$

$$\boxed{\mathcal{E} = \frac{M I_0}{\tau} e^{-t/\tau}}$$

$$i_{loop} = \frac{\mathcal{E}}{R} = \frac{M I_0}{R\tau} e^{-t/\tau}$$

c) $F_1 = \dfrac{\mu_0\, i_{wire}\, i_{loop}}{2\pi d} L$ $\qquad$ $F_3 = \dfrac{\mu_0\, i_{wire}\, i_{loop}}{2\pi (d+L)} L$

$$F_1 = \frac{\mu_0 i_{wire} i_{loop}}{2\pi d} L \qquad F_3 = \frac{\quad}{2\pi (d+L)} L$$

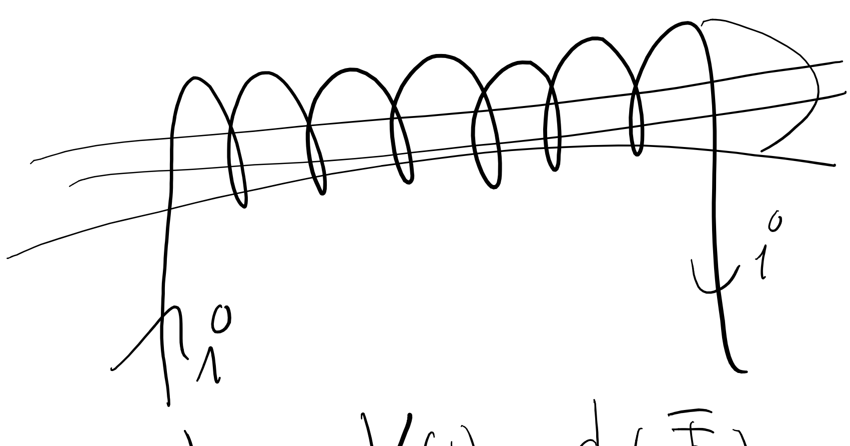
$$F_{net} = F_1 - F_3 = \frac{\mu_0 i_{wire} i_{loop}}{2\pi} L \left(\frac{1}{d} - \frac{1}{d+L} \right)$$

$$= \frac{\mu_0 i_{wire} i_{loop}}{2\pi} \frac{L^2}{d(d+L)}$$

$$= \frac{\mu_0}{2\pi} I_0 e^{-t/\tau} \frac{M I_0 e^{-t/\tau}}{R\tau} \frac{L^3}{d(d+L)}$$

$$= \frac{\mu_0 I_0^2 e^{-2t/\tau} L^3}{2\pi R \tau d(d+L)} \mu_0 \frac{L}{2\pi} \ln\left(\frac{L+d}{d}\right)$$

$$\boxed{F_{net} = \frac{\mu_0^2 I_0^2}{(2\pi)^2 R\tau} \frac{L^3}{d(d+L)} \ln\left(\frac{L+d}{d}\right)}$$



$$\Phi_{total} = L\, i$$

$$\downarrow$$

inductance

$$V(t) = \frac{d}{dt}(\Phi) = L \frac{di}{dt}$$

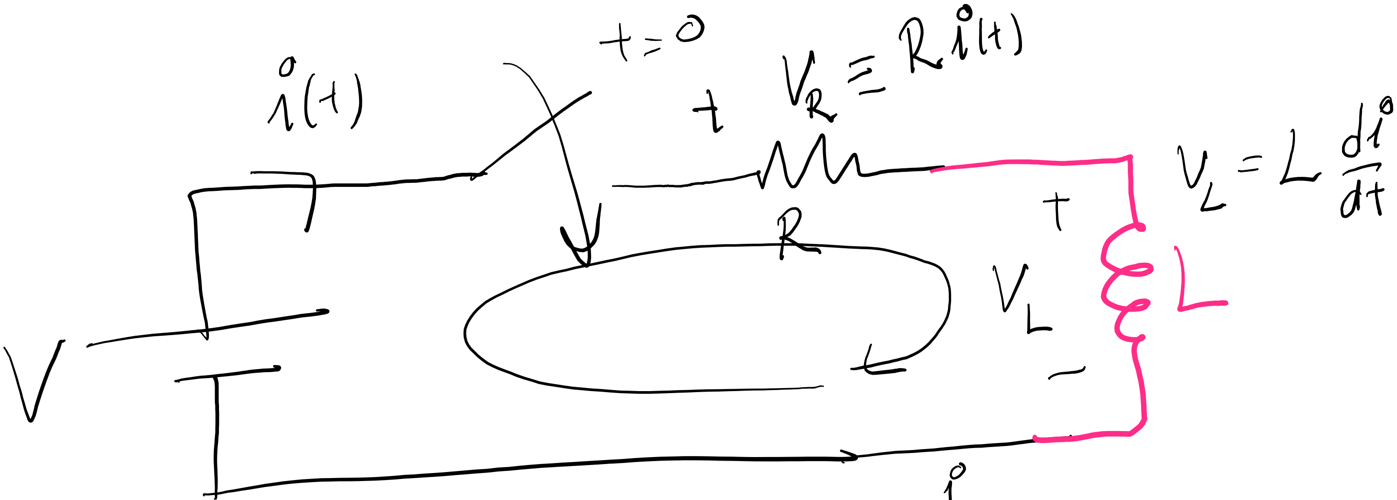
inductor

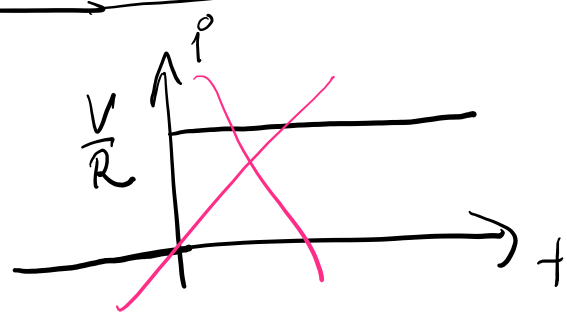
$$V(t) = L\frac{di}{dt}$$

inductor

$$V = Ri$$

$$i = C\frac{dV}{dt}$$





$$V = V_R + V_L$$

$$\boxed{V = R\,i(t) + L\frac{di(t)}{dt}}$$

$$i(t=0) = 0$$

$$J(t) = i(t) - \frac{V}{R} \Rightarrow \frac{dJ}{dt} = \frac{di}{dt}$$

$$V = R\left(J + \frac{V}{R}\right) + L\frac{dJ}{dt}$$

$$\cancel{V} = RJ + \cancel{V} + L\frac{dJ}{dt}$$

$$\boxed{\frac{dJ}{dt} = -\frac{R}{L}J}$$

$$J(t) = A e^{-\alpha t}$$

$$\frac{dJ}{dt} = -\alpha A e^{-\alpha t}$$

$$-\alpha \cancel{A e^{-\alpha t}} = -\frac{R}{L}\cancel{A e^{-\alpha t}} \Rightarrow \boxed{\alpha = \frac{R}{L}}$$

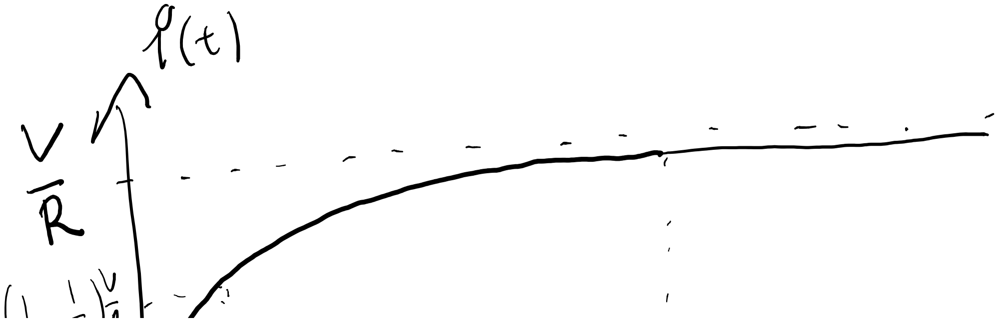
$$J(t=0) = \underbrace{i(t=0)}_{0} - \frac{V}{R} = -\frac{V}{R} = A\underbrace{e^{-\alpha 0}}_{1}$$

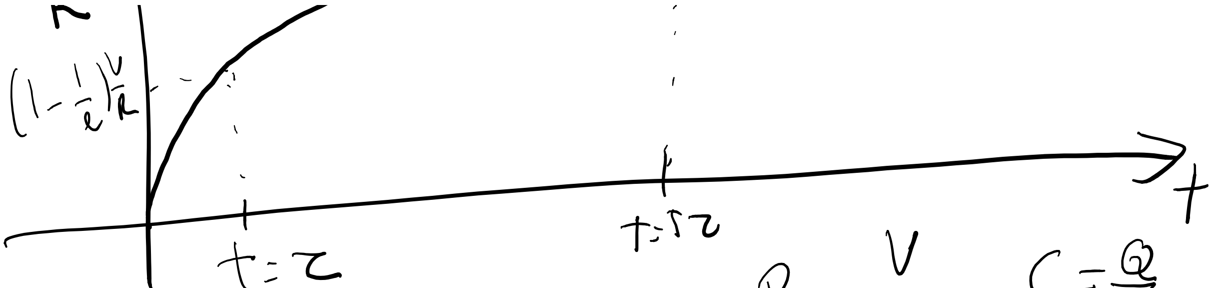
$$J(t) = -\frac{V}{R} e^{-t/(L/R)}$$

$$i(t) = -\frac{V}{R} e^{-t/(L/R)} + \frac{V}{R}$$

$$i(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right) \qquad \boxed{\tau = \frac{L}{R}}$$

time constant





$R = \frac{V}{I}$ $\quad C = \frac{Q}{V}$

$[\tau_{RC}] = [R][C]$

$s = \Omega\, F$

$[F] = \frac{sec}{\Omega}$

$[R][C] = \frac{\cancel{[V]}}{\frac{[Q]}{[t]}} \frac{[Q]}{\cancel{[V]}} = sec$

$[\tau_{RL}] = \frac{[L]}{[R]}$

$[L] \longrightarrow Henry = \Omega\, sec$

$[R] \longrightarrow \Omega$

$\Phi = L\, i$

$\Phi \to BA = \underbrace{m^2\, T}_{Weber}$

$[\tau_{RL}] = sec$ ✓

$[L] = \frac{m^2\, T}{Ampere} \rightarrow \boxed{Henry}$

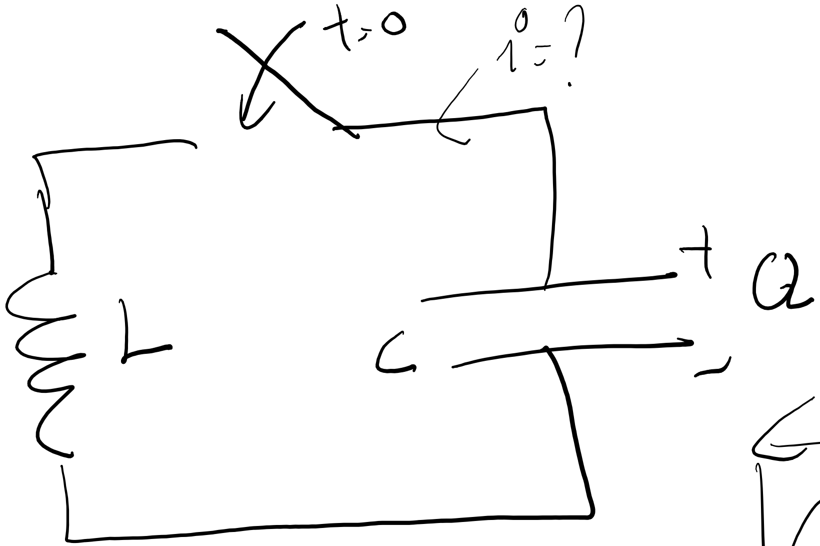
$\left[\frac{d\Phi}{dt}\right] = Volt = \frac{[\Phi]}{[t]}$

$[\Phi] = Weber = sec\ Volt$

$[L] = \frac{[\Phi]}{[i]} = Henry = \frac{sec\ Volt}{Ampere}$

$= \boxed{sec\ \Omega}$

$[LC] = ? = \cancel{\Omega}\,\sec\,\frac{\sec}{\cancel{\Omega}}$



$T \cong \sqrt{LC}$