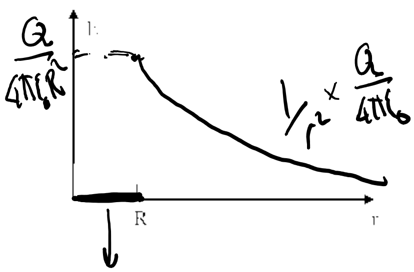
Week 5 Notes (Feb 22-26)

Wednesday, January 27, 2016 4:44 PM

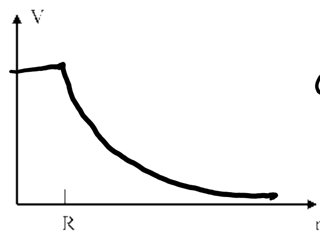
QUIZ-7

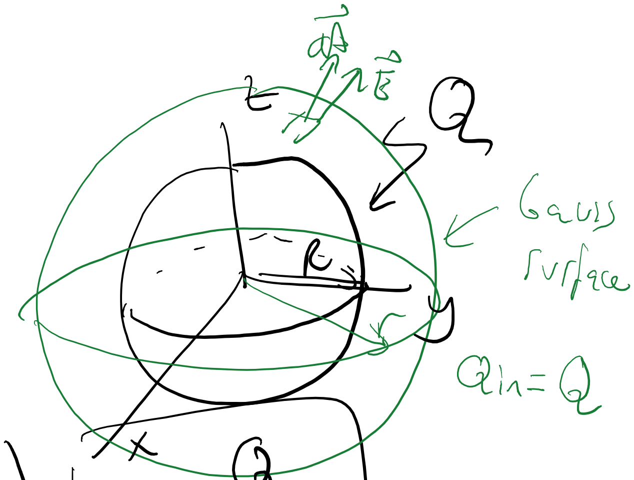
A conducting sphere of radius **R** is charged with a total charge **Q**.

a) What is the surface charge density on the surface of the sphere.
b) Find and plot the magnitude of the electric field **E** inside and outside the sphere.
c) Find and plot the potential **V**, inside and outside the sphere.



$E=0$ inside a conductor





a) $$\boxed{\sigma = \frac{Q}{4\pi R^2}}$$

b) $|\vec{E}(r)| = 0 \quad r < R$, because conductor!

$r > R$

$$\oint \underbrace{\vec{E} \cdot d\vec{A}}_{\vec{E} \parallel d\vec{A}} = \frac{Q_{in}}{\varepsilon_0}$$

$$\oint |\vec{E}| \, dA = \frac{Q}{\varepsilon_0}$$

$$|\vec{E}| \underbrace{\oint dA}_{4\pi r^2} = \frac{Q}{\varepsilon_0} \Longrightarrow \boxed{|\vec{E}(r)| = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

c) $$\Delta V = -\int_i^f \vec{E} \cdot d\vec{l}$$, Take $V(\infty) = 0$

$r_f > R$

$$V(r_f) = -\int^{r_f} E(r) \, dr = -\int^{r_f} \frac{Q}{4\pi\varepsilon_0 r^2} \, dr$$

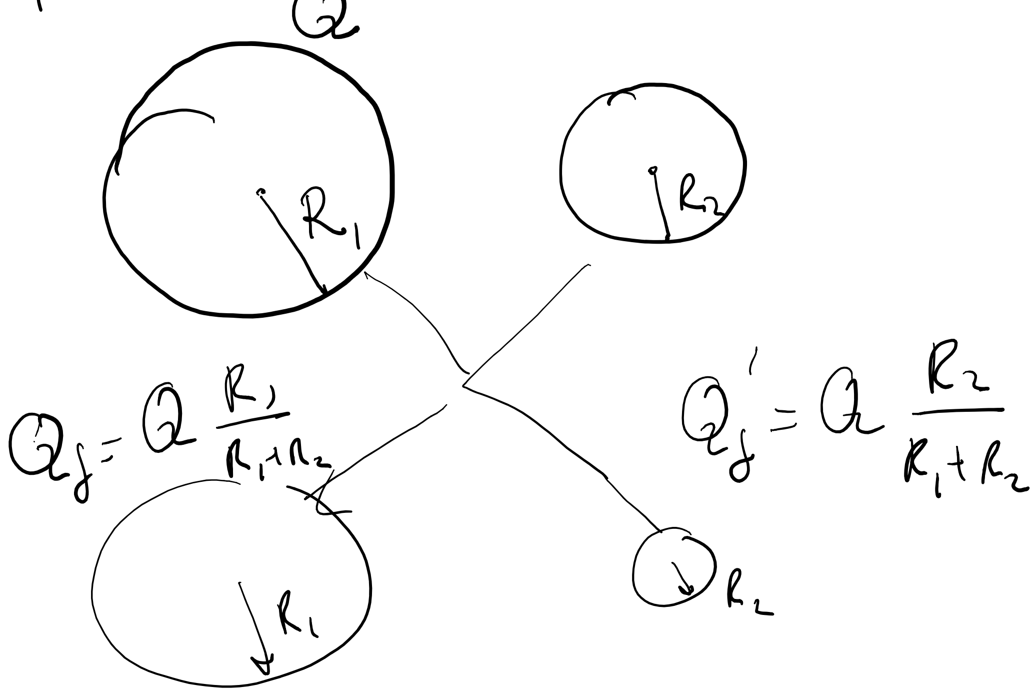
$$V(r_f) = -\int_{\infty} E(r)\,dr = -\int \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{r_f} \frac{1}{r^2}\,dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r_f} = \boxed{\frac{Q}{4\pi\varepsilon_0 r_f}}$$

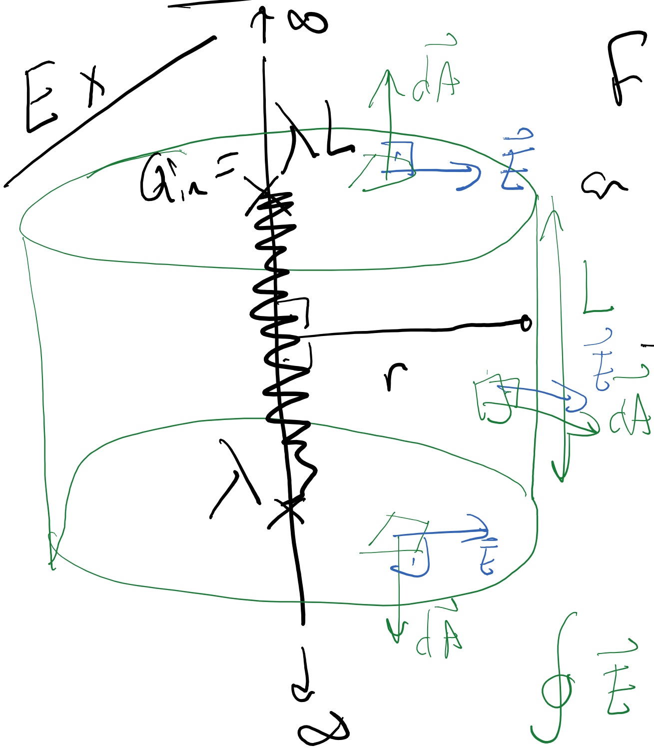
$$\boxed{V(r) = \frac{Q}{4\pi\varepsilon_0 r}} \quad r > R$$

The body of a conductor is at the same potential throughout!



$$Q_f = Q\frac{R_1}{R_1+R_2} \qquad Q_f' = Q\frac{R_2}{R_1+R_2}$$

Ex



Find the potential r away from an infinite line charge with charge density λ.

1°) $\vec E$ from gauss' Law

2°) find V!

$\vec E \perp d\vec A$

$$\oint \vec E\cdot d\vec A = \underbrace{\int_{side} \vec E\cdot d\vec A}_{\vec E \parallel d\vec A} + \overbrace{\cancel{\int_{top} \vec E\cdot d\vec A}}^{0} + \overbrace{\cancel{\int_{bottom} \vec E\cdot d\vec A}}^{0}$$

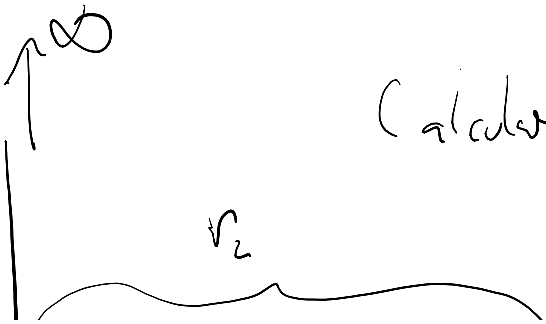
$$\oint \vec E\cdot d\vec A = |\vec E|\int_{side} dA = 2\pi r L |\vec E|$$

$Q_{in} = \lambda L$

$$\oint \vec E\cdot d\vec A = \frac{Q_{in}}{\varepsilon_0}$$

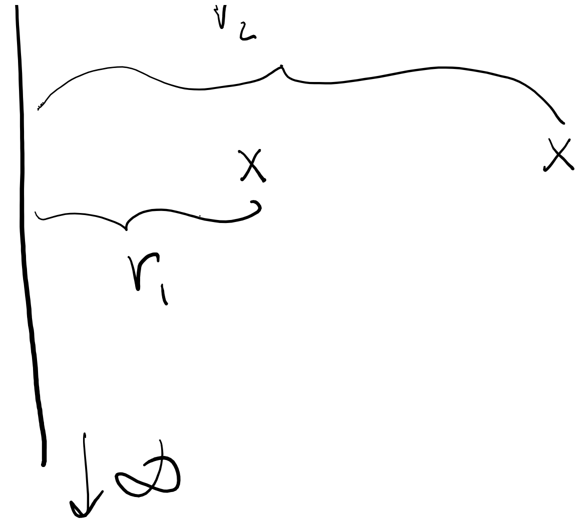
$$2\pi r \cancel{L} |\vec E| = \frac{\lambda \cancel{L}}{\varepsilon_0}$$

$$\boxed{|\vec E| = \frac{\lambda}{2\pi\varepsilon_0 r}} \qquad \vec E \parallel \hat r$$



Calculate ΔV between two points r_1, r_2

$$\Delta V(r_2, r_1) = -\int \vec E\, d\vec l$$



$$\Delta V(r_2, r_1) = -\int \vec{E} \cdot d\vec{l}$$

$$= -\int_{r_2}^{r_1} \frac{\lambda}{2\pi\varepsilon_0 r}\, dr$$

$$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0} \int_{r_2}^{r_1} \frac{1}{r}\, dr$$

$$= -\frac{\lambda}{2\pi\varepsilon_0} \ln(r) \Big|_{r_2}^{r_1}$$

$$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \left[\ln(r_2) - \ln[r_1]\right]$$

$$\boxed{\Delta V(r_1, r_2) = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_2}{r_1}\right)}$$

$$V(\infty) = ? \qquad \ln(\infty) = \infty$$

I need to fix my reference at some other point.

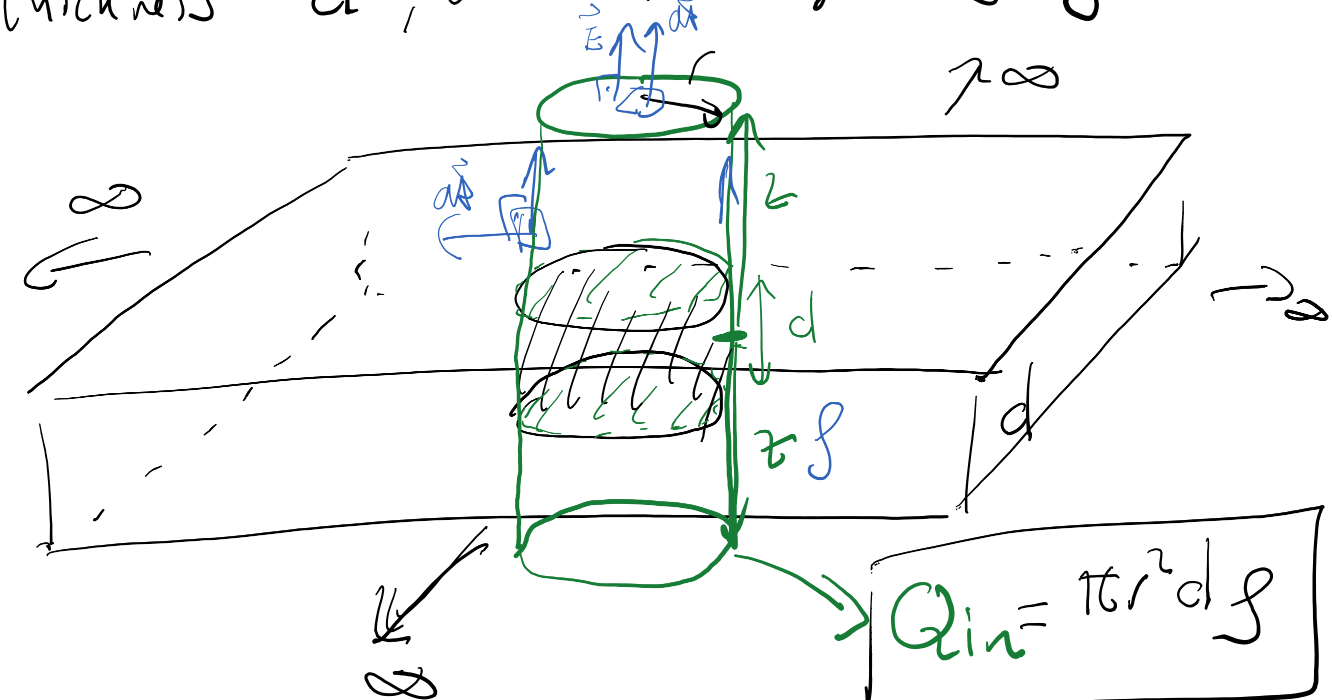
Arbitrarily choose a point R so that

$$V(R) = 0$$

$$V(r) - \frac{\lambda}{} \ln\left(\frac{r}{}\right)$$

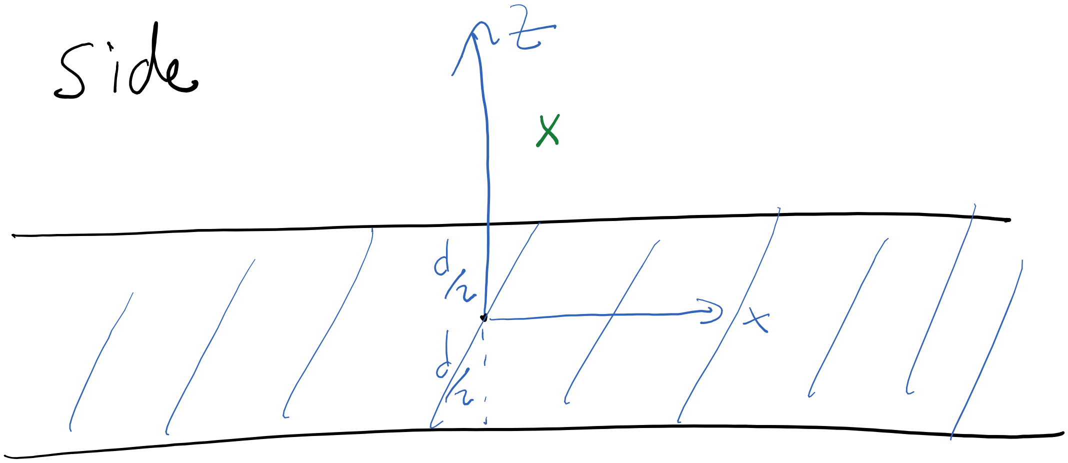
$$V(r) = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r}{R}\right)$$

~~Ex~~ Calculate the electric field and the potential for a charged slab, with thickness d, and 3D charge density ρ.



$$\boxed{Q_{in} = \pi r^2 d \rho}$$

Side



1°) Use Gauss' Law to find the electric field!

1°) Use Gauss' Law to find the electric field,

$z > \frac{d}{2}$ (outside the slab)

$z < \frac{d}{2}$ (inside the slab)

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{top} \vec{E}\cdot d\vec{A} + \int_{bottom} \vec{E}\cdot d\vec{A}}_{\vec{E} \parallel d\vec{A}} + \cancelto{0}{\int_{side} \vec{E}\cdot d\vec{A}} \quad \vec{E} \perp d\vec{A}$$

$$= |\vec{E}|\, 2\pi r^2$$

$Q_{in} = \pi r^2 d \rho$

$d\rho \simeq \sigma$

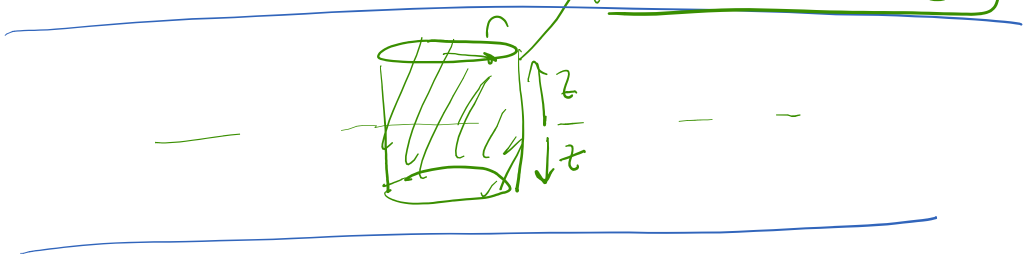
$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$|\vec{E}|\, 2\cancel{\pi}\cancel{r^2} = \frac{\cancel{\pi}\cancel{r^2} d\rho}{\varepsilon_0} \Rightarrow \boxed{|\vec{E}| = \frac{d}{2}\frac{\rho}{\varepsilon_0}}$$

side

outside

$\boxed{Q_{in} = \pi r^2 2z \rho}$

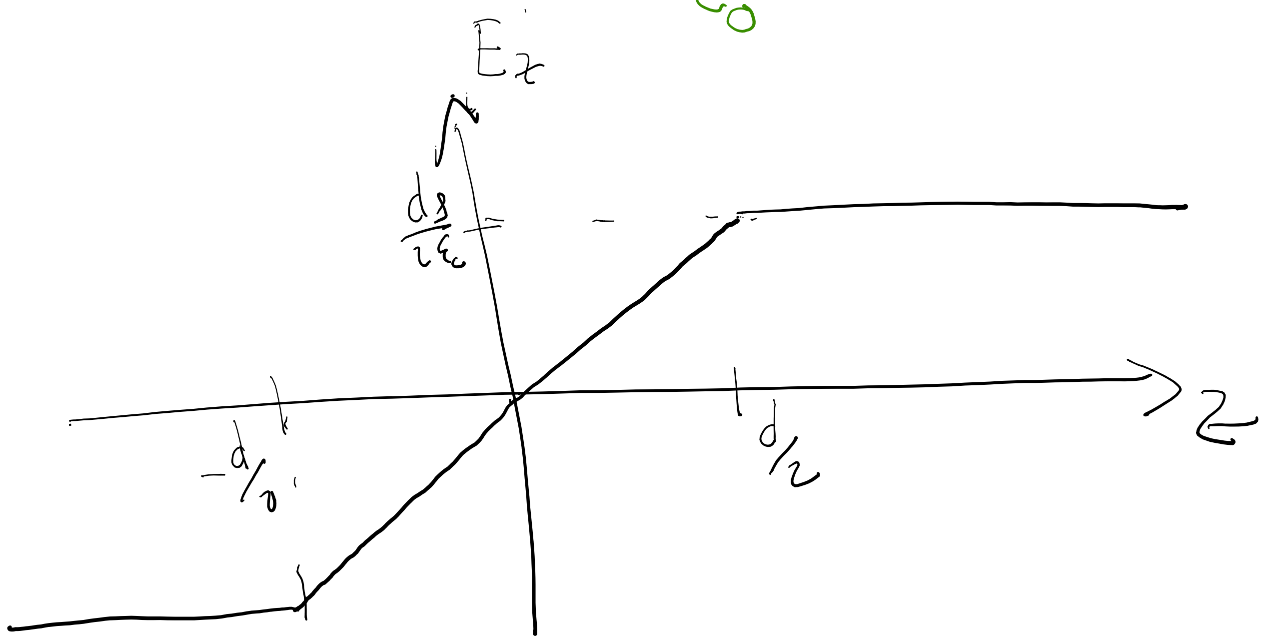


inside

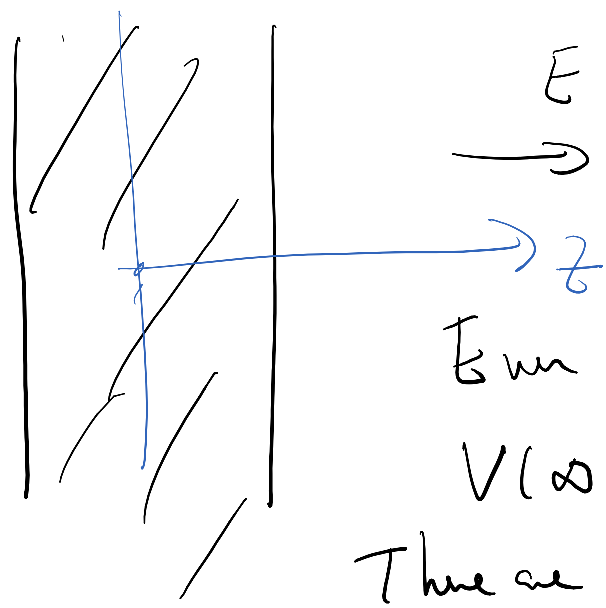
$$\oint \vec{E}\cdot d\vec{A} = 2\pi r^2 |\vec{E}| = \frac{Q_{in}}{\varepsilon_0}$$

$$\cancel{2}\cancel{\pi} r^2 |\vec{E}| = \frac{\cancel{2} z \cancel{\pi} r^2 \rho}{\varepsilon_0}$$

$$|\vec{E}| = \frac{z\rho}{\varepsilon_0}$$



b) Potential!



Even without calculation

$V(\infty) \rightarrow \infty$

There are charges at ∞!!!

Choose

Choose

$$V(z=d) = 0$$

$$V(z) = ? \qquad \underbrace{V(z) - V(0)}_{0} = -\int_0^z E(z')\, dz'$$

$z < d/2$ (inside)

$$V(z) = -\int_0^z \frac{z'\rho}{\varepsilon_0}\, dz' = \boxed{-\frac{\rho}{\varepsilon_0}\frac{z^3}{2}}$$

$z > d/2$

$$V(z) = -\int_0^z |\vec{E}(z)|\, dz$$

$$= -\int_0^{d/2} |E(z')|\, dz' - \int_{d/2}^z |E(z')|\, dz'$$

$$= -\int_0^{d/2} \frac{\rho z'}{\varepsilon_0}\, dz' - \int_{d/2}^z \frac{\rho d}{2\varepsilon_0}\, dz'$$

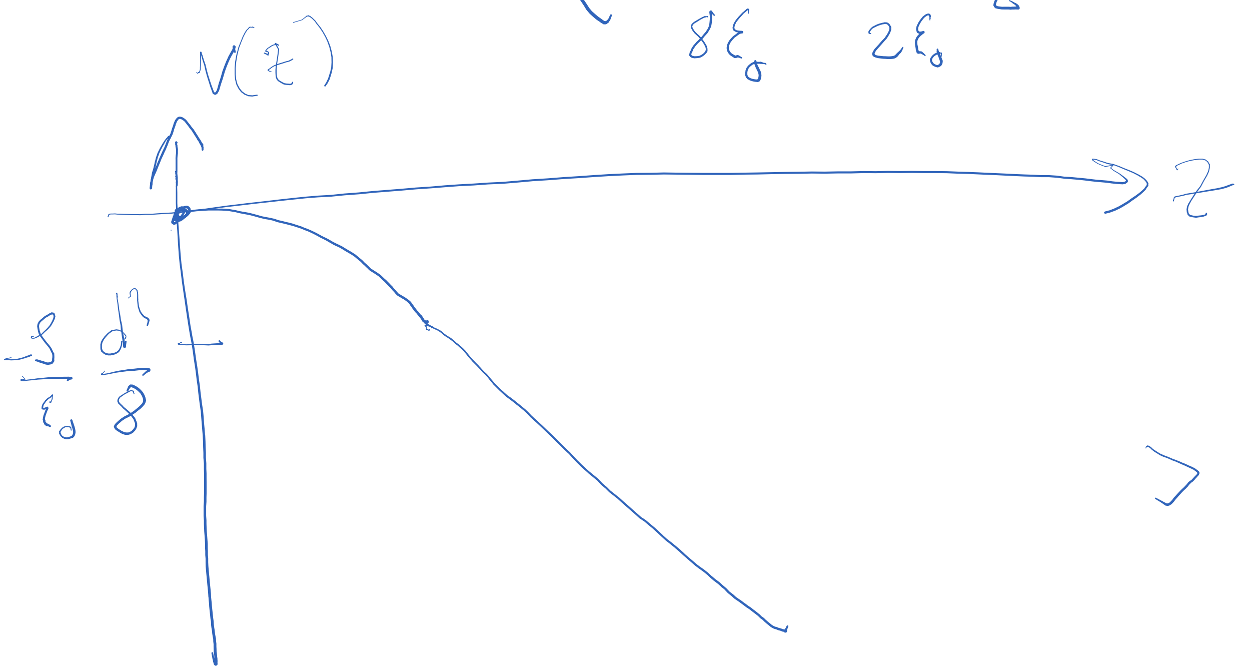
$$= -\frac{\rho}{\varepsilon_0}\frac{z'^3}{2}\Big|_0^{d/2} - \frac{\rho d}{2\varepsilon_0}\, z'\Big|_{d/2}^z$$

$$= -\frac{\rho}{\varepsilon_0}\frac{z^2}{2}\Big|_0 - \frac{\rho d}{2\varepsilon_0}\, z\Big|_{d/2}$$

$$= -\frac{\rho}{\varepsilon_0}\frac{d^2}{8} - \frac{\rho d}{2\varepsilon_0}\left(z - \frac{d}{2}\right)$$

$$V(z) = -\frac{\rho d^2}{8\varepsilon_0} + \frac{\rho d^2}{4\varepsilon_0} - \frac{\rho d}{2\varepsilon_0} z = +\frac{\rho d^2}{8\varepsilon_0} - \frac{\rho d}{2\varepsilon_0} z$$

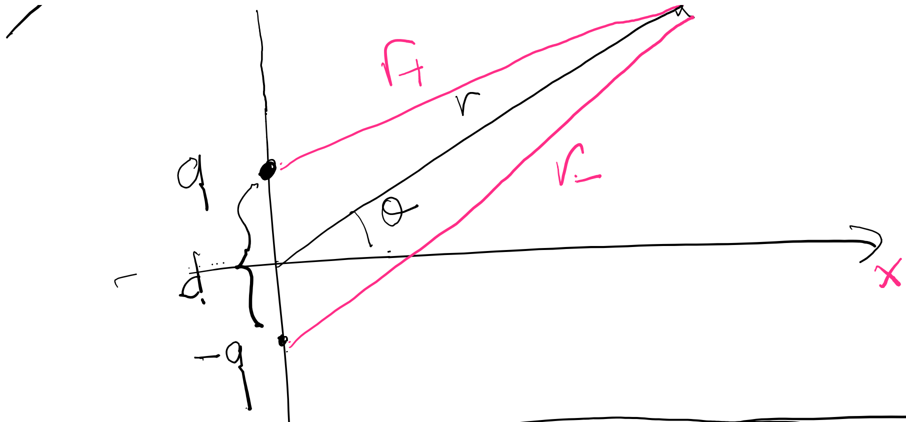
$$V(z) = \begin{cases} -\dfrac{\rho}{\varepsilon_0}\dfrac{z^2}{2} & z < d/2 \\[2ex] \dfrac{\rho d^2}{8\varepsilon_0} - \dfrac{\rho d}{2\varepsilon_0} z & z > d/2 \end{cases}$$



Ex

Dipole

Calculate $\vec{E}$ field



Find the $\vec{E}$ field direction and magnitude

$$\boxed{r \gg d}$$

Solution: First find $V(x,y) \Rightarrow$ Then $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$

$$r_+ = \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2}$$

$$r_- = \sqrt{x^2 + \left(y + \frac{d}{2}\right)^2} \qquad V(x,y) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$

let's use $r \gg d$ to find approximate results

$$r_\pm = \sqrt{\qquad yd + \frac{d^2}{4}} = r\sqrt{1 \mp \frac{yd}{r^2} + \cancel{\frac{d^2}{4r^2}}}$$

neglect $\qquad \frac{d}{r} \ll 1 \qquad \frac{d^2}{r^2} \ll \frac{d}{r}$

$$\simeq r\sqrt{1 \mp \frac{yd}{r^2}} \qquad (1+x)^n \simeq 1 + nx \quad x \ll 1$$

$$r_\pm \simeq r\left(1 \mp \frac{yd}{2r^2}\right)$$

$$\frac{1}{1+x} = (1+x)^{-1} \simeq 1 - x \quad x \ll 1$$

$$\frac{1}{r_\pm} \simeq \frac{1}{r}\left(1 \pm \frac{yd}{2r^2}\right)$$

$p \rightarrow$ dipole moment

$$V(x,y) \quad \frac{q}{4\pi\varepsilon_0 r}\left[\cancel{1} + yd \right. \qquad = \frac{qdy}{4\pi\varepsilon_0 r^3}$$

$$y \quad]$$

$$= -\frac{qd}{4\pi\varepsilon_0} \frac{(x^2+y^2)^{3/2} - y\,\frac{3}{\cancel{2}}\,\cancel{2}y\,(x^2+y^2)^{1/2}}{}$$

$$= -\frac{qd}{4\pi\varepsilon_0} \frac{x^2+y^2-3y^2}{(x^2+y^2)^{5/2}} = \frac{qd}{4\pi\varepsilon_0} \frac{2y^2-x^2}{r^5}$$

$$\boxed{\vec{E} = \frac{qd}{4\pi\varepsilon_0 r^5}\left[\qquad \hat{i} + (2y^2-x^2)\,\hat{j}\right]}$$

PHYSICS 102 - Instructor: M. Özgür OKTEL- Spring 2016

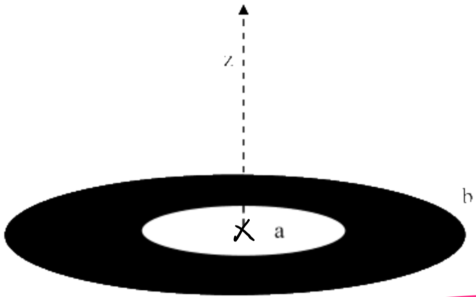
QUIZ 8

A non-conducting thin annulus of inner radius **a** and outer radius **b** is placed on the x-y plane. If the annulus is uniformly charged with total charge **Q**.

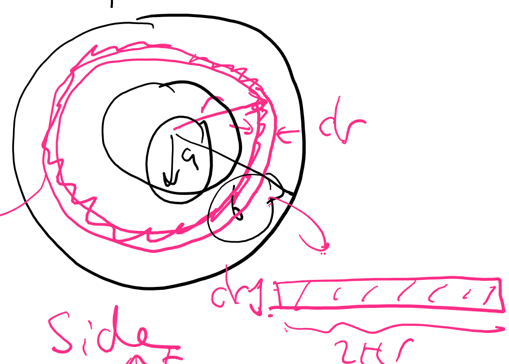
a) Find the surface charge distribution **σ** on the annulus. $\quad \sigma = \dfrac{Q}{\pi(b^2-a^2)}$

b) Find the potential **V**, at a point **z** above the axis of the annulus.

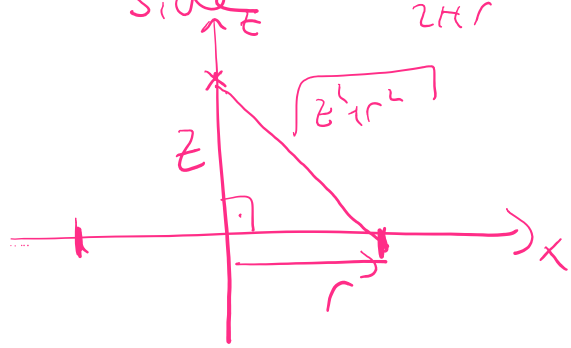
c) Find the electric field at the same point.



Top view



Side



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{z^2+r^2}}$$

$$dq = \sigma\, dA = 2\pi r\, dr\, \sigma$$

$$dV = \frac{\sigma}{4\pi\varepsilon_0}\frac{2\pi r}{\sqrt{z^2+r^2}}\, dr$$

$$V = \int dV = \int_a^b \frac{\sigma}{\cancel{4\pi}_{2}\varepsilon_0}\frac{\cancel{2\pi} r}{\sqrt{z^2+r^2}}\, dr$$

$$= \frac{\sigma}{2\varepsilon_0}\int_a^b \frac{r}{\sqrt{z^2+r^2}}\, dr$$

$u = z^2+r^2$ $\quad r=b \to u = z^2+b^2 \quad r=a \to u = z^2+a^2$

$du = 2r\,dr$

$$= \frac{\sigma}{\ldots}\int^{z^2+b^2} \frac{\frac{1}{2}}{\ldots}\, du$$

$$= \frac{\sigma}{2\varepsilon_0} \int_{z^2+a^2} \frac{1/2}{\sqrt{u}}\, du \qquad du = 2r\ldots$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$= \frac{\sigma}{\cancel{4}_2 \varepsilon_0} \left. \frac{u^{1/2}}{1/\cancel{2}} \right|_{z^2+a^2}^{z^2+b^2}$$

1°) Goal ✓

2°) Units ✓

$$\left[\frac{\sigma}{2\varepsilon_0}\right] = N/C = V/m$$

3°) Limits

$$\begin{pmatrix} a \to 0 \\ b \to \infty \end{pmatrix}$$

$$\frac{\sigma}{2\varepsilon_0} \checkmark$$

$$\boxed{V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+b^2} - \sqrt{z^2+a^2}\right]}$$

$$= \frac{Q}{2\varepsilon_0 \pi (b^2-a^2)}\left[\sqrt{z^2+b^2} - \sqrt{z^2+a^2}\right]$$

c) By symmetry $\vec{E} = E_z \hat{k}$

$$E_z = -\frac{\partial}{\partial z} V(z) = -\frac{\partial}{\partial z}\left[\frac{\sigma}{2\varepsilon_0}\left(\sqrt{b^2+z^2} - \sqrt{z^2+a^2}\right)\right]$$

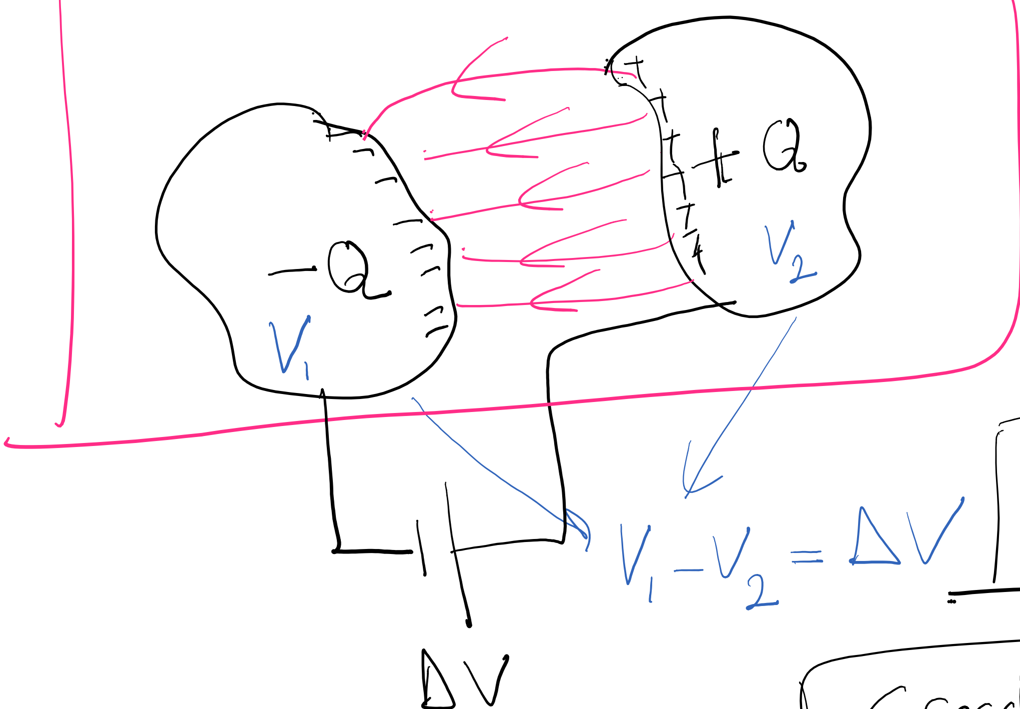
$$= \frac{\sigma}{2\varepsilon_0}\left[-\frac{1}{\cancel{2}}\frac{\cancel{2}z}{\sqrt{b^2+z^2}} + \frac{1}{\cancel{2}}\frac{\cancel{2}z}{\sqrt{z^2+a^2}}\right]$$

$$\boxed{E_z = \frac{\sigma}{2\varepsilon_0} z \left[\frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}}\right]}$$

$$E_z(z=0) = 0 \checkmark$$

Capacitors

Capacitor



$V_1 - V_2 = \Delta V$

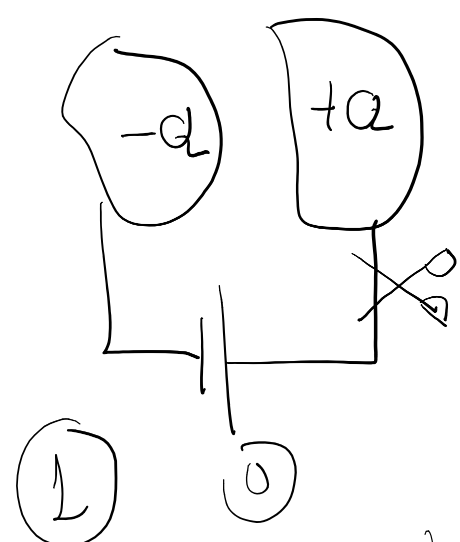
$Q \propto \Delta V$

$\boxed{Q = C\,\Delta V}$

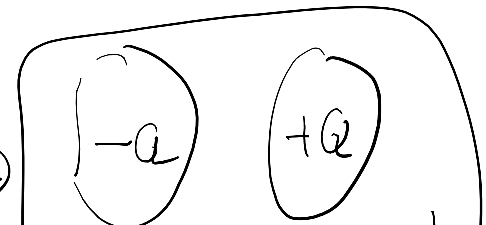
Capacitance

1°) Capacitance depends on the geometry not on ΔV or Q!

2°)

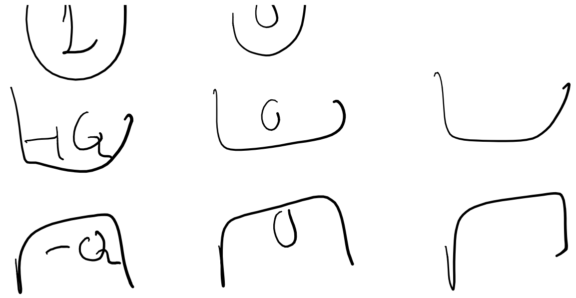


$\Longrightarrow$

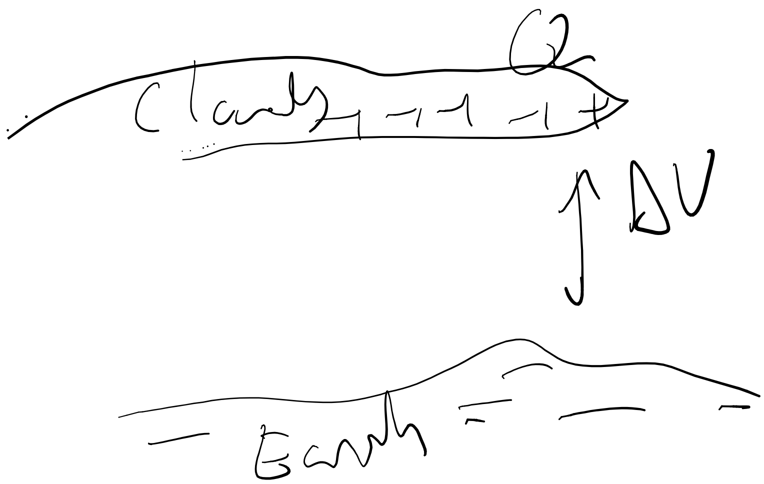


Store charge!

Store energy



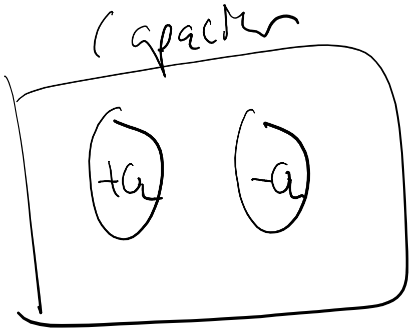
Store Information



3°) Units $\quad Q = C\,\Delta V$

$$[C] = \frac{\text{Coulomb}}{\text{Volt}} = \text{Farad}$$

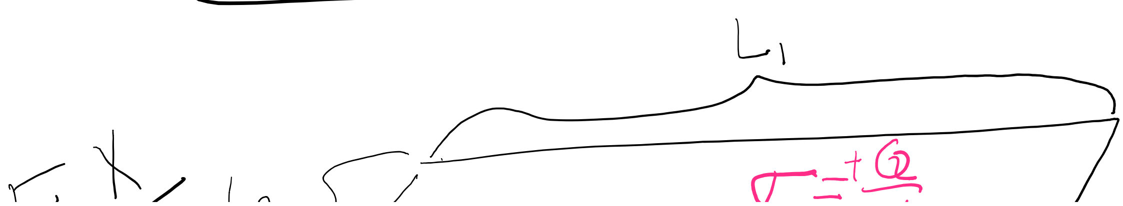
5°) $\boxed{C > 0}$ μF, nF more common

4°) 

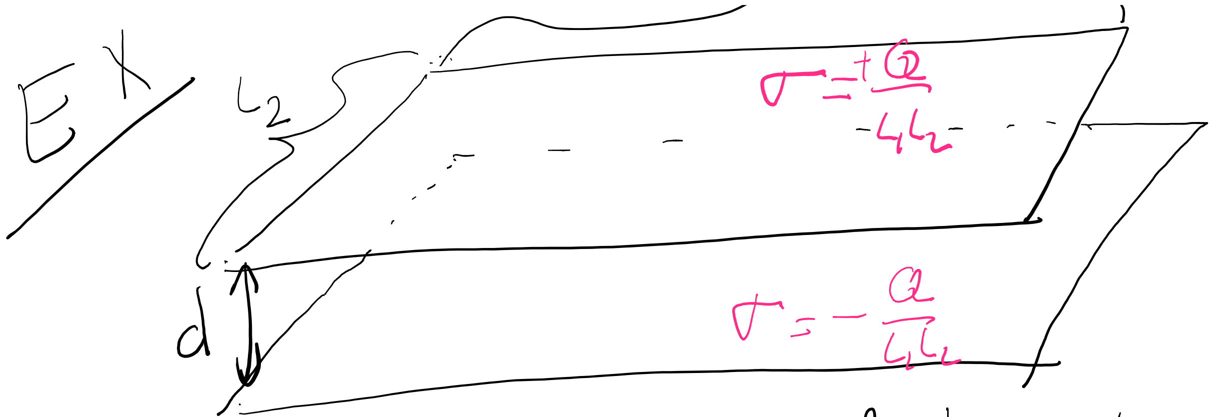
Total charge ≠ Stored charge

$\Downarrow$ $\qquad\qquad$ $\Downarrow$

0 $\qquad\qquad$ Q



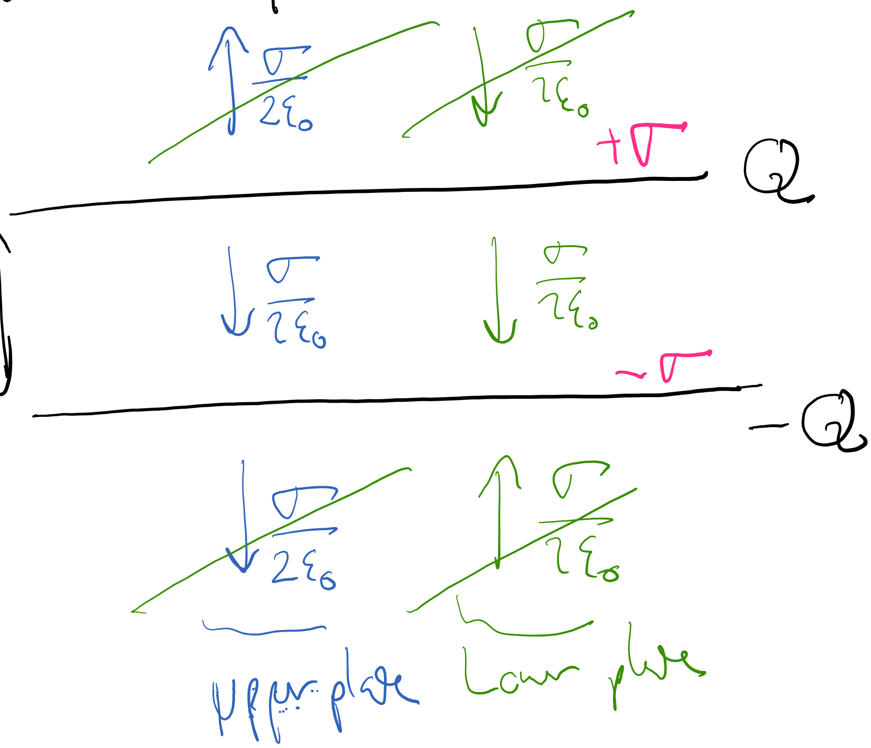
Ex

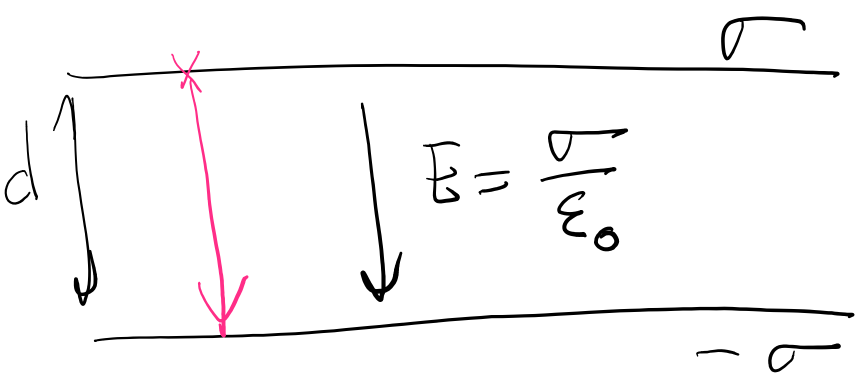


What is the capacitance of this system

side

$L_1, L_2 \gg d$





$$\left|\Delta V\right| = \left|\cancel{-}\int_i^f \vec{E}\cdot d\vec{l}\right|$$

$$= \left|\,|\vec{E}| \int_i^f dl\right|$$

$$= |\vec{E}| d$$

$$\Delta V = \frac{\sigma}{\varepsilon_0} d$$

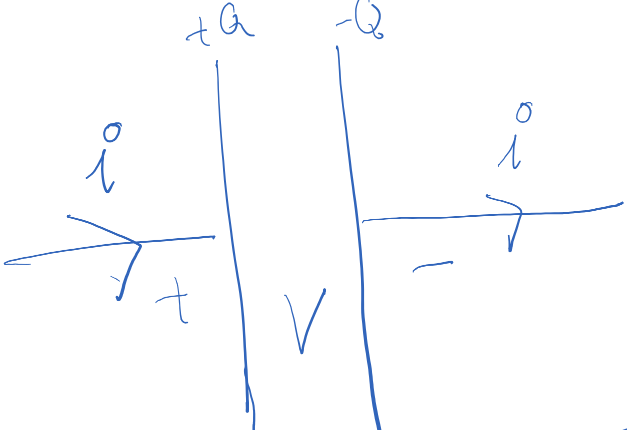
$$Q = C \Delta V$$

$$\Delta V = \frac{Q}{\varepsilon_0 L_1 L_2} d \Rightarrow Q = \varepsilon_0 \frac{L_1 L_2}{d} \Delta V$$

$$\boxed{C = \varepsilon_0 \frac{L_1 L_2}{d}}$$

$$[\varepsilon_0] \Rightarrow F/m$$

$$\boxed{C = \varepsilon_0 \frac{A}{d}}$$



$$Q(t) \Rightarrow i(t)$$

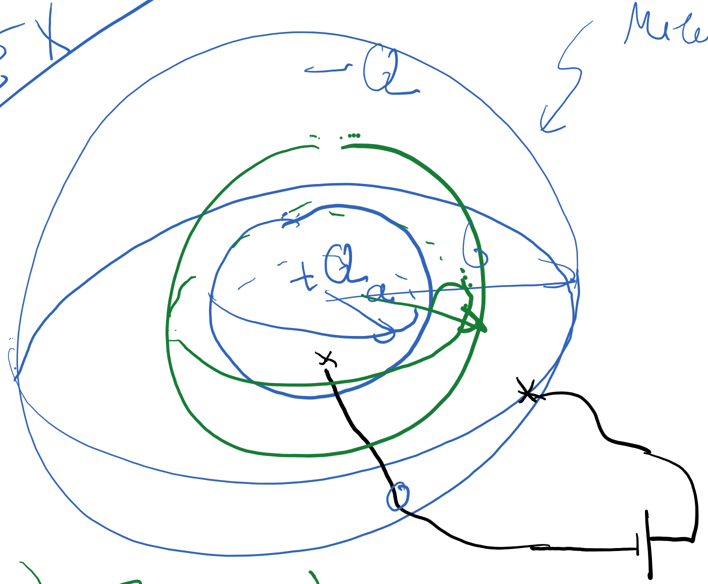
$$i(t) = \frac{dQ}{dt}$$

$$Q(t) = C\, V(t)$$

$$\boxed{i(t) = C \frac{d\,V(t)}{dt}}$$

$$i(t) = C\frac{d\,\cdot}{dt}$$

EX



Capacitance of this spherical capacitor?

$$\oint \underbrace{\vec{E}\cdot d\vec{A}}_{\vec{E}\,//\,d\vec{A}} = \frac{Q_{in}}{\varepsilon_0}$$

1°) Find $\vec{E} \Rightarrow \Delta V$

$$4\pi r^2 |\vec{E}| = \frac{Q}{\varepsilon_0}$$

$$|\vec{E}| = \frac{Q}{4\pi\varepsilon_0 r^2} \quad a<r<b$$

$$\Delta V = \left|-\int \vec{E}\cdot d\vec{l}\right| = \left|\int_a^b E(r)\,dr\right|$$

$$= \frac{Q}{4\pi\varepsilon_0}\underbrace{\left|\int_a^b \frac{1}{r^2}dr\right|}_{-\frac{1}{r}\Big|_a^b}$$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\,\frac{b-a}{ab}$$

$$Q = \underbrace{\left(4\pi\varepsilon_0\,\frac{ab}{b-a}\right)}\Delta V$$

$$\boxed{C = 4\pi\varepsilon_0\,\frac{ab}{b-a}}$$