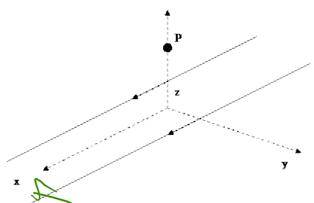
Week 10 Extra (April 2nd)

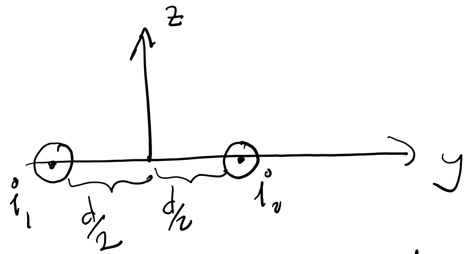
Wednesday, March 30, 2016 4:55 PM

QUIZ-18

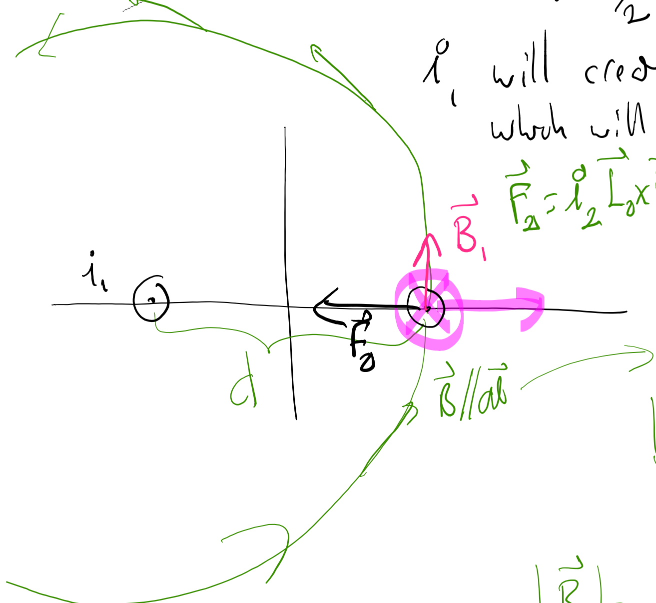
Two infinite wires carry the same current I in the same direction, along x. One wire is placed at y=d/2, z=0; while the other is placed at y=-d/2, z=0, as shown in the figure.

a) Calculate the force per unit length that the wires apply to each other, indicate the direction of the forces on the figure.
b) For which point P on the z axis is the magnitude of the magnetic field is maximum? (Give its z coordinate)





i_1 will create a magnetic field which will apply a force on i_2.



$\vec{F}_2 = i_2 \vec{L}_2 \times \vec{B}_1$

Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$$

$$\underbrace{|\vec{B}| \oint dl}_{2\pi d |\vec{B}|} = \mu_0 i_1$$

$$2\pi d |\vec{B}| = \mu_0 i_1$$

$$|\vec{B}| = \frac{\mu_0 i}{2\pi d}$$

$$\vec{F}_2 = i_2 \vec{L}_2 \times \vec{B}_1 \qquad |\vec{F}_2| = i_2 L_2 \frac{\mu_0 i_1}{2\pi d} = L_2 \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$\boxed{\frac{|\vec{F}_2|}{L_2} = \mu_0 \frac{i\, i}{2\pi d}}$$

attractive

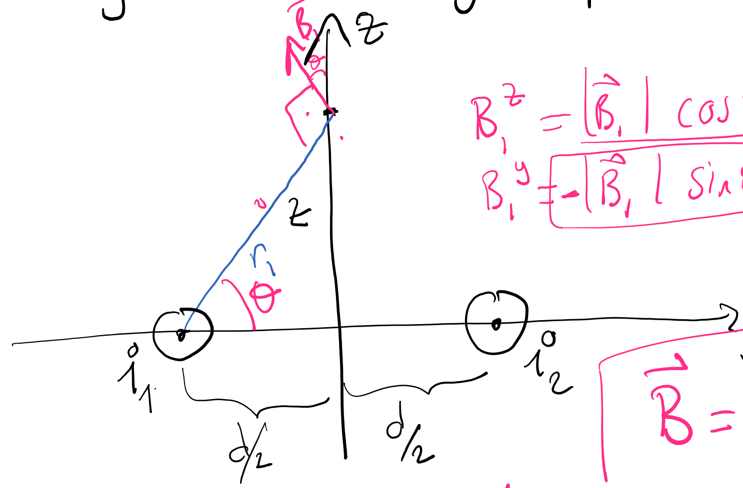
Aside what would be the direction of the force if i_2 was reversed $\Rightarrow$ F would be reversed!

Opposite direction currents repel each other

b) For which point on the z axis is the magnitude of the magnetic field maximum?

$\vec{B}$ z $\rightarrow$

magnitude of the magnetic field maximum.



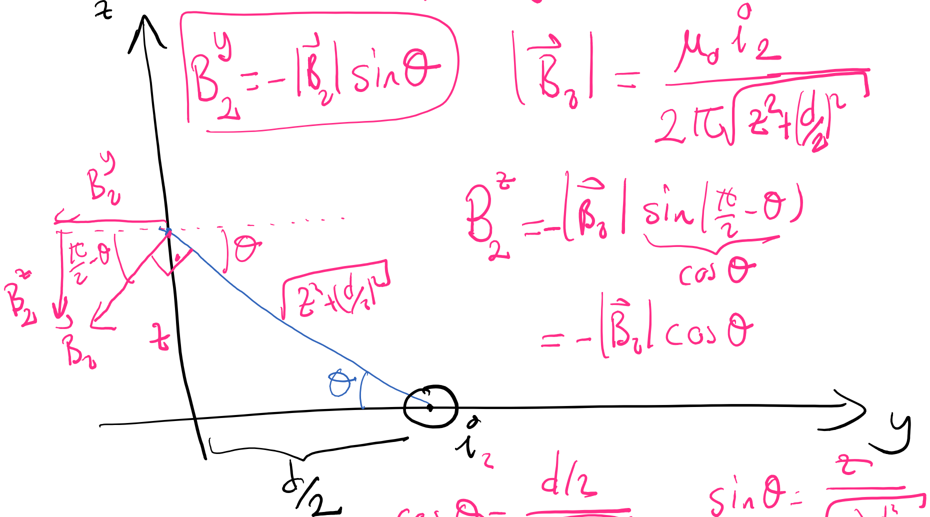
$$B_1^z = |\vec{B}_1| \cos\theta \qquad B_2^z = -|\vec{B}_2| \cos\theta$$

$$\boxed{B_1^y = -|\vec{B}_1| \sin\theta} \qquad \boxed{B_2^y = -|\vec{B}_2| \sin\theta}$$

$$\boxed{\vec{B} = \vec{B}_1 + \vec{B}_2}$$

z components cancel!

Let's do these separately.



$$\boxed{B_2^y = -|\vec{B}_2| \sin\theta} \qquad |\vec{B}_2| = \frac{\mu_0 i_2}{2\pi\sqrt{z^2+\left(\frac{d}{2}\right)^2}}$$

$$B_2^z = -|\vec{B}_2| \underbrace{\sin\left(\frac{\pi}{2}-\theta\right)}_{\cos\theta}$$

$$= -|\vec{B}_2| \cos\theta$$

$$\cos\theta = \frac{d/2}{\sqrt{z^2+d^2/4}} \qquad \sin\theta = \frac{z}{\sqrt{z^2+d^2/4}}$$

$$B_2^z = -\frac{\mu_0 i_2}{2\pi} \frac{1}{\sqrt{z^2+\frac{d^2}{4}}} \frac{d/2}{\sqrt{z^2+d^2/4}} = -\frac{\mu_0 i_2}{2\pi} \frac{d/2}{z^2+\frac{d^2}{4}}$$

$$B_2^y = -\frac{\mu_0 i_2}{2\pi} \frac{z}{z^2+d^2/4}$$

$$\vec{B} = 2 B_2^y \hat{J} = -\frac{\mu_0 i}{\pi} \frac{z}{z^2+d^2/4} \hat{J}$$

$$|\vec{B}(z)| = \frac{\mu_0 i}{\pi} \frac{z}{z^2+d^2/4}$$

$$\frac{\partial|\vec{B}|}{} \quad \mu_0 i \ \partial\left(\frac{z}{}\right)$$

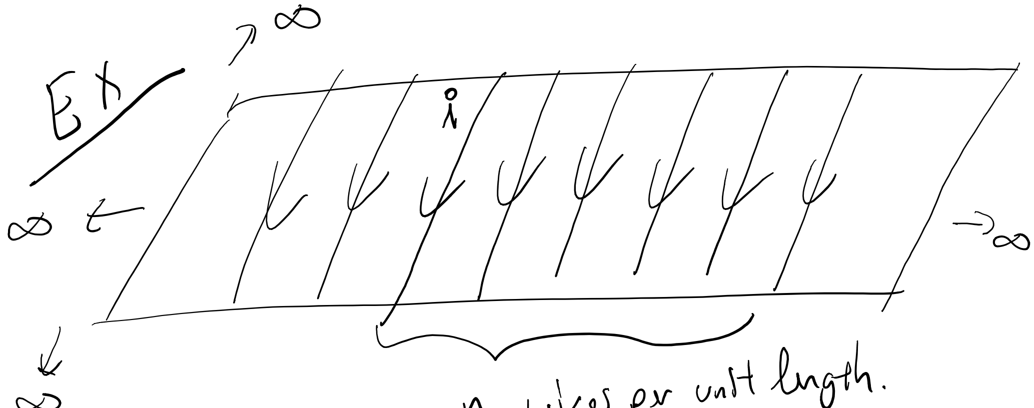
$$\frac{\partial |\vec{B}|}{\partial z} = 0 = \frac{\mu_0 i}{\pi} \frac{\partial}{\partial z}\left(\frac{z}{z^2 + d^2/4}\right)$$

$$0 = \cancel{\frac{\mu_0 i}{\pi}} \; \frac{\left(z^2 + d^2/4\right) - z(2z)}{\left(z^2 + d^2/4\right)^2}$$

$$z^2 + \frac{d^2}{4} - 2z^2 = 0$$

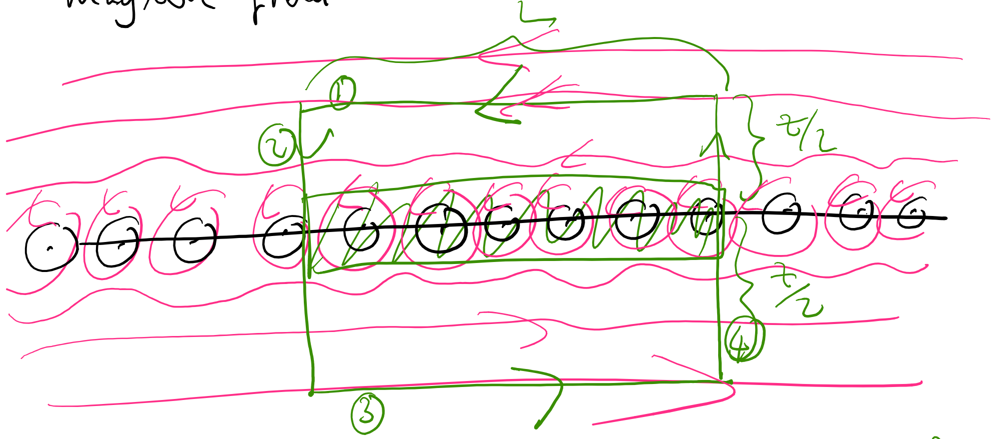
$$\boxed{z = \pm \frac{d}{2}}$$

EX



n wires per unit length.

Assume that the wires are very close, find the magnetic field above and below the plane!



$$\oint \vec{B}\cdot d\vec{l} = \underbrace{\int_{①} \vec{B}\cdot d\vec{l}}_{\vec{B} \parallel d\vec{l}} + \underbrace{\int_{②} \vec{B}\cdot d\vec{l}}_{=0 \;\; \vec{B}\perp d\vec{l}} + \underbrace{\int_{③} \vec{B}\cdot d\vec{l}}_{\vec{B} \parallel d\vec{l}} + \underbrace{\int_{④} \vec{B}\cdot d\vec{l}}_{=0 \;\; \vec{B}\perp d\vec{l}}$$

top and

$\vec{B} \parallel d\vec{l}$ $\quad = 0 \quad \vec{B} \perp d\vec{l} \quad \vec{B} \parallel d\vec{l} \quad = 0 \quad \vec{B} \perp d\vec{l}$

top and bottom are equal

$$= 2|\vec{B}| \int_{1} dl = 2|\vec{B}| L$$

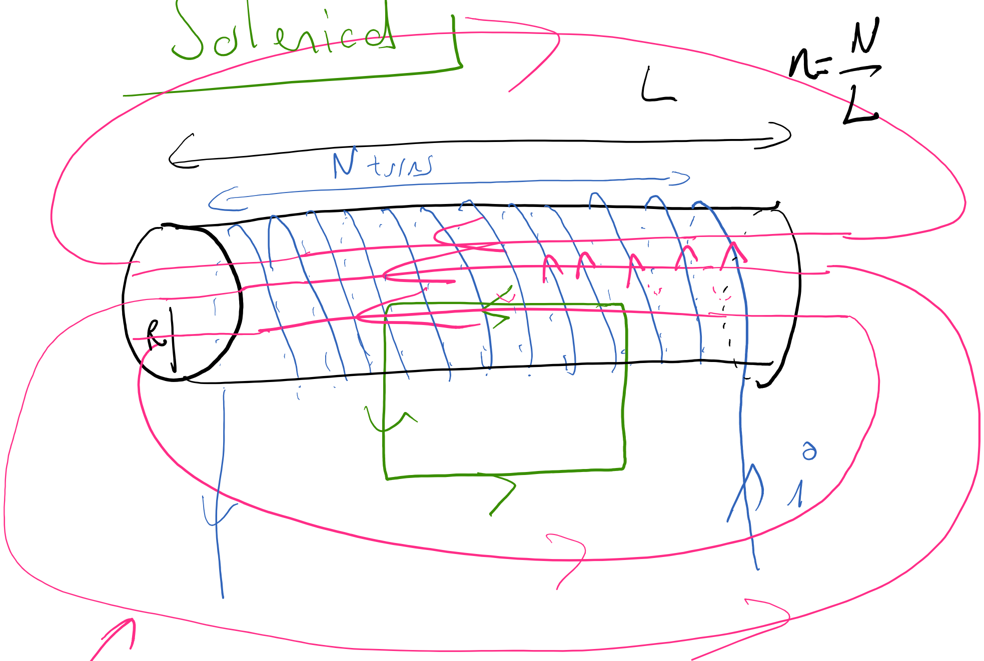
$$\mu_0 I_{in} = \mu_0 L n i$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$$

$$2|\vec{B}| \cancel{L} = \mu_0 \cancel{L} n i$$

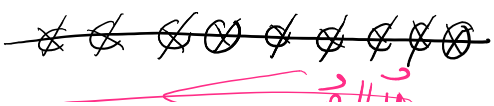
$$\boxed{|\vec{B}| = \frac{\mu_0 n i}{2}}$$

Solenoid

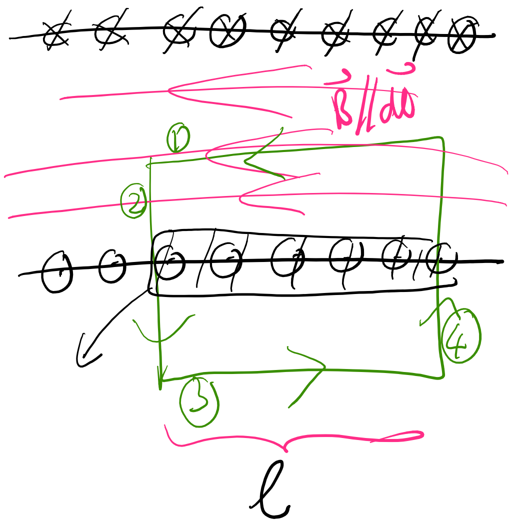


Outside magnetic fields (Fringing fields) are small for a long solenoid.

Cross section



$\oint \vec{B} \cdot d\vec{l}$



$\oint \vec{B}\cdot d\vec{l}$

2, 4 $\int \vec{B}\cdot d\vec{l} = 0$

$\vec{B} \perp d\vec{l}$

3 $\vec{B} = 0$

$$\oint \vec{B}\cdot d\vec{l} = \int_{1} \vec{B}\cdot d\vec{l}$$

$$= |\vec{B}|\, l$$

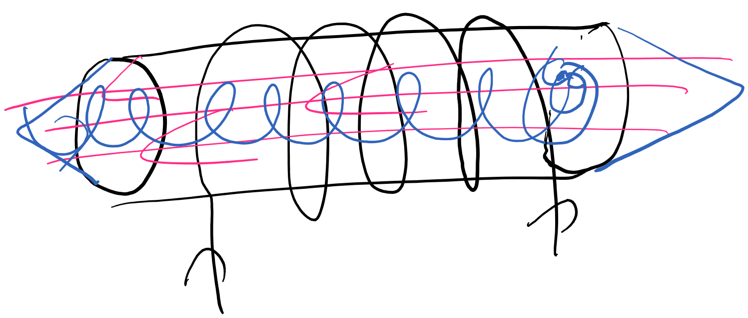
$I_{in} = n l i$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{in}$$

$$|\vec{B}|\cancel{l} = \mu_0 n \cancel{l} i$$

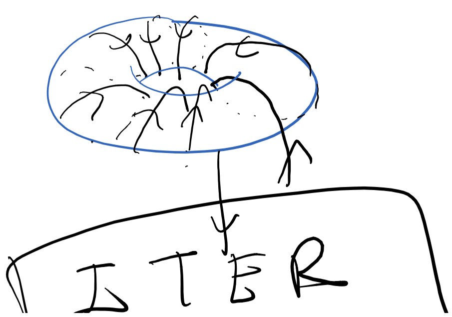
$$|\vec{B}| = \mu_0 n i$$

$\frac{N}{L}$ (turns per unit length of the solenoid.)



Torus

Toroid

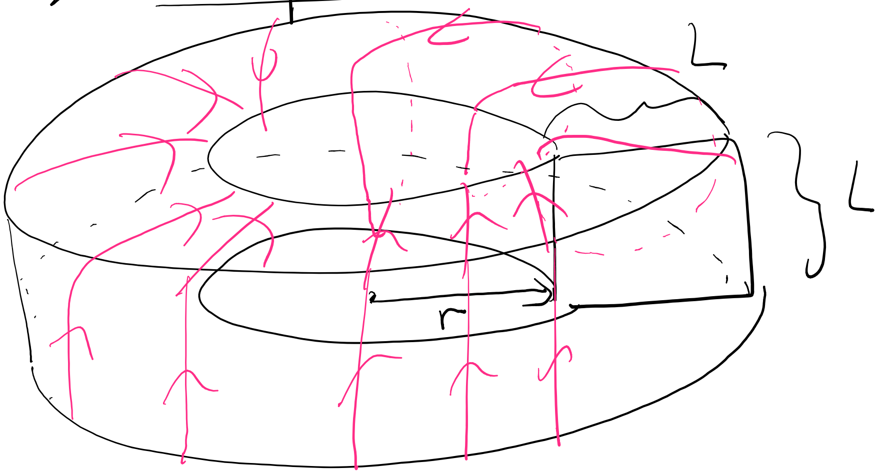


Fusion Reactor

ITER

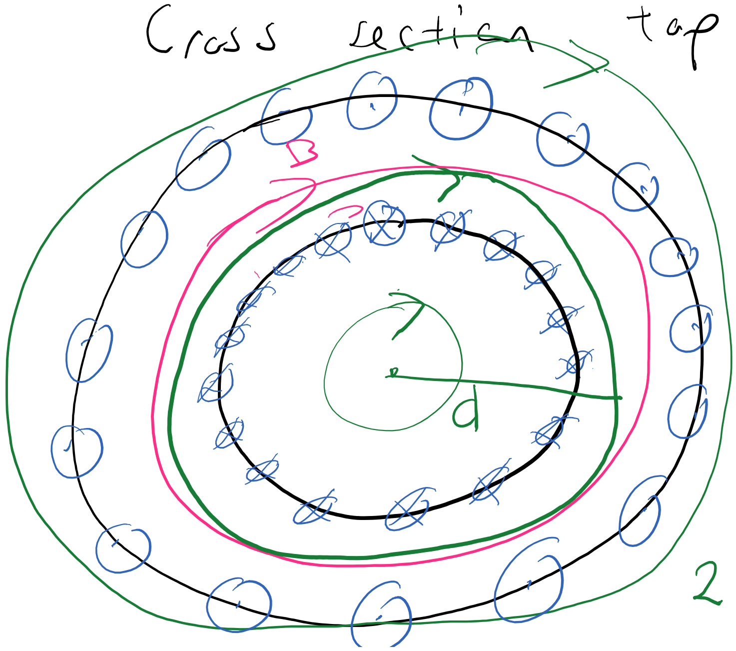
H H → He

Ex/ Square toroid



N turns of wire are wound around the square torus. The wire is carrying a current i. Find the magnetic field inside the toroid.

Cross section top



$$\oint \underbrace{\vec{B} \cdot d\vec{l}}_{\vec{B} \parallel d\vec{l}} = 2\pi d |\vec{B}|$$

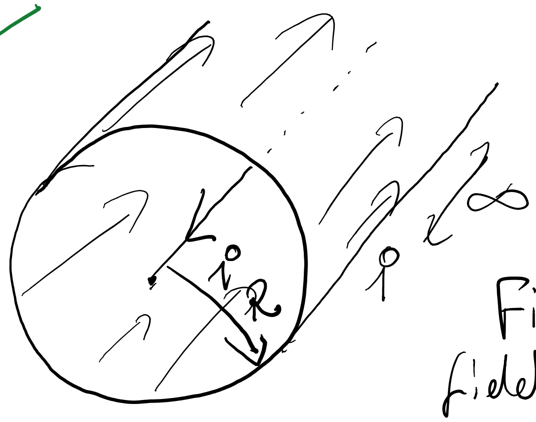
$$I_{in} = N i$$

$$2\pi d |\vec{B}| = \mu_0 N i$$

$$|\vec{B}| = \frac{\mu_0 N i}{2\pi d}$$

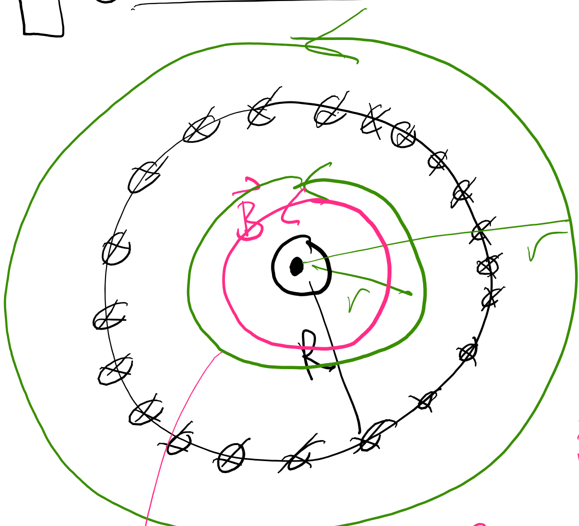
$$(L+\phi) > d > r$$

Ex



Coaxial wire

Find the magnetic field inside and outside the coaxial wire.



Find $\vec{B}$ outside $r > R$

$$\oint \vec{B}\cdot d\vec{l} = I_{in}\mu_0$$

$$I_{in} = 0$$

$\vec{B}$ inside?

$$\underbrace{\oint \vec{B}\cdot d\vec{l}}_{\vec{B}\parallel d\vec{l}} = |\vec{B}|\,2\pi r = \mu_0 I_{in}$$

$I_{in} = i$

$$\boxed{|\vec{B}| = \frac{\mu_0 i}{2\pi r}} \quad r < R$$

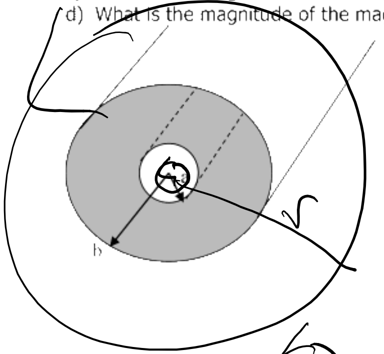
QUIZ-19

An infinite hallow wire is used to carry current. The inside of the wire (**r<a**) is empty, while the region between the inner and outer radii (**a<r<b**) is filled.

If the wire is carrying a total current of **I**, which is uniformly distributed, An infinite hallow wire is used to carry current. The inside of the wire (**r<a**) is empty, while the region between the inner and outer radii (**a<r<b**) is filled.

If the wire is carrying a total current of **I**, which is uniformly distributed,

a) What is the current density **J** within the wire?
b) What is the magnitude of the magnetic field for **r<a**?
c) What is the magnitude of the magnetic field for **a<r<b**?
d) What is the magnitude of the magnetic field for **r>b**?



a) $J = \dfrac{I}{A} = \dfrac{I}{\pi(b^2-a^2)}$

b) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{in}$

$I_{in} = 0 \quad r<a \Rightarrow \boxed{\vec{B} = 0 \quad r<a}$

d) $\underbrace{\oint \vec{B}\cdot d\vec{l}}_{\vec{B} \parallel d\vec{l}} = \mu_0 I$

$\boxed{|\vec{B}(r)| = \dfrac{\mu_0 I}{2\pi r} \quad r>b}$

Cross section

(1) $\oint \vec{B}\cdot d\vec{l} = 0 \qquad I_{in} = 0$

(3) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

$\boxed{\vec{B}(r) = \dfrac{\mu_0 I}{2\pi r}} \quad r>b$

$I_{in} = \pi(r^2-a^2)J$

(2) $\underbrace{\oint \vec{B}\cdot d\vec{l}} = 2\pi r|\vec{B}| = \dfrac{\pi(r^2-a^2)}{\pi(b^2-a^2)} I$

$\vec{B} \parallel d\vec{l}$

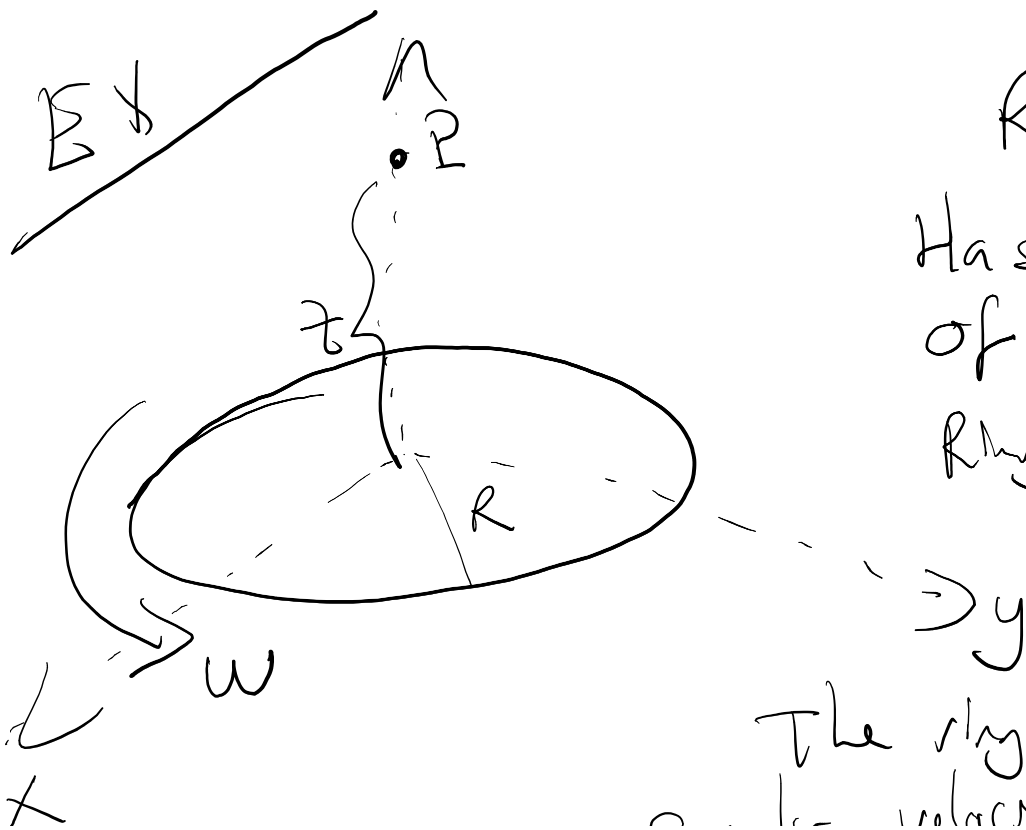
(2)

$$I_{in} = \frac{(r^2 - a^2)}{(b^2 - a^2)} I$$

$$\oint \vec{B}\, d\vec{l} = \mu_0 I_{in}$$

$$2\pi r |\vec{B}| = \mu_0 \frac{(r^2 - a^2)}{(b^2 - a^2)} I$$

$$\boxed{|\vec{B}| = \frac{\mu_0 I}{2\pi r} \frac{r^2 - a^2}{b^2 - a^2}} \qquad a < r < b$$

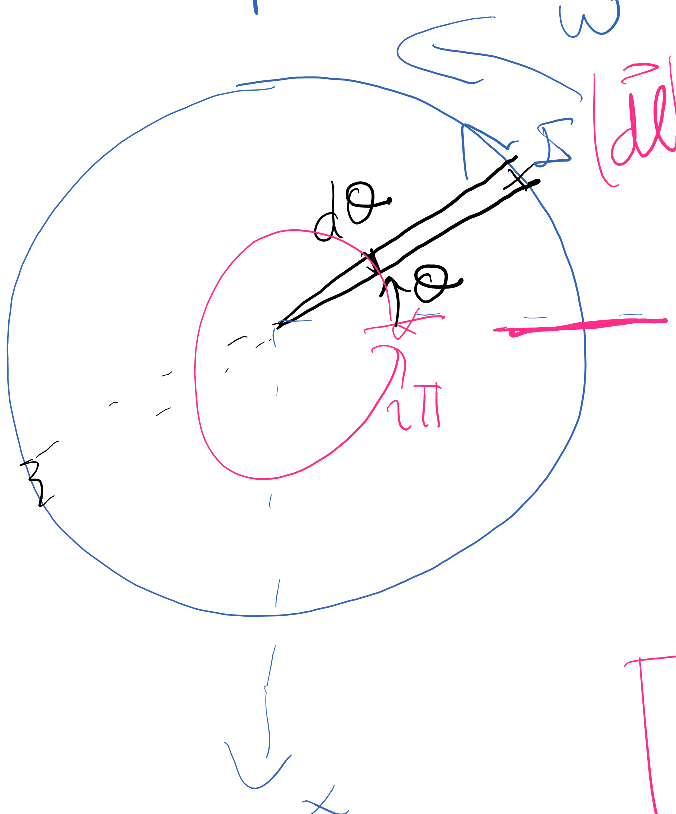


Ring of radius R
Has a total charge
of Q distributed uniform
Ring is on the x-y
plane with center
at the origin.

The ring is rotating with
angular velocity ω as shown.

Find the magnetic field at point P.

Top



$I = ?$

$|\vec{dl}| = R\,d\theta$

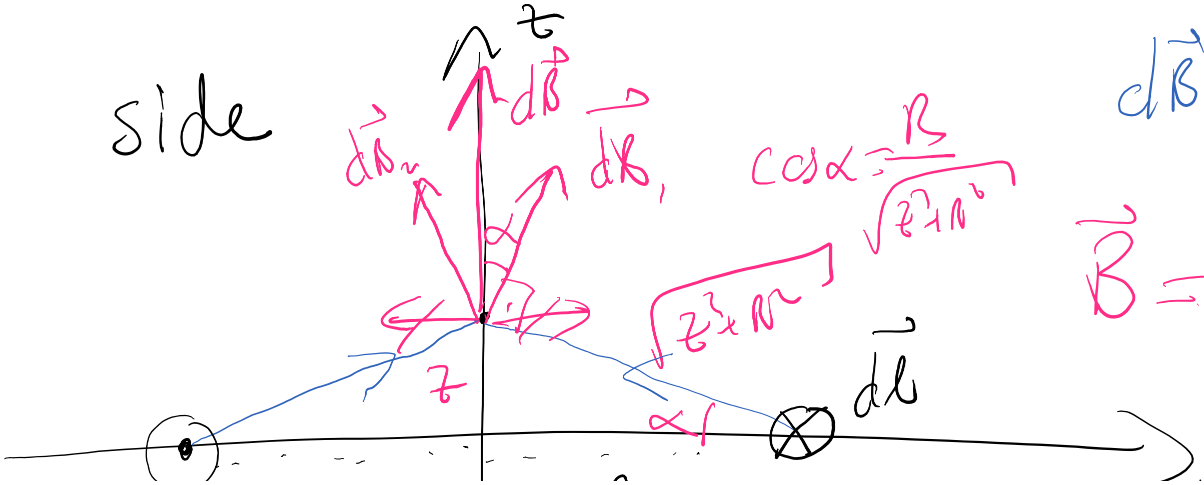
$$I = \frac{\Delta Q}{\Delta t} = \frac{q}{y} = \frac{Q}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$\boxed{I = \frac{Q\omega}{2\pi}}$$

Biot-Savart Law!!

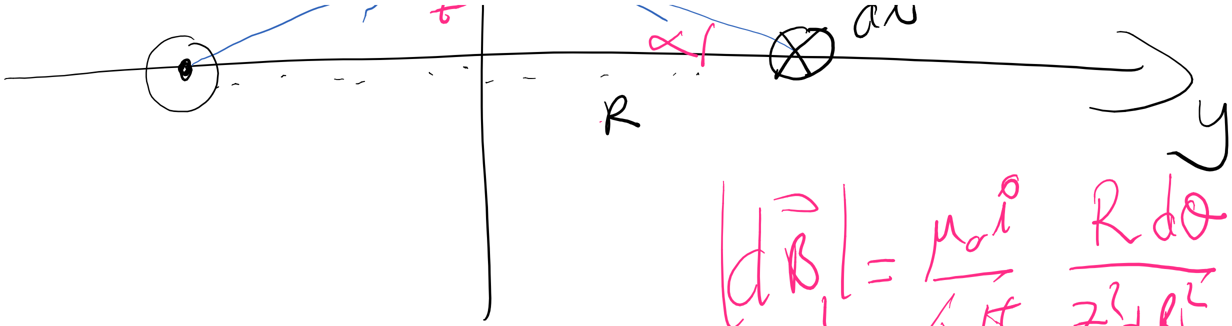
side



$$\cos\alpha = \frac{R}{\sqrt{z^2+R^2}}$$

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\,\frac{\vec{dl}\times\hat{r}}{r^2}$$

$$\vec{B} = |\vec{B}|\,\hat{n}$$



$$|d\vec{B}_1| = \frac{\mu_0 i}{4\pi} \frac{R\,d\theta}{z^2+R^2}$$

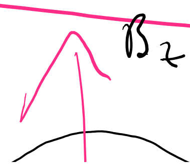
$$dB_z = |d\vec{B}_1| \cos\alpha = \frac{\mu_0 i}{4\pi} \frac{R\,d\theta}{z^2+R^2} \frac{R}{\sqrt{z^2+R^2}}$$

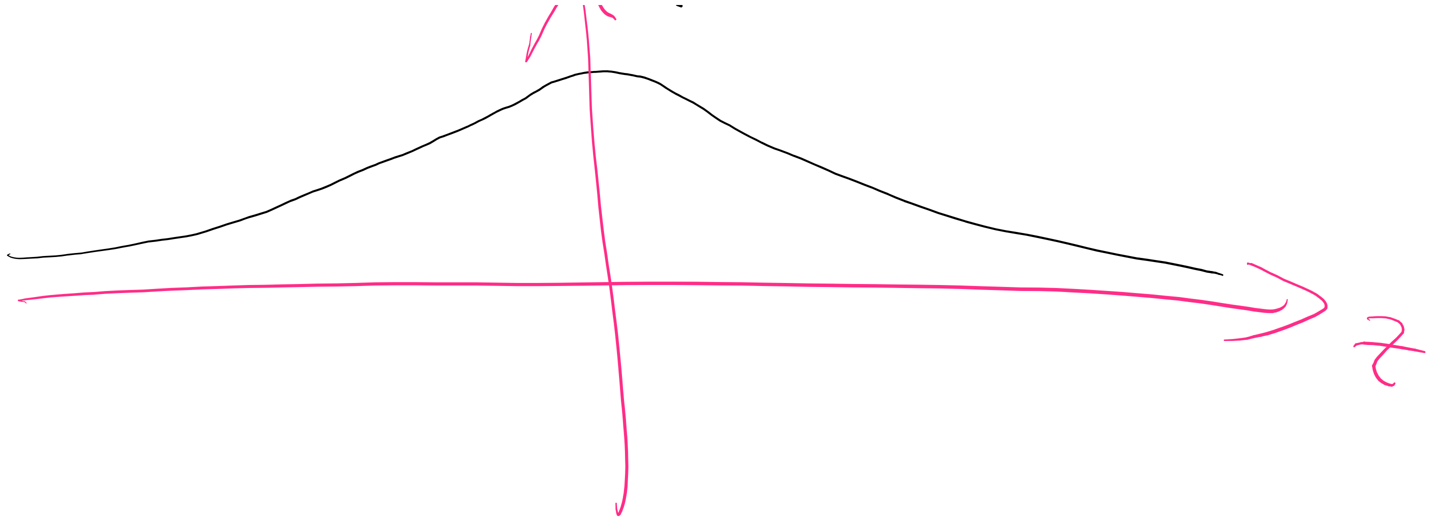
$$dB_z = \frac{\mu_0 i}{4\pi} \frac{R^2\,d\theta}{(z^2+R^2)^{3/2}}$$

$$B_z = \int dB_z = \int_0^{2\pi} \frac{\mu_0 i}{4\pi} \frac{R^2}{(z^2+R^2)^{3/2}}\,d\theta$$

$$= \frac{\mu_0 i}{\cancel{4\pi}_{\,2}} \frac{R^2}{(z^2+R^2)^{3/2}} \underbrace{\int_0^{2\pi} d\theta}_{\cancel{2\pi}}$$

$$\boxed{B_z = \frac{\mu_0 i\, R^2}{2(z^2+R^2)^{3/2}}}$$





z