Week 11 Notes (Apr 4-8)

Wednesday, January 27, 2016 4:48 PM

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{in}$$

$$2\pi r|\vec{B}| = \mu_0 i$$

Single wire $\boxed{|\vec{B}| = \frac{\mu_0 i}{2\pi r}}$

QUIZ-20

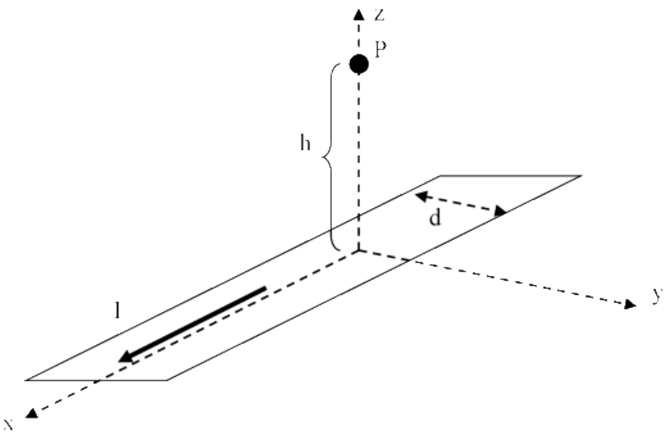
An infinite, thin metal strip of width d is placed on the x-y plane so that it extends from **−d/2** to **d/2** along the y axis and from -**∞** to **∞** along the x axis. The strip carries a uniformly distributed total current of **I** along +x direction (see figure).

Find the magnetic field at a point **P**, which is at a height **h** above the midpoint of the strip (Coordinates **x=0,y=0, z=h**). Give

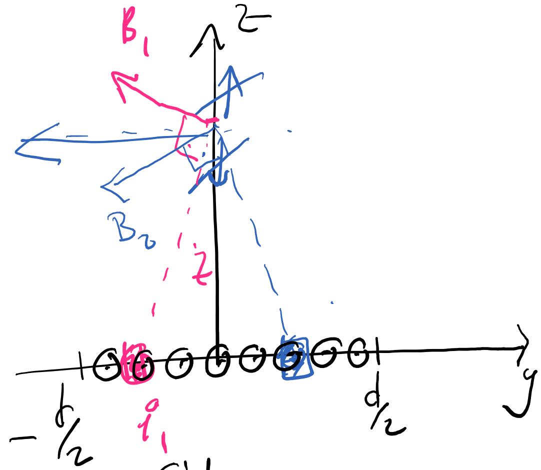
a) The direction of the magnetic field → By symmetry $\boxed{\vec{B} = |\vec{B}|(-\hat{j})}$

b) The magnitude of the magnetic field

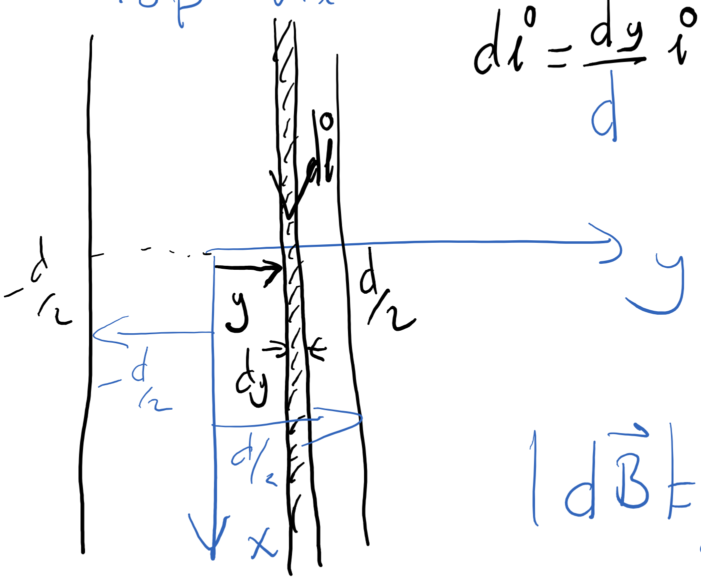
(Hint: Think of the metal strip as made up of wires! Check your result by thinking of the d=0 limit.)



Side view

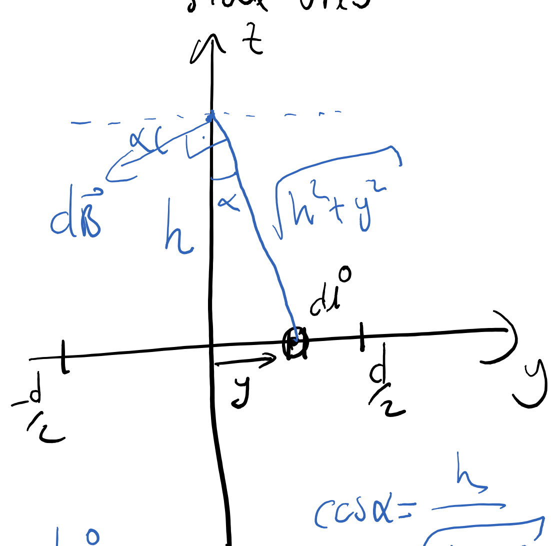


Top view



$$di = \frac{dy}{d}\, i$$

Side view



$$\cos\alpha = \frac{h}{\sqrt{h^2+y^2}}$$

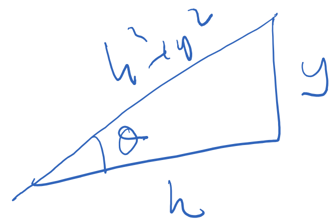
$$|d\vec{B}| = \frac{\mu_0\, di}{2\pi\sqrt{h^2+y^2}}$$

$$dB_y = -\frac{\mu_0}{2\pi}\frac{di}{\sqrt{h^2+y^2}}\cos\alpha$$

$$dB_y = -\frac{\mu_0}{2\pi}\frac{h}{h^2+y^2}\frac{i}{d}\,dy$$

$$B_y = \int dB_y = -\frac{\mu_0 h i}{2\pi d}\int_{-d/2}^{d/2}\frac{1}{h^2+y^2}\,dy$$

$$= \frac{\mu_0 h i}{2\pi d}\int_{-\arctan\frac{d}{2h}}^{\arctan\frac{d}{2h}}\frac{1}{\frac{\cancel{h^2}}{\cancel{\cos^2\theta}}}\,\frac{\cancel{h}}{\cancel{\cos^2\theta}}\,d\theta$$



$$\tan\theta = \frac{y}{h}$$

$$\frac{1}{\cos^2\theta}\,d\theta = \frac{1}{h}\,dy$$

$$\cos\theta = \frac{h}{\sqrt{h^2+y^2}}$$

$$h^2+y^2 = \frac{h^2}{\cos^2\theta}$$

$$= \frac{\mu_0 i}{2\pi d}\,2\arctan\frac{d}{2h}$$

$$\boxed{|\vec{B}| = \frac{\mu_0 i}{2\pi d}\,2\arctan\frac{d}{2h}}$$

1°) Good? ✓

2°) Units $\left[\frac{\mu i}{2\pi d}\right] \to$ Tesla

3°) $\lim_{d\to 0}\left(\frac{\mu_0 i}{2\pi d}\,2\underbrace{\arctan\frac{d}{2h}}\right)$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\theta \ll 1$$

$$\tan\theta \simeq \frac{\theta}{1}$$

$$\text{So } \lim_{d\to 0}\left(\frac{\mu_0 i}{2\pi d} \cdots \underbrace{\qquad\quad}\ \frac{\cdots}{2h}\right)$$

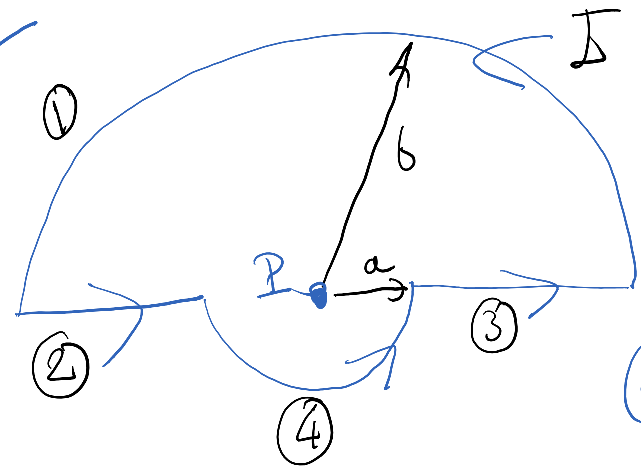
$$\tan\theta \simeq \frac{\theta}{1}$$

$$\arctan\theta \simeq \theta$$

$$\simeq \left(\frac{\mu_0 i}{2\pi \cancel{d}}\cancel{2}\ \frac{\cancel{d}}{\cancel{2}h}\right)$$

$$B = \frac{\mu_0 i}{2\pi h}$$

Ex



A wire is bent into two semicircles of radius a and b as shown.

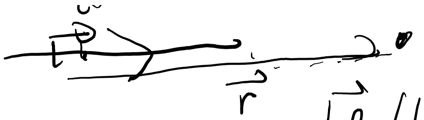
(a) Find the magnetic field at the center P if the loop carries a current I

(b) Find the magnetic moment of the loop.

(a) Calculate the contribution of each section!

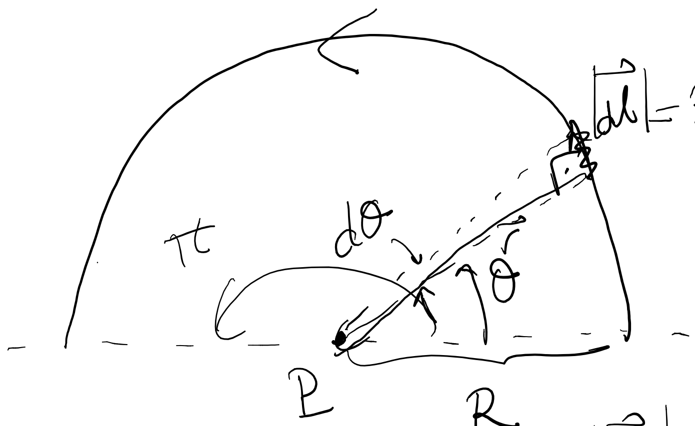
Let's look at the straight parts

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\,\frac{\vec{dl}\times\hat{r}}{r^2}$$



$d\vec{l} \parallel \vec{r}$ | $\boxed{d\vec{l}\times\hat{r} = 0}$

so contributions from ② and ③ are zero.



$|d\vec{l}| = R\,d\theta$

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi}\frac{|d\vec{l}|}{R^2}$$

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi}\frac{\cancel{R}\,d\theta}{R^{\cancel{2}}}$$

$$|\vec{B}|_{\text{semicircle}} = \frac{\mu_0 i}{4\pi R}\int_0^{\pi} d\theta = \frac{\mu_0 i}{4R}$$

$\vec{B}$ is $\odot$

$$|\vec{B}| = \frac{\mu_0 i}{4b} + \frac{\mu_0 i}{4a} = \boxed{\frac{\mu_0 i}{4}\,\frac{a+b}{ab}}$$

Exam Review

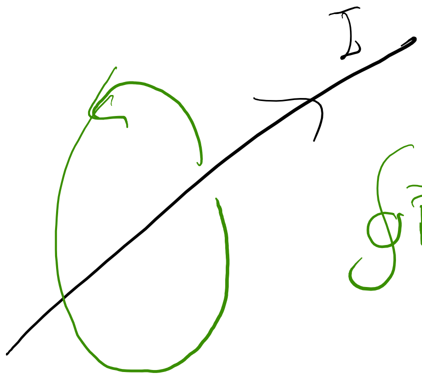
Sources of Magnetic Field.

$\vec{B}$ $\vec{A}$

fields

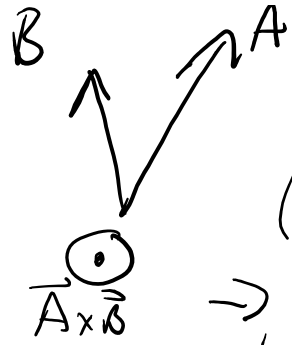
Currents cause magnetic fields

Ampère's Law



has a sign convention

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{in}$$



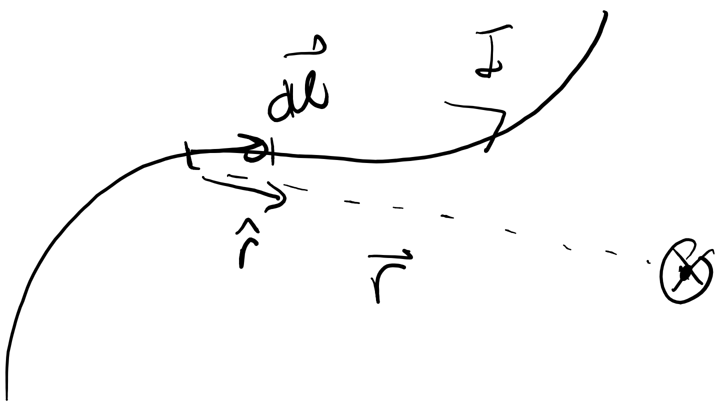
$(\vec{A}\times\vec{B})$

→ is $\perp$ to both $\vec{A}$ and $\vec{B}$

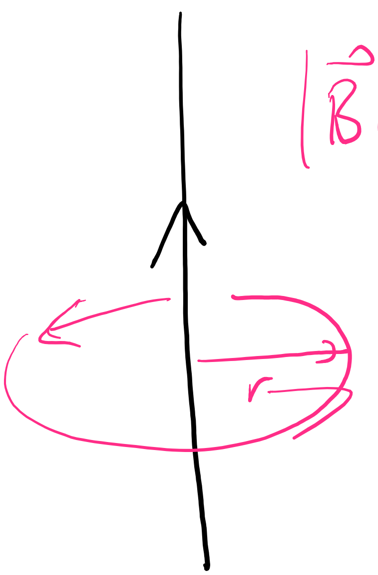
→ Sense is given by r.h.r.

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

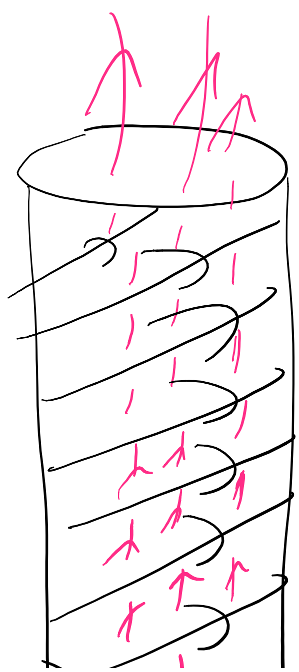
Biot-Savart Law



$$d\vec{B} = \frac{\mu_0 i}{4\pi}\,\frac{d\vec{l}\times\hat{r}}{r^2}$$

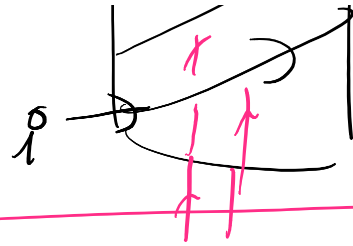


$$|\vec{B}(r)| = \frac{\mu_0 i}{2\pi r}$$



n turns per unit length

$$|\vec{B}| = \mu_0 n i$$

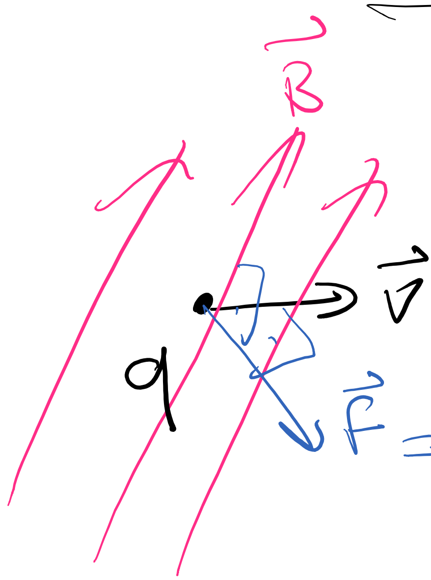


$$|\vec{B}| = \mu_0 n i$$

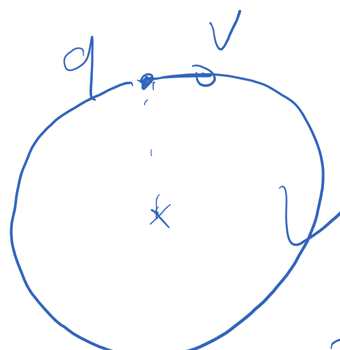
Superposition applies to

$\vec{B}$ is a vector.

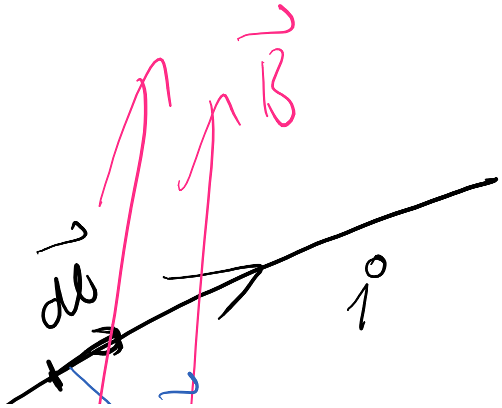
Magnetic Field



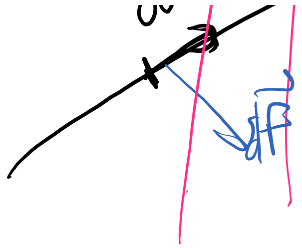
$$\vec{F} = q\,\vec{v} \times \vec{B}$$

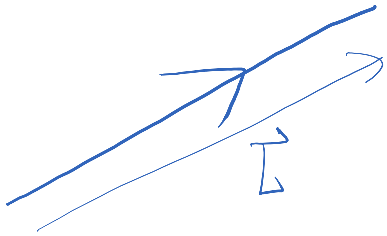


$$\omega_c = \frac{qB}{m}$$



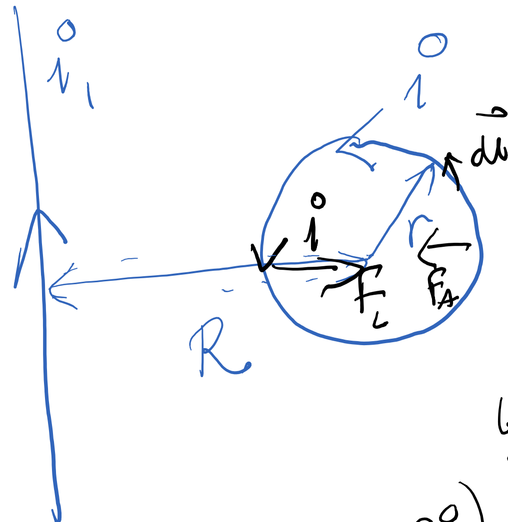
$$\boxed{d\vec{F} = i\, d\vec{l} \times \vec{B}}$$





$$\vec{F} = i\,\vec{L} \times \vec{B}$$

Ex

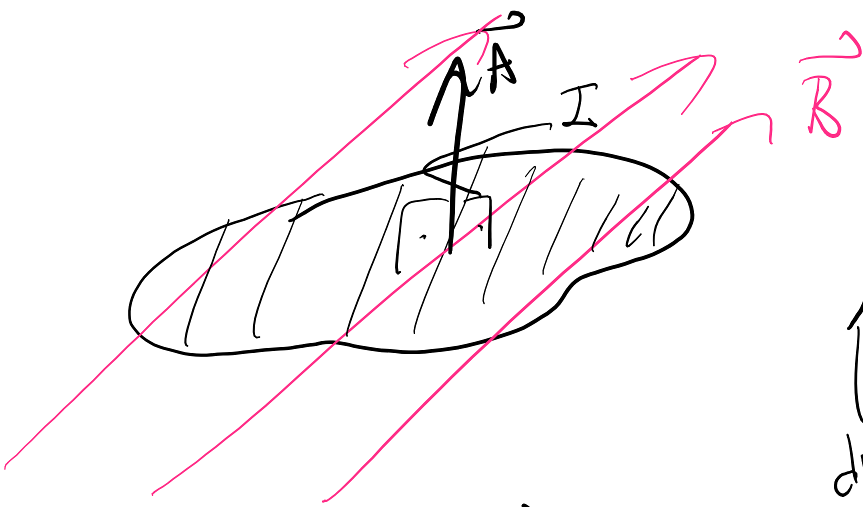


What is the force on the loop?

1°) Calculate $\vec{B}(r)$ by Ampere's law

2°) $\vec{F}$ by $d\vec{F} = i\, d\vec{l} \times \vec{B}$

Current loop in a uniform magnetic field



$$\vec{\mu} = I\,\vec{A}$$

$|\hat{A}| \Rightarrow$ area

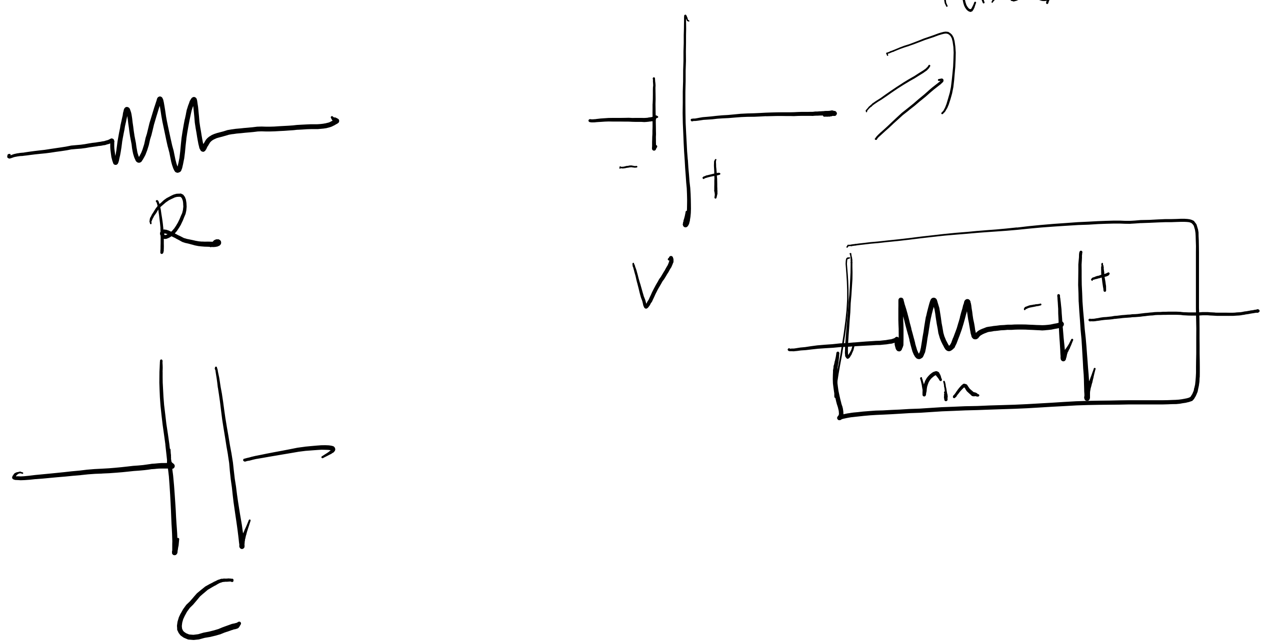
direction by r.h.r

$$\vec{F} = 0$$

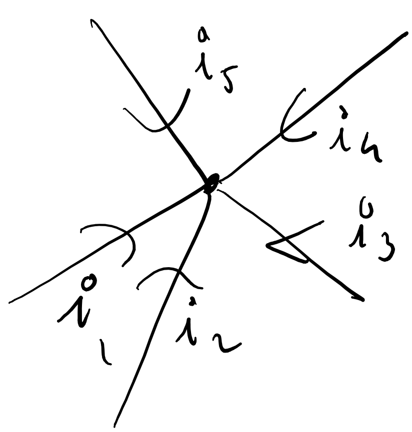
$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

$$\boxed{\vec{\tau} = \vec{\mu} \times B}$$

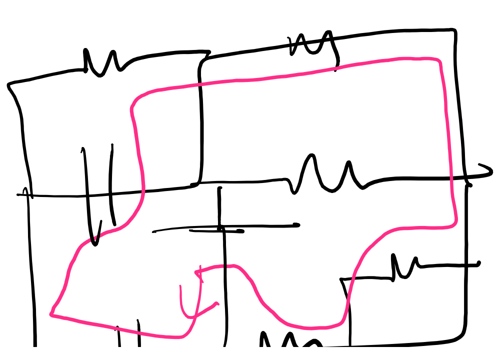
DC circuits



Kircheff's Laws



$$i_1 + i_2 + i_3 + \dots = 0$$



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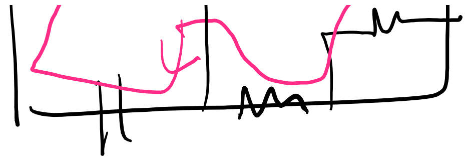
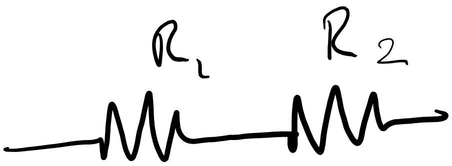


$$\sum_{loop} V = 0$$



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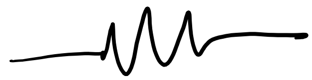
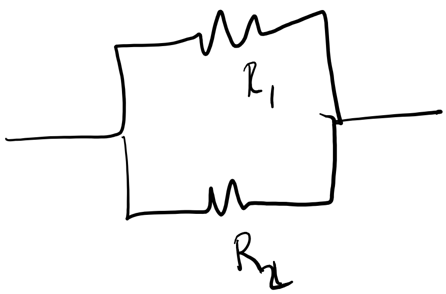


$R_1 + R_2$



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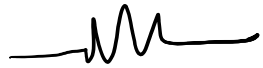
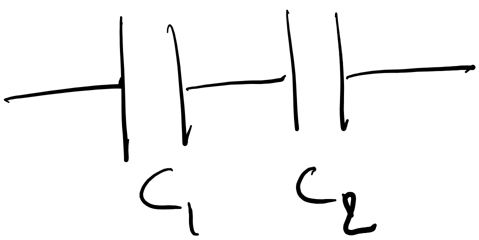
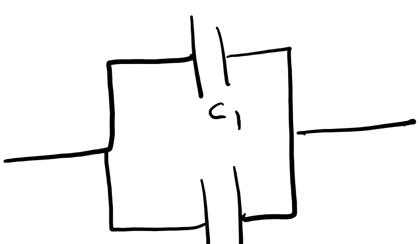


$$R_{eq}^{-1} = R_1^{-1} + R_2^{-1}$$



≡

$$C_{eq}^{-1} = C_1^{-1} + C_2^{-1}$$



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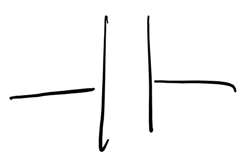
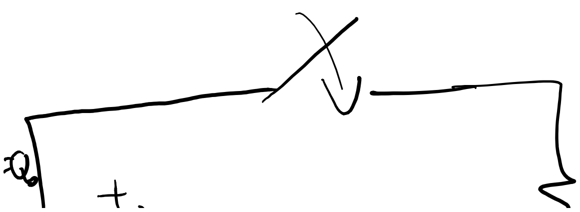
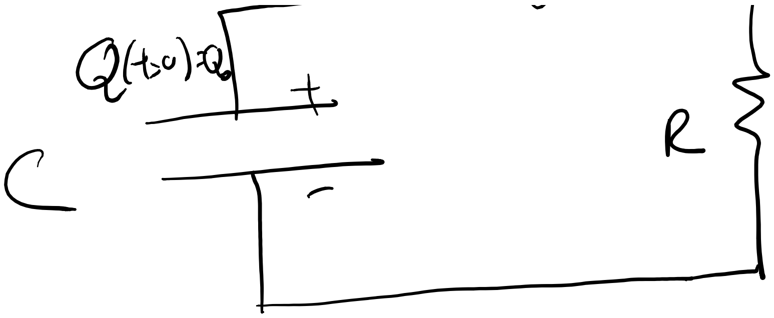


$C_1 + C_2$

$Q(t=0) = Q_0$



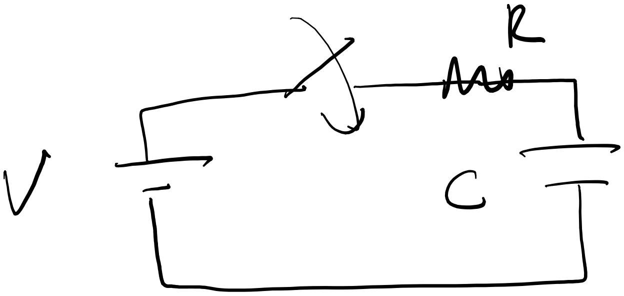
$$Q(t) = Q_0 e^{-t/\tau}$$



$$Q(t) = Q_0 e^{-t/\tau}$$

$$\tau = RC$$

time constant



$$Q(t) = VC\left(1 - e^{-t/\tau}\right)$$

$$\tau = RC$$

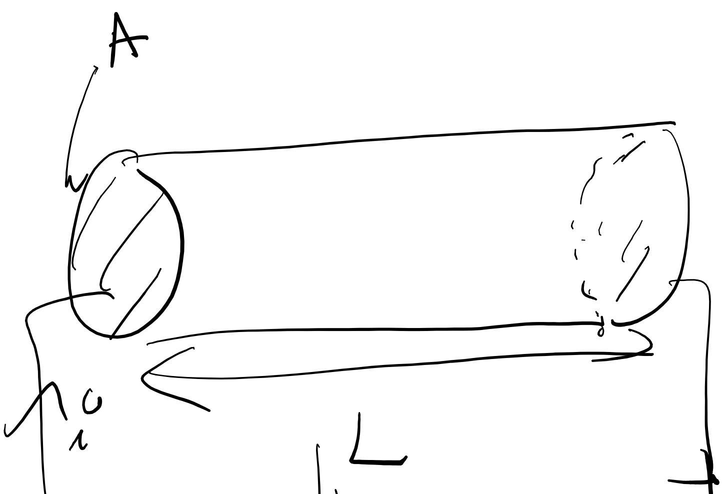
Current and Resistance

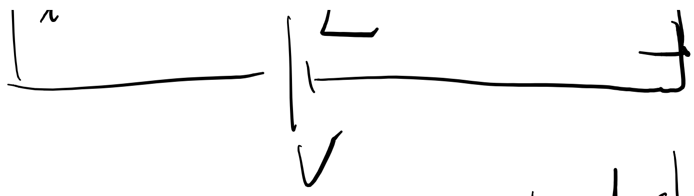
$R = \frac{V}{I}$ Ω

$$R = \rho \frac{L}{A}$$

material property

resistivity



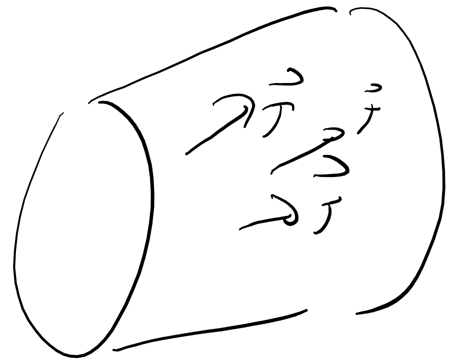


resistivity

total charge passing a plane

$$I = \frac{dQ}{dt}$$

$$\vec{J} = n q \vec{v}_d$$



$$\int_S \vec{J} \cdot d\vec{A} = I$$

$V = iR$

$$P = Ri^2 = \frac{V^2}{R}$$

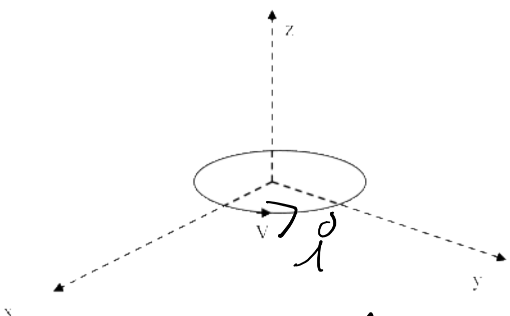
$$U = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$\boxed{i = \frac{dQ}{dt} = C \frac{dV}{dt}}$$

$$C \qquad \boxed{i = \frac{dq}{dt} = C\frac{dV}{dt}}$$

A ring of radius **R** carries a total charge **Q** which is uniformly distributed. The ring is placed on the x-y plane with its center at the origin. The ring is rotating with velocity **V** as shown in the figure.

Calculate the magnetic field (both direction and magnitude) at the origin.

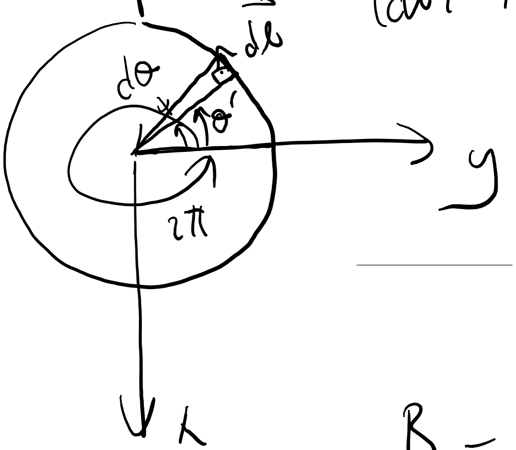


$$i = ?$$

$$i = \frac{Q}{T} = \frac{Q}{\frac{2\pi R}{V}} = \boxed{\frac{QV}{2\pi R}}$$

$$\vec{B} = |\vec{B}|\,\hat{k}$$

top



$$|\vec{dl}| = R\,d\theta$$

$$\vec{dl} \times \hat{r} = |\vec{dl}|\,\underbrace{|\hat{r}|}_{1}\,\underbrace{\sin\frac{\pi}{2}}_{1} = R\,d\theta$$

$$dB = \frac{\mu_0 i}{4\pi}\frac{R\,d\theta}{R^2} = \frac{\mu_0 i}{4\pi R}\,d\theta$$

$$B = \int dB = \int_0^{2\pi} \frac{\mu_0 i}{4\pi R}\,d\theta = \frac{\mu_0 i}{2R}$$

$$\left[\frac{\mu_0 i}{r}\right] \to T \qquad [\mu_0]\frac{C}{s\,m} \to T \qquad B = \frac{\mu_0}{2R}\frac{QV}{2\pi R} = \boxed{\frac{\mu_0 QV}{4\pi R^2}}$$

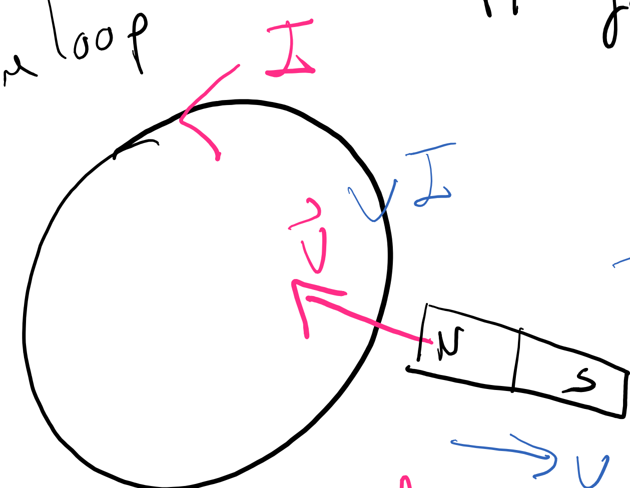
Induction

static

current $\Longrightarrow$ magnetic field

applies a force

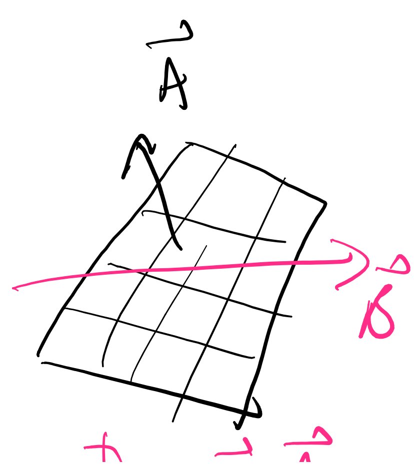
wire loop



$\Longrightarrow$ induction

change of magnetic flux through a loop ~~Change of magnetic field~~ ~~can~~ will induce currents.

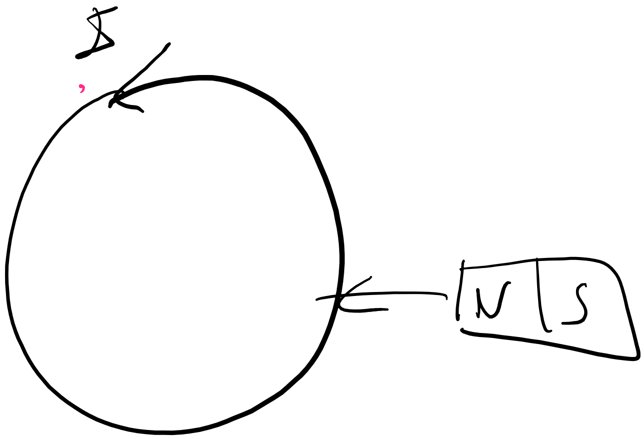
$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$



$\Phi_B = \int_S$

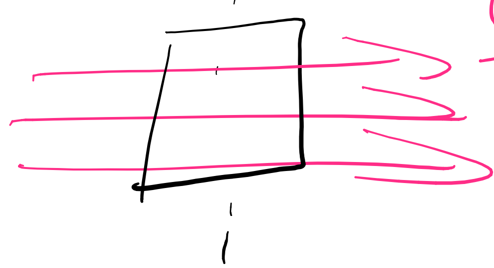
$\Phi_B = \vec{B}\cdot\vec{A}$

$\underbrace{\qquad\qquad}_{|\vec{B}||\vec{A}|\cos\theta}$



Tesla

Generator Φ_B



Direction of the current?

Lenz's Law:

Loops resist the change of flux through them!

$\vec{B}_{ind} \uparrow$

$\Phi_B \uparrow$

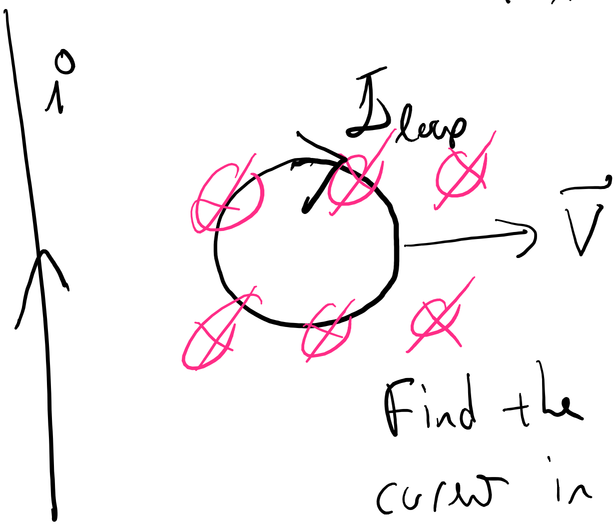
B_{loop} must oppose $\vec{B}_{ext}$

B_{loop} must oppose $\vec{B}_{ext}$

$\vec{B}_{loop}$ $\odot$

i_{loop} C.C.W.

Ex



Find the direction of the current in the circular loop.

$\vec{B}_{ext}$ $\otimes$

$\Phi_B \downarrow$

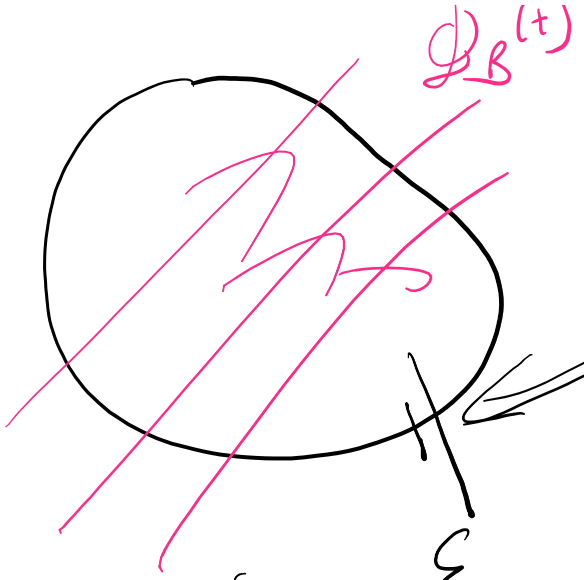
$\vec{B}_{loop}$ must be in the same direction as $\vec{B}_{ext}$

$\vec{B}_{loop}$ $\otimes$

i_{loop} C.W.

Faraday's Law of Induction

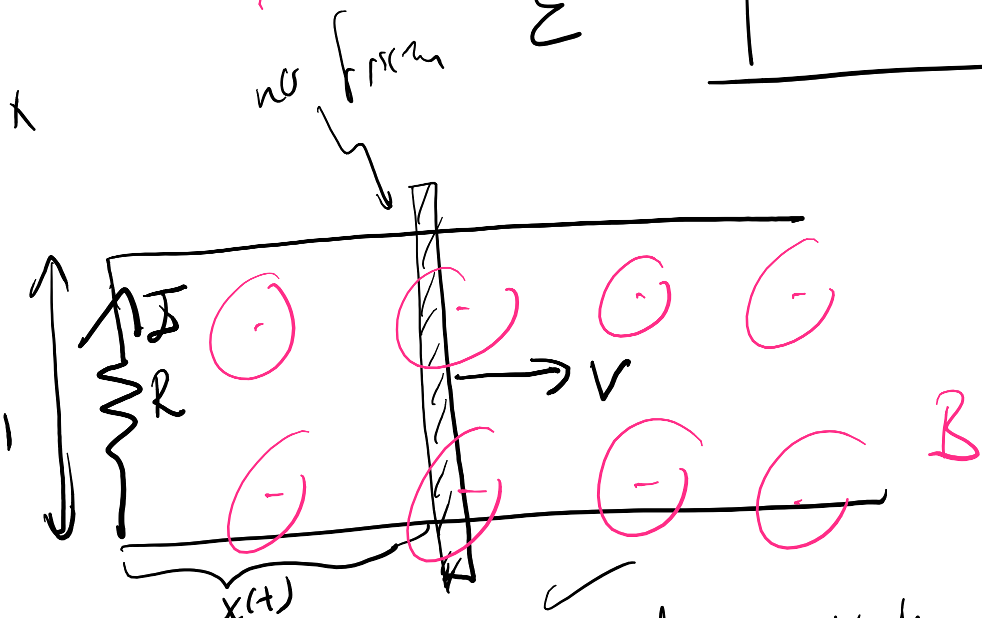
$\Phi_B(t)$



$$\frac{d\Phi_B}{dt}$$

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

Ex



Find the direction and magnitude of the current over the resistor.

$$\boxed{BLv = \mathcal{E}} \qquad \to v$$

$$\Phi_B(t) = B\,L_1\,x(t)$$

$$\frac{d\Phi_B}{dt} = B L_1 \underbrace{\frac{dx(t)}{dt}}_{v} = \boxed{B L_1 v}$$

$$I = \frac{\frac{d\Phi_B}{dt}}{\;} = \frac{B L_1 v}{\;}$$

$$I = \frac{\frac{d\Phi}{dt}}{R} = \boxed{\frac{B L_1 v}{R}}$$