Week 4 Notes (Feb 15-19)

Wednesday, January 27, 2016 4:43 PM

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Coulomb 1780

$$F_{12} = G \frac{m_1 m_2}{r^2}$$

Newton 1680

Cavendish ~1780

shake

EM waves ~1870

Maxwell

Hertz ~1890

⋮

⋮

m_1 m_2 shake

Einstein 1916

Gravitational Waves.

1970 → 2016

LIGO

Detects them!

Gauss' Law & Conductors

Conductor vs Insulator

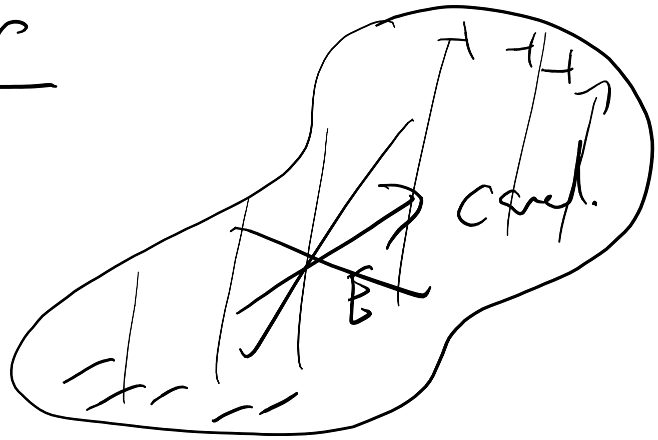
⇓

allows charges to flow freely!

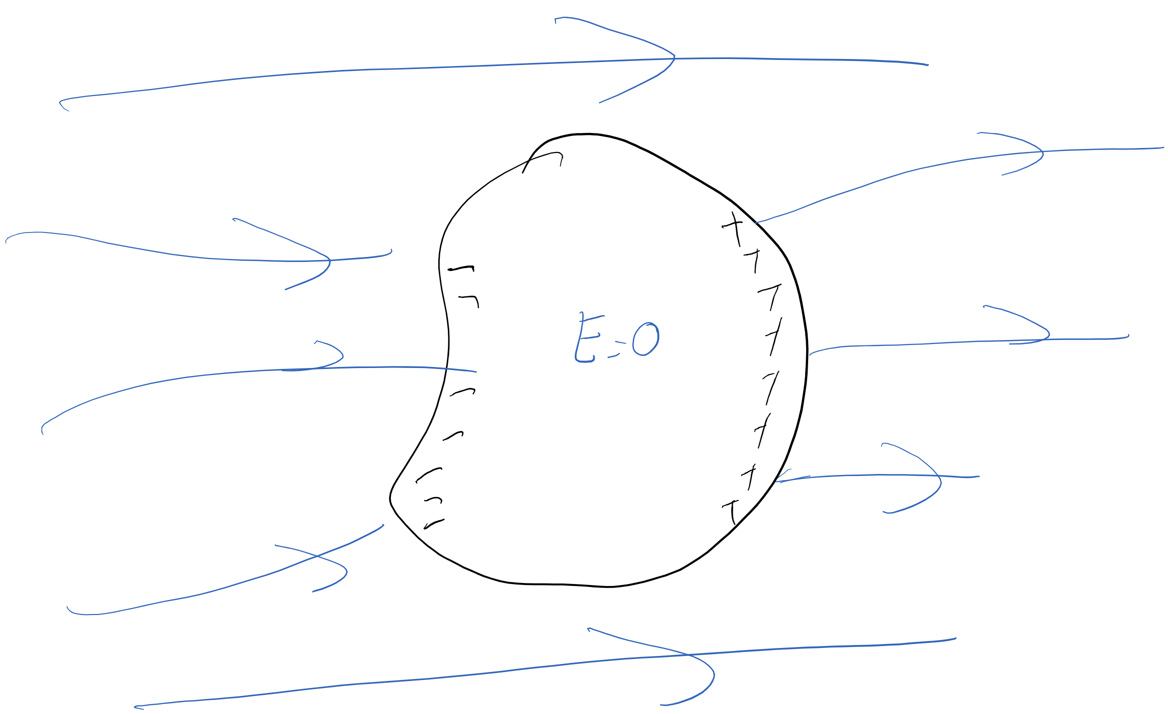
allows charges to flow freely!

perfect conductor

Static Electric field inside a perfect conductor is ZERO!

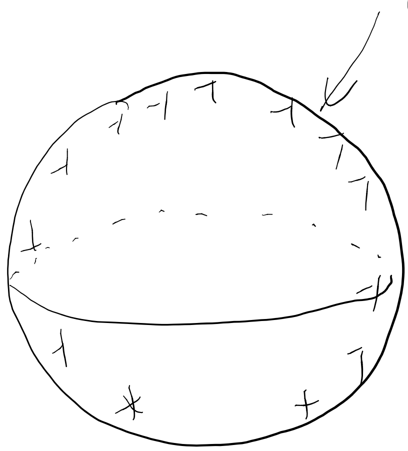


Charges will move until $\vec{E}$ is zero inside.



Ex – A [illegible] radius R is

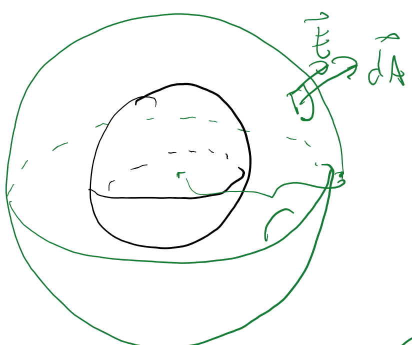
Ex A conducting sphere of radius R is charged with a total charge Q. Find the charge density σ on the surface and electric field inside and outside the sphere.



1°) All the charge will go the surface (to get away from all the others!)

$$\boxed{\sigma = \frac{Q}{4\pi R^2}}$$

2°) Electric field outside.



$$\oint \underbrace{\vec{E} \cdot d\vec{A}}_{\vec{E} \parallel d\vec{A}} = \oint \underbrace{|\vec{E}|}_{\text{constant on the surface}} dA = |\vec{E}| \oint dA = 4\pi r^2 |\vec{E}|$$

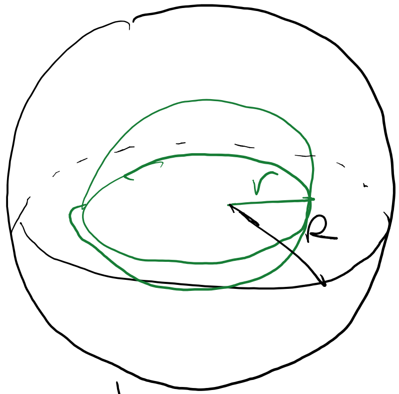
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$Q_{in} = Q$$

$$|\vec{E}|\, 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$$

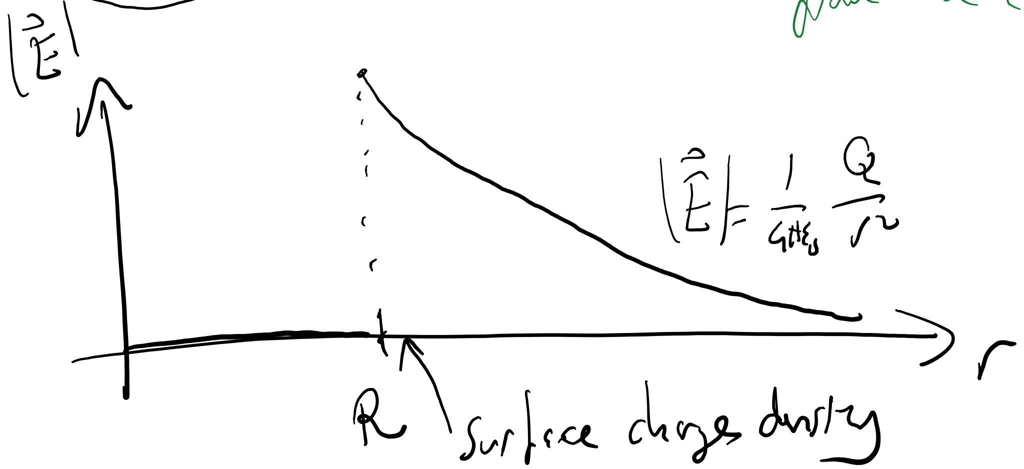
3°) Inside?



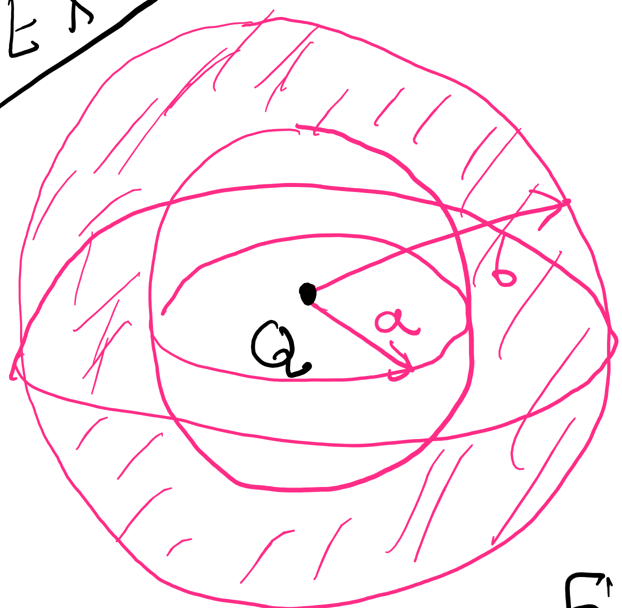
$$\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 |\vec{E}|$$

$$Q_{in} = 0 \Rightarrow |\vec{E}| = 0$$

what I expect from a conductor!



Ex/

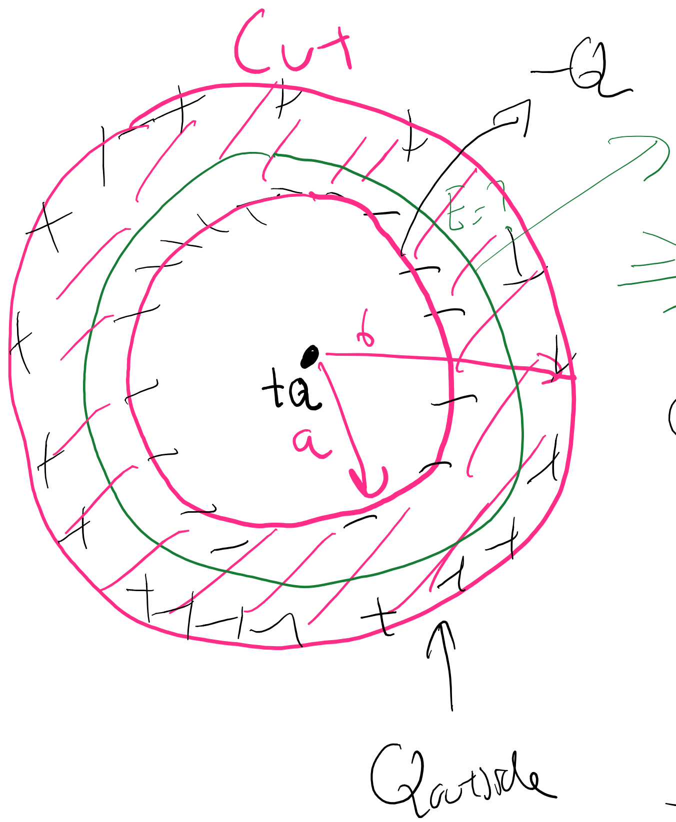


A point charge Q is enclosed in a spherical shell of inner radius a and outer radius b. The shell is a conductor and it has $\boxed{\text{total charge } 2Q}$.

Find 1°) σ_{in} → the charge density on the inner surface

2°) $\sigma_{out} \to$ the charge density on the outer surface

3°) Electric field Magnitude everywhere.



Gauss Surface $\oint \vec{E}\cdot d\vec{A} = 0$

$\Rightarrow Q_{in} = 0$

$Q_{in} = +Q + Q_{inside}$

$\Rightarrow \boxed{Q_{inside} = -Q}$

$$\boxed{\sigma_{in} = \frac{-Q}{4\pi a^2}}$$

$$2Q = \underbrace{Q_{inside}}_{-Q} + Q_{outside}$$

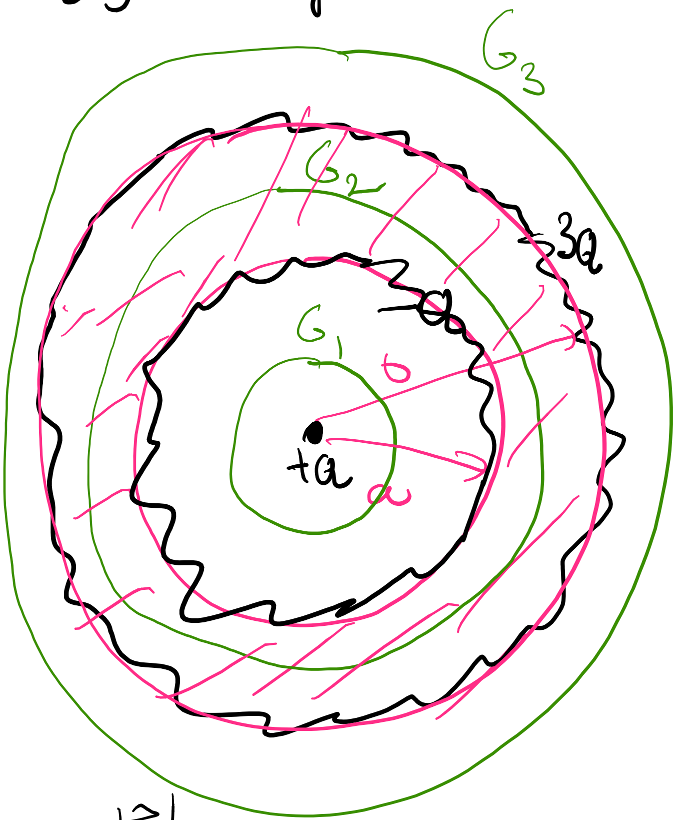
$\Rightarrow \boxed{Q_{outside} = 3Q}$

$$\boxed{\sigma_{out} = \frac{3Q}{4\pi b^2}}$$

2°) $\vec{E}$ field

$\oint \vec{E}\cdot d\vec{A} = \frac{+Q}{\varepsilon}$

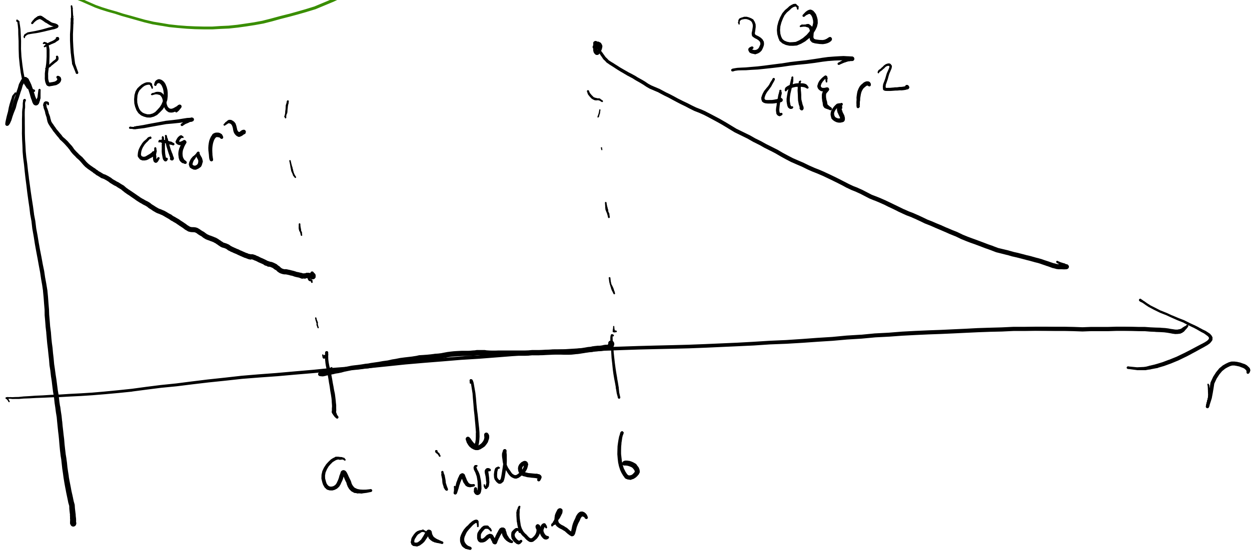
2) E field

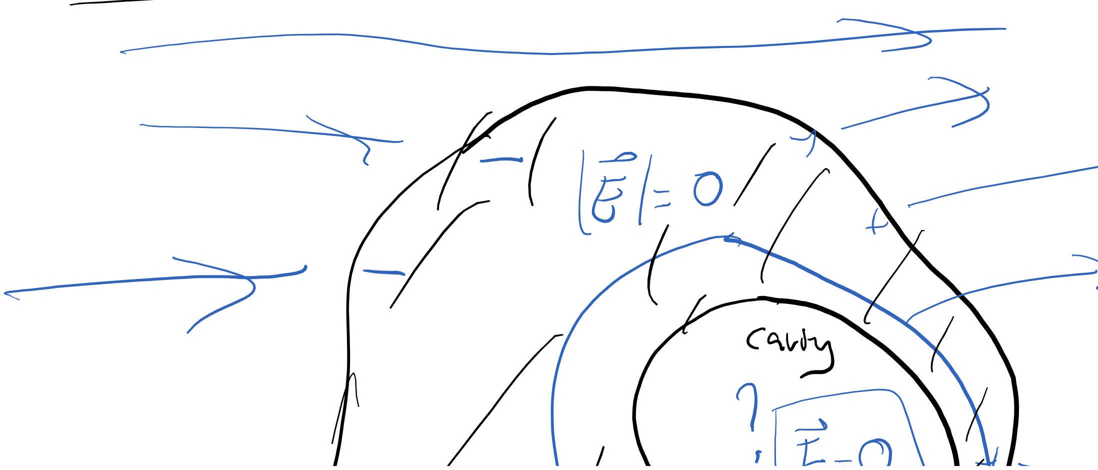


$$\oint_{G_1} \underbrace{\vec{E}\cdot d\vec{A}}_{4\pi r^2|\vec{E}|} = \frac{+Q}{\varepsilon_0}$$

$$\oint_{G_2} \underbrace{\vec{E}\cdot d\vec{A}}_{4\pi r^2|\vec{E}|} = 0$$

$$\oint_{G_3} \underbrace{\vec{E}\cdot d\vec{A}}_{4\pi r^2|\vec{E}|} = \frac{3Q}{\varepsilon_0}$$





$\oint \vec{E}\cdot d\vec{A} = 0$

$Q_{in} = 0$

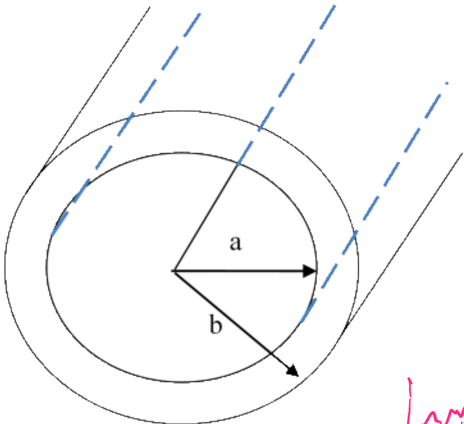
Cavity inside a conductor is shielded from the $\vec{E}$ field outside!

(Faraday Cage!)

QUIZ-6

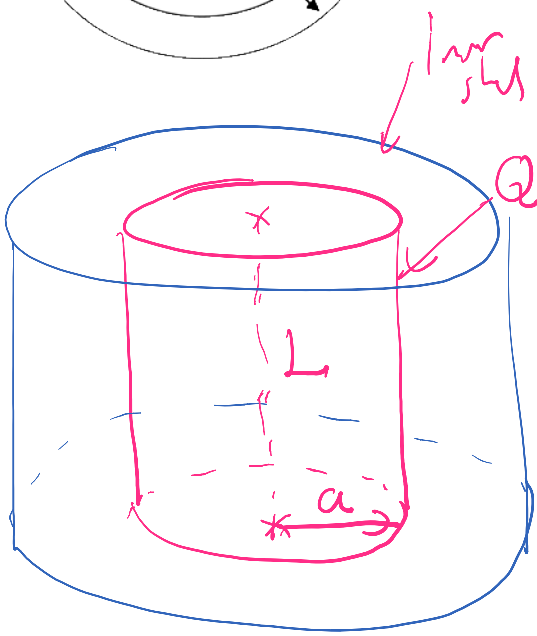
An infinite thin wire is charged with line charge density **λ.** A metal cylindrical shell of inner radius **a** and outer radius **b** is concentric with the thin wire. If the total charge in the metal shell is zero.

a) Find the magnitude of the electric field at a distance r from the wire for **r<a**
b) Find the magnitude of the electric field inside the metal shell **a<r<b**. $\Longrightarrow \vec{E}(r) = 0 \quad a<r<b$
c) Find the magnitude of the electric field outside the metal shell **b<r**.
d) What is the surface charge density **σ** in the inner surface of the metal shell, and the outer surface of the metal shell?



$\oint \vec{E}\cdot d\vec{A} = 0$

$Q_{in} = 0$

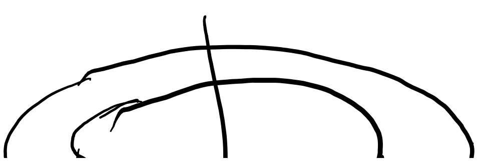


$$\sigma_{inside} = \frac{-\lambda \cancel{L}}{2\pi a \cancel{L}} = \boxed{-\frac{\lambda}{2\pi a}}$$

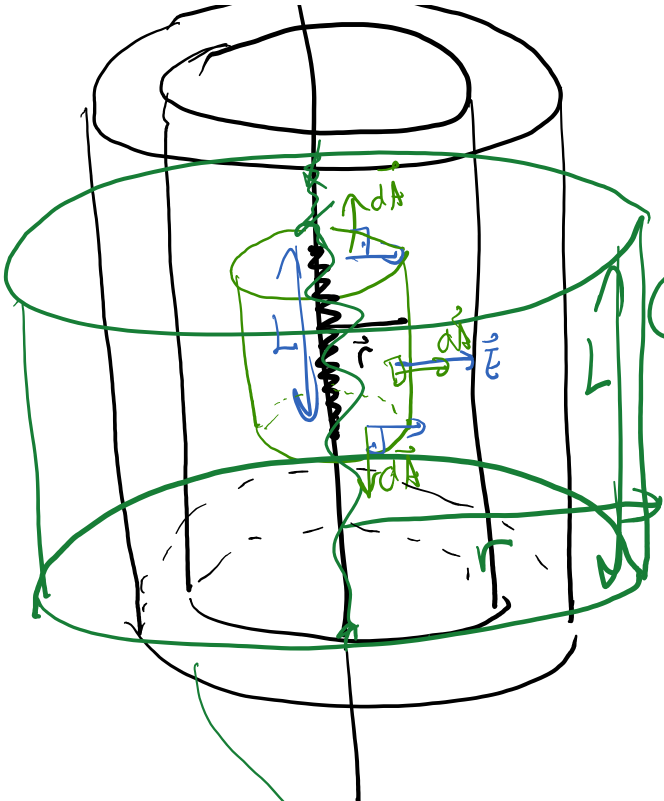
$$\sigma_{out} = \frac{\lambda \cancel{L}}{2\pi b \cancel{L}} = \boxed{\frac{\lambda}{2\pi b}}$$

a) c)

Electric field.



$r < a \quad 0 \quad 0$



$Q_{in} =$

$$\oint \vec{E}\cdot d\vec{A} = \cancel{\int_{top} \vec{E}\cdot d\vec{A}} + \cancel{\int_{bottom} \vec{E}\cdot d\vec{A}} + \underbrace{\int_{side} \vec{E}\cdot d\vec{A}}_{\vec{E} // d\vec{A}}$$

$\vec{E} \perp d\vec{A}$ $\quad$ $\vec{E} \perp d\vec{A}$

$$= |\vec{E}| \int dA = |\vec{E}|\, L\, 2\pi r$$

$$Q_{in} = \lambda L$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$|\vec{E}|\, 2\pi r \cancel{L} = \frac{\lambda \cancel{L}}{\varepsilon_0}$$

$$\boxed{|\vec{E}| = \frac{\lambda}{2\pi r \varepsilon_0}} \qquad |\vec{r}| < a$$

$r > b$

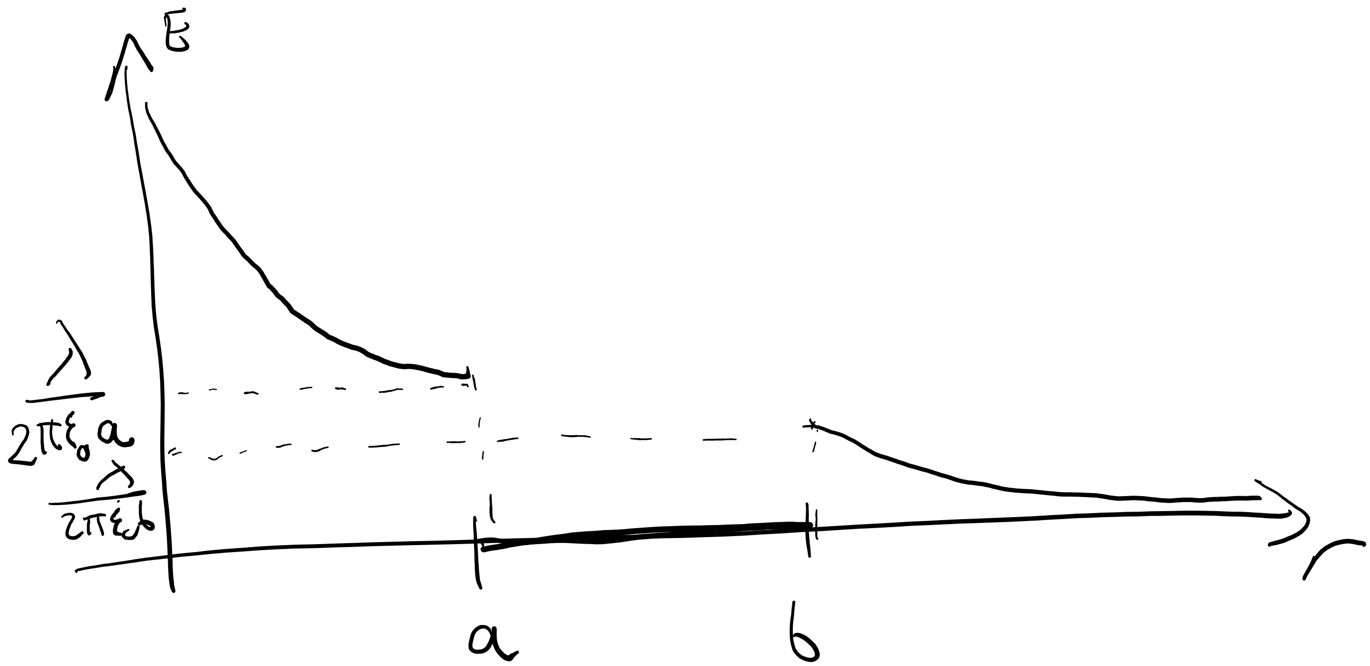
$$\oint \vec{E}\cdot d\vec{A} = 2\pi r L |\vec{E}|$$

$$Q_{in} = \lambda L$$

$$|\vec{E}| = \frac{\lambda \cancel{L}}{2\pi \varepsilon_0 r \cancel{L}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 r}}$$

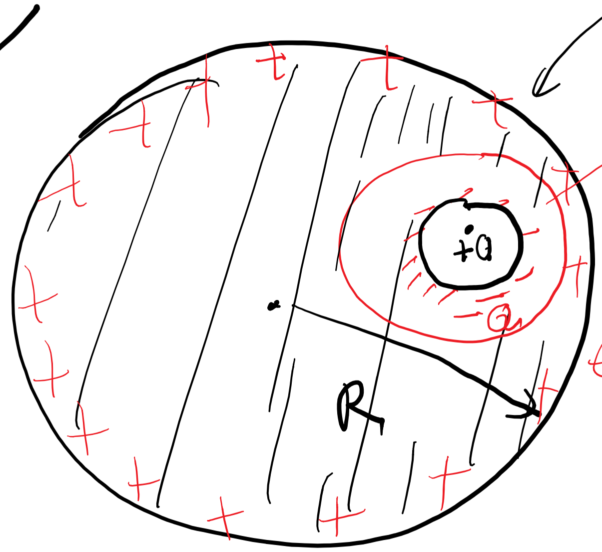
$r > b$ $\quad$ Net $Q_{in} = (\lambda - \lambda + \lambda) = \lambda L$

$\wedge E$



Ex

sphere with radius R



$\oint \vec{E}\cdot d\vec{A} = 0$

Net +Q on the outer surface.

How is this charge distributed?

For a full sphere surface charge is uniformly distributed.

This case is not different!

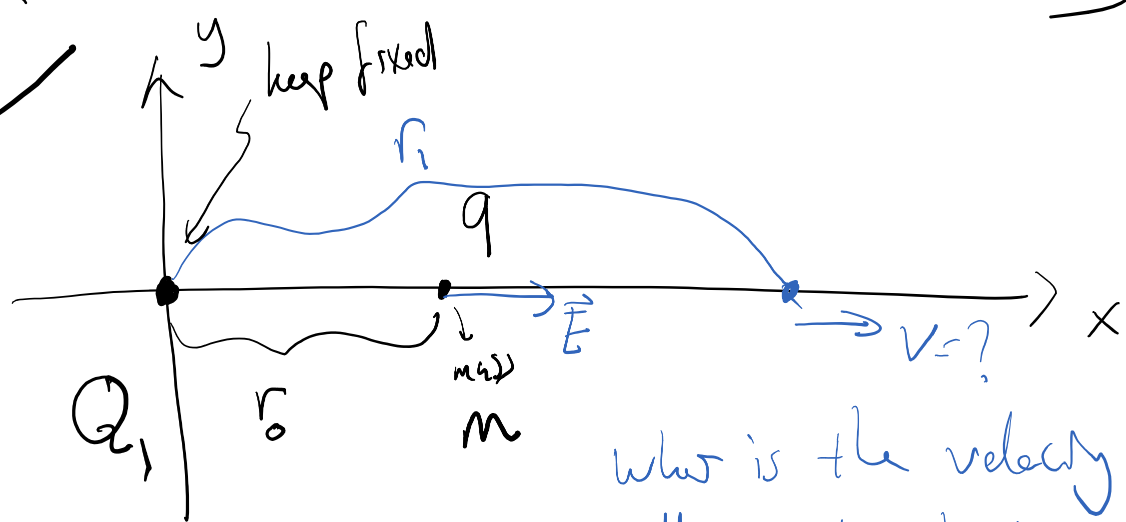
differs !

On the outside $\sigma = \dfrac{Q}{4\pi R^2}$ uniformly.

$$|\vec{F}| = \frac{1}{4\pi\varepsilon_0}\,\frac{Q}{r^{\varepsilon}}$$

$$\varepsilon = 2.\underbrace{001 \mp 0.02}_{\text{deviation from } \underline{2} \text{ is related to rest mass of photons}}$$

Ex



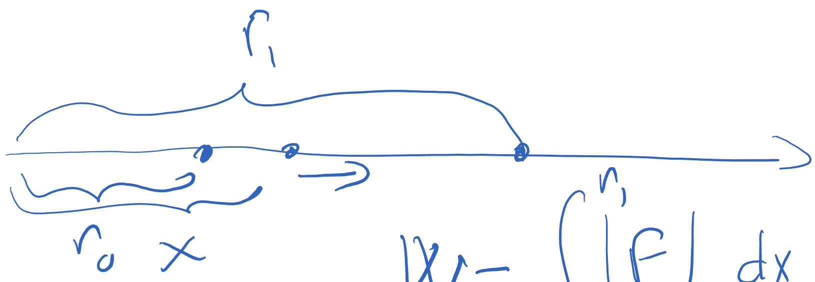
what is the velocity of the small particle when it reaches r_1?

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\,\frac{Q_1 q}{r^2} \Rightarrow |\vec{F}| = q|\vec{E}| \Rightarrow m\vec{a} = \vec{F}$$

$$\Rightarrow |\vec{a}| = \frac{q}{m}|\vec{E}|$$

$x(t) = ?\,;\; v(t) = ?$

Let's use Work - Energy theorem.



$$|F| = \frac{Q\,q}{4\pi\varepsilon_0 x^2}$$

$$W = \int_{r_0}^{r_1} |F|\, dx$$

$$W = \int_{r_0}^{r_1} \frac{Q\,q}{4\pi\varepsilon_0 x^2}\, dx = \frac{Q\,q}{4\pi\varepsilon_0} \underbrace{\int_{r_0}^{r_1} \frac{1}{x^2}\, dx}_{-\frac{1}{x}\Big|_{r_0}^{r_1}}$$

$$\underset{\text{work done by electrical force}}{W} = \frac{Q\,q}{4\pi\varepsilon_0}\left[\frac{1}{r_0} - \frac{1}{r_1}\right] = \Delta K$$

$$= U(r_0) - U(r_1) \qquad = \frac{1}{2} m v^2 = K_f$$

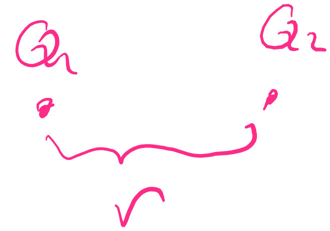
$$\underset{\text{Initial}}{U(r_0)} = \underset{\text{final}}{U(r_1) + K_f}$$

$$v = \sqrt{\frac{2}{m}}\sqrt{\frac{Q\,q}{4\pi\varepsilon_0}\left[\frac{1}{r_0} - \frac{1}{r_1}\right]}$$

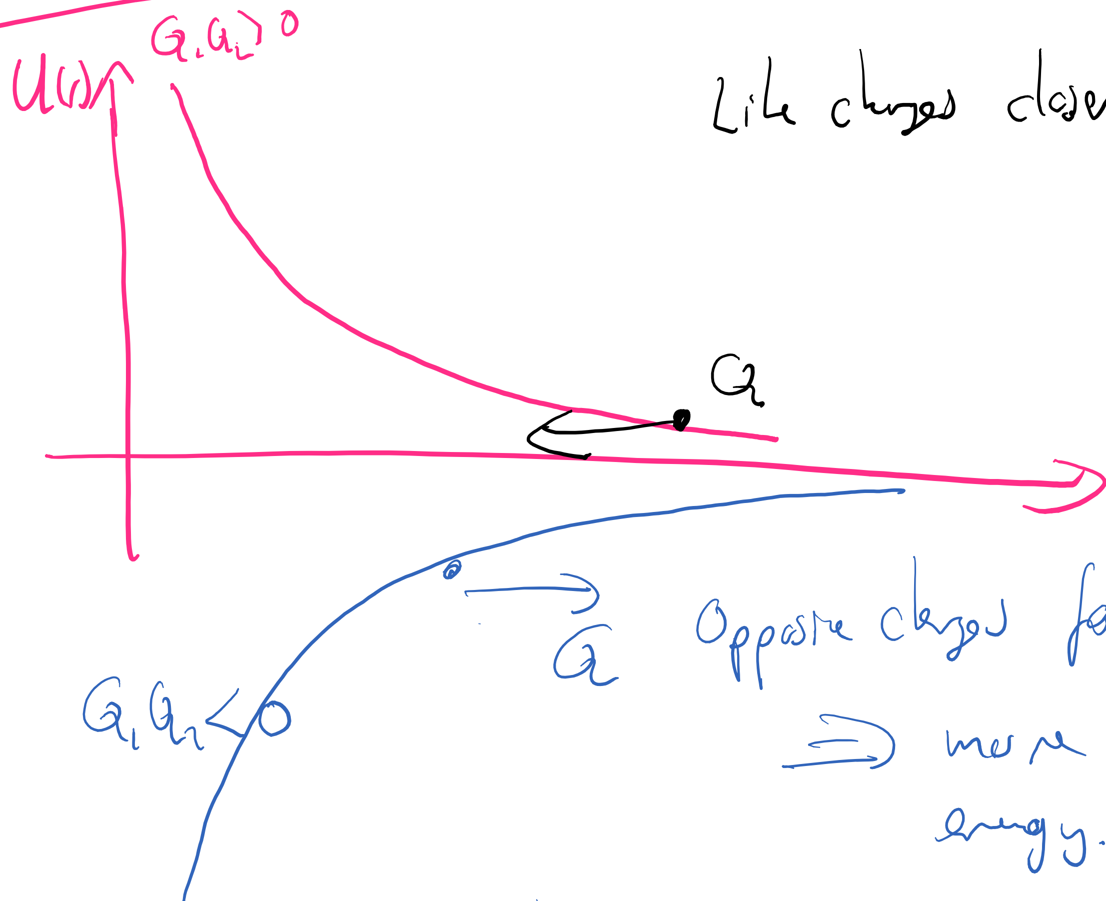
(Static) Electrical Forces are Conservative!

$$\boxed{\oint \vec{E}\cdot d\vec{l} = 0}$$

I can define a potential Energy



$$\boxed{U(r) = \frac{1}{4\pi\epsilon_0}\,\frac{Q_1 Q_2}{r}}$$

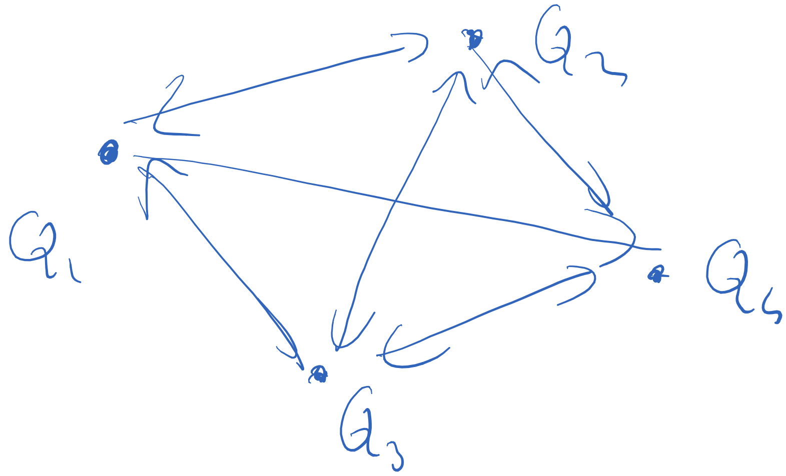


Like charges closer ⇒ more potential energy

Opposite charges farther ⇒ more potential energy.

More than two charges

Q_3

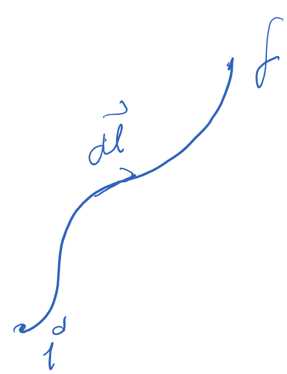


$$U_{total} = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$\boxed{U_{total} = \frac{1}{2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} U_{ij}}$$

Electric potential Energy

$$\Delta U = -\int_i^f \vec{F}_e \cdot d\vec{l}$$



Two point charges Q q

$$\vec{F}_e = q\vec{E}$$

$$U(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q\,q}{r}$$

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q\,?}{r}$$

$F_e = q\,?$

$$\Delta U = -\int_i^f q\,\vec{E}\cdot\vec{dl}$$

$$\underbrace{\Delta U}_{\text{potential energy}} = \underbrace{(q)}_{(\text{charge})}\underbrace{\left[-\int_i^f \vec{E}\cdot\vec{dl}\right]}_{\Delta V}$$

ΔV the change in $\boxed{\text{electric potential}}$

$$\Delta V = -\int_i^f \vec{E}\cdot\vec{dl} = \frac{\Delta U}{q}$$

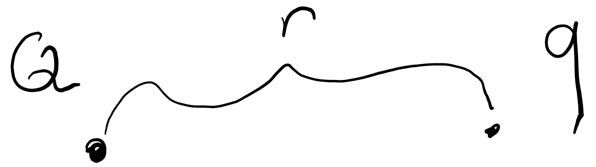
$[\Delta U] \rightarrow$ Joule

$[V] \rightarrow J/C \Rightarrow$ Volt $\boxed{\text{potential}}$

$[q] \rightarrow C$

$[\vec{E}] \rightarrow N/C$

$$\left[\Delta V = -\int_i^f \vec{E}\cdot\vec{dl}\right] \Rightarrow [V] = N/C\, m = \frac{Nm}{C} = J/C$$



$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q\,q}{r} = \underbrace{V(r)}_{\text{potential created by } Q}\, q$$

1°) Potential is additive

$$V_P = V_{Q_1} + V_{Q_2} + \dots$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_2} \dots$$

2°) Potential is a $\boxed{\text{scalar}}$

(Generally easier to calculate)

3°) Potential is always calculated with respect

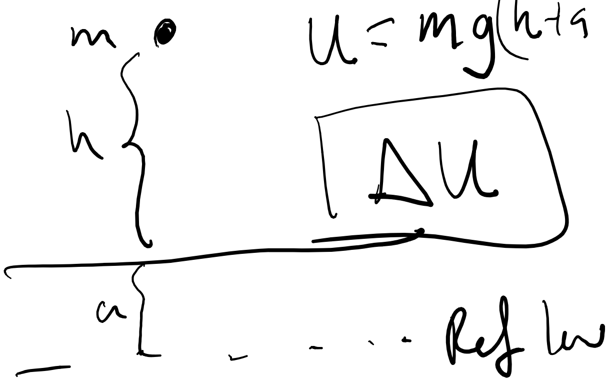
[illegible] point. ground !!

to some refrence point.

At some point we fix

$$V = 0$$

Recall PHYS101

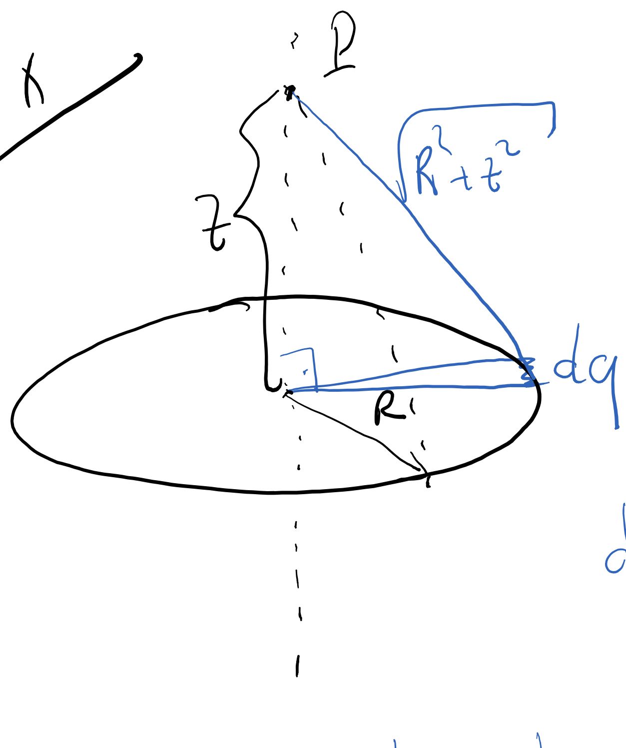


$\Longrightarrow$ In many cases we take $V(\infty) = 0$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Ex

A ring of Radius R carries a total charge of Q. Find the potential on its axis.



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R^2+z^2}}$$

$$\int dV = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{R^2+z^2}}\int dq = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}}$$

1°) Goal ✓

2°) Units $[V] = \frac{1}{[\varepsilon_0]} \frac{C}{m}$ ✓

3°) Limit $z \gg R$ $\quad \sqrt{z^2 + \cancel{R^2}} \simeq z \quad V \to \frac{Q}{4\pi\varepsilon_0 z}$ ✓

neglig

$$V(z) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{R^2 + z^2}} \qquad V(x, y, z)$$

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{\ell}$$

$$\Downarrow$$

$$\vec{E}(x,y,z) = E_x(x,y,z)\,\hat{i} + E_y(x,y,z)\,\hat{j} + E_z(x,y,z)\,\hat{k}$$

$$E_x = -\frac{\partial V}{\partial x}$$

[$\frac{\partial V}{\partial x}$ means take a derivative treating y and z as constants]

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$_z = -\frac{}{\partial z}$$

Let's check

$$V(z) = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+z^2}}$$

$$E_z = -\frac{\partial}{\partial z}V(z) = -\frac{\partial}{\partial z}\left[\frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+z^2}}\right]$$

$$= -\frac{Q}{4\pi\epsilon_0}\frac{\partial}{\partial z}\frac{1}{(R^2+z^2)^{1/2}}$$

$$= +\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{\cancel{2}}\right)\cancel{2}z\,(R^2+z^2)^{-3/2}$$

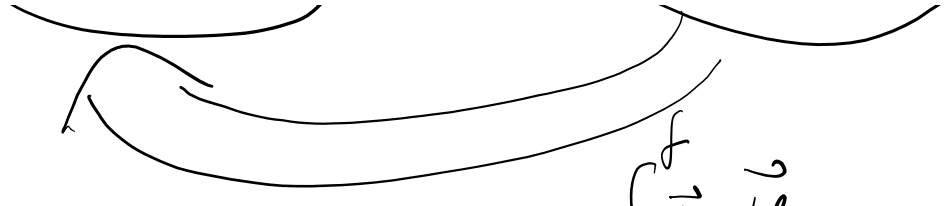
$$E_z = \frac{Q}{4\pi\epsilon_0}\frac{z}{(R^2+z^2)^{3/2}}$$

$$\boxed{\vec{E} = E_z\hat{k}}$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$\boxed{\vec{E} = -\vec{\nabla}V}$$

$V(x,y,z)$ $\vec{E}(x,y,z)$



$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{l}$$

Another unit electron-Volt eV

(A unit energy)

= The energy needed to move an electron to a point with 1 V energy difference.

CERN ~ TeV

$$m_e c^2 \cong 0.5 \text{ MeV}$$

Quiz hint

Gauss

Spherically symmetric system → $\vec{E}(r)$

$V(r) = ?$ ← integration. $\boxed{V(\infty)=0}$