Wednesday 12:35 - 1:40

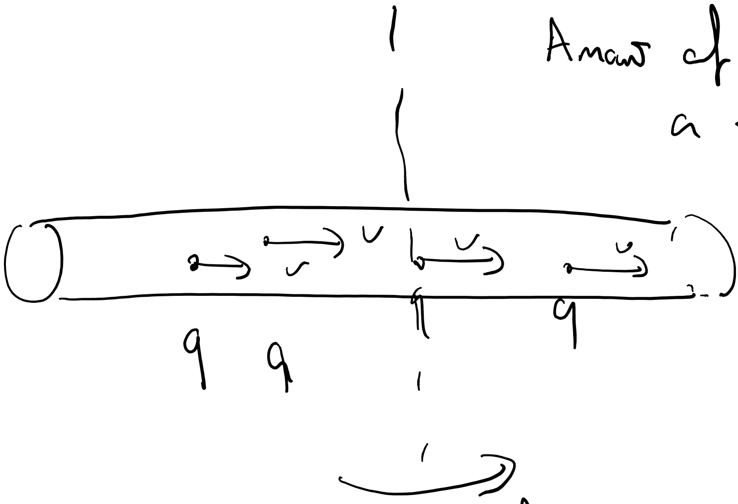
See your exam papers in my room
PHYS A227
→ bring a copy of the solutions!

Next Week 12th to 19th of March

No PHYS 102 Section 03 classes.

Current & Resistance & DC Circuits

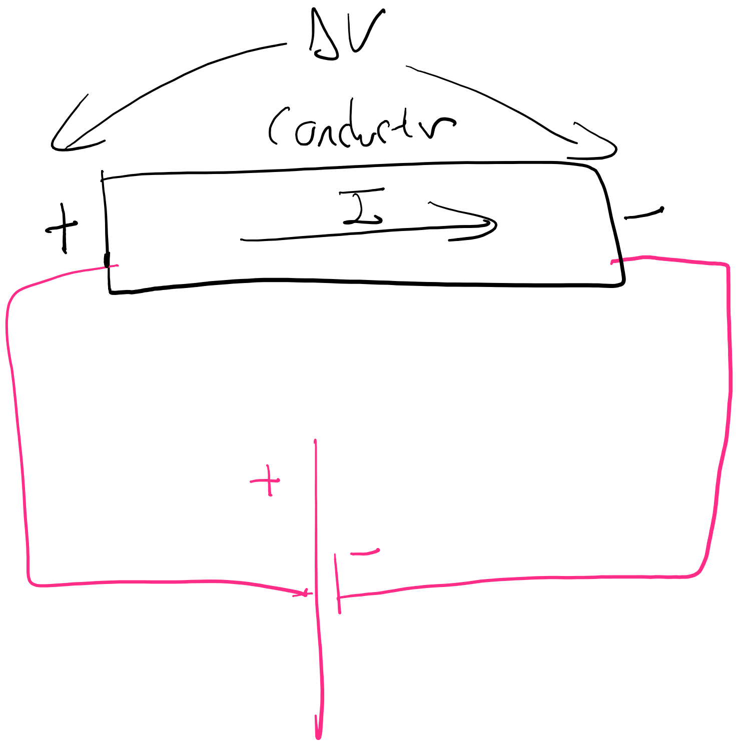
Amount of charge passing through a surface per unit time.



electric current

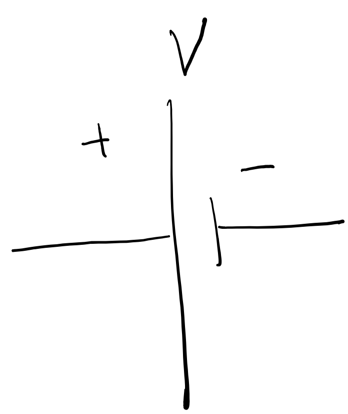
$$I_{avg} = \frac{\Delta q}{\Delta t} \implies i = \frac{dq}{dt}$$

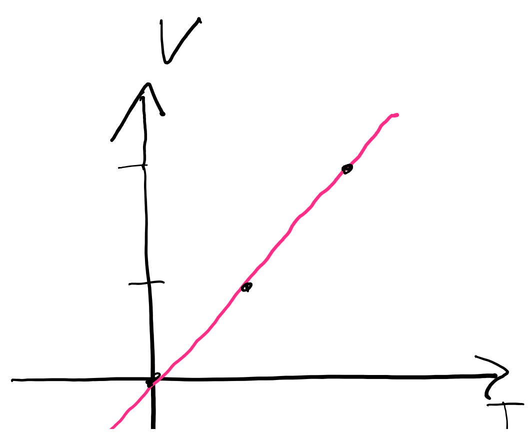
$$[i] = \frac{[q]}{[t]} = C/s = A$$

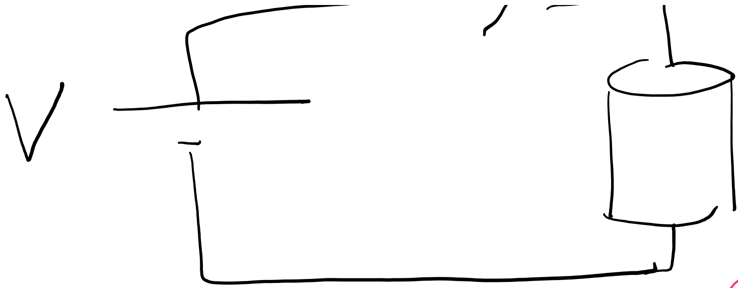


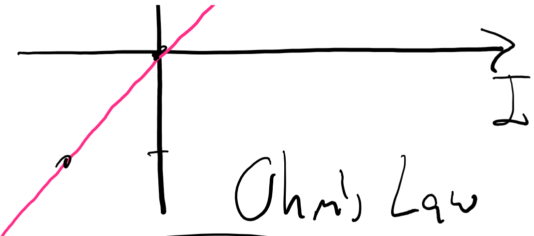
How does the battery ~~create~~ maintain this potential difference?

$\Rightarrow$ CHEMISTRY





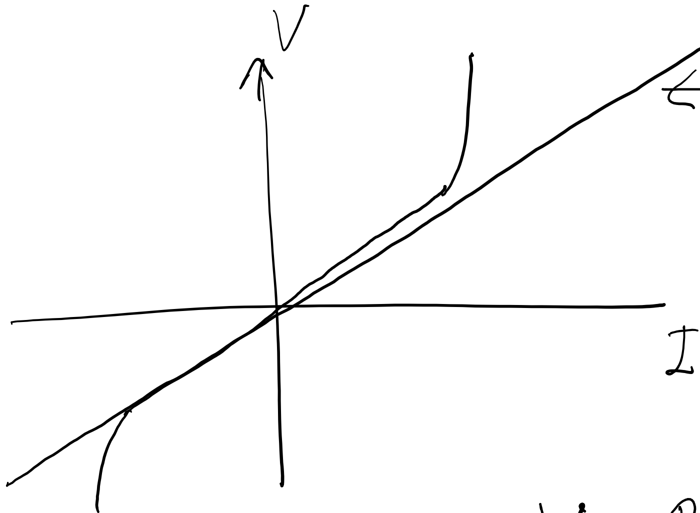




Ohm's Law

$$V = R I$$

Resistance



Good approximation for most conductors

$$V = R I$$

$$[R] = \frac{[V]}{[I]} = V/A = \Omega$$

"Ohm"

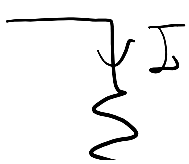
Aside

$$I = G V$$

$$G = \frac{1}{R} \Rightarrow \text{conductance} \qquad [G] = \mho$$

"Mho"

Siemens



$$V = R I$$

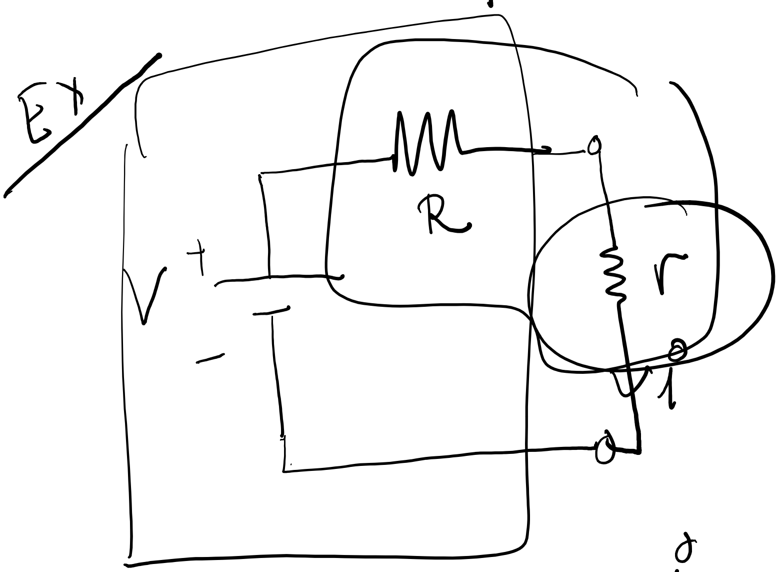
$$dW = V\,dq$$

$$\frac{dW}{dt} = P = V\frac{dq}{dt} = VI$$

$P = VI \Rightarrow$ energy converted to heat per unit time across a resistor.

$$P = I^2 R = \frac{V^2}{R}$$

EX



How should we choose r so that the power lost on r is maximum.

$$i = \frac{V}{R+r}$$

$$P = i^2 r = \frac{V^2}{(R+r)^2} r$$

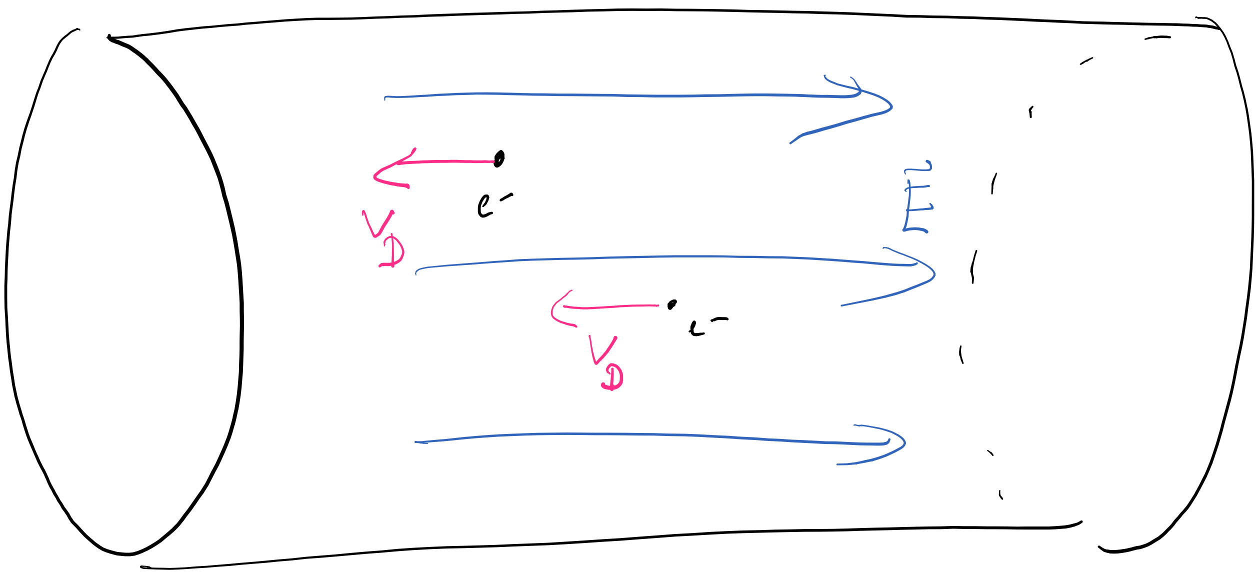
$$\frac{\partial P}{\partial r} \quad V^2\,(R+r)^2 - r\,2(R+r) \quad = 0$$

$$\frac{\partial P}{\partial r} = V^2 \frac{(R+r)^2 - r\,2(R+r)}{(R+r)^4} = 0$$

$$R^2 + r^2 + \cancel{2rR} - \cancel{2rR} - 2r^2 = 0$$

$$R^2 = r^2 \Rightarrow \boxed{r = R}$$

Microscopic view of current



$V_D \rightarrow$ drift velocity

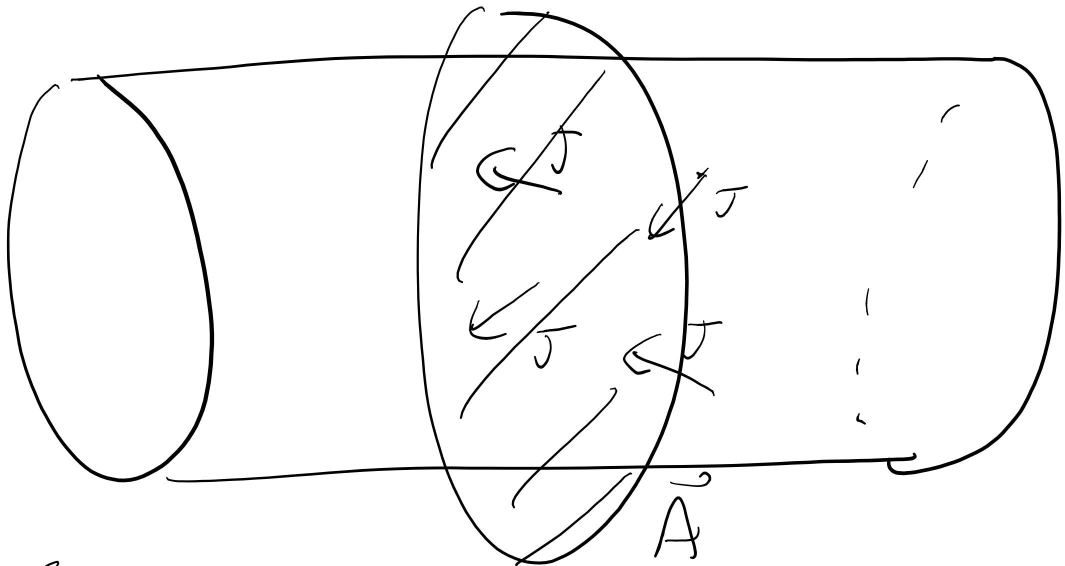
n electrons per unit volume

each of them have charge $-e$

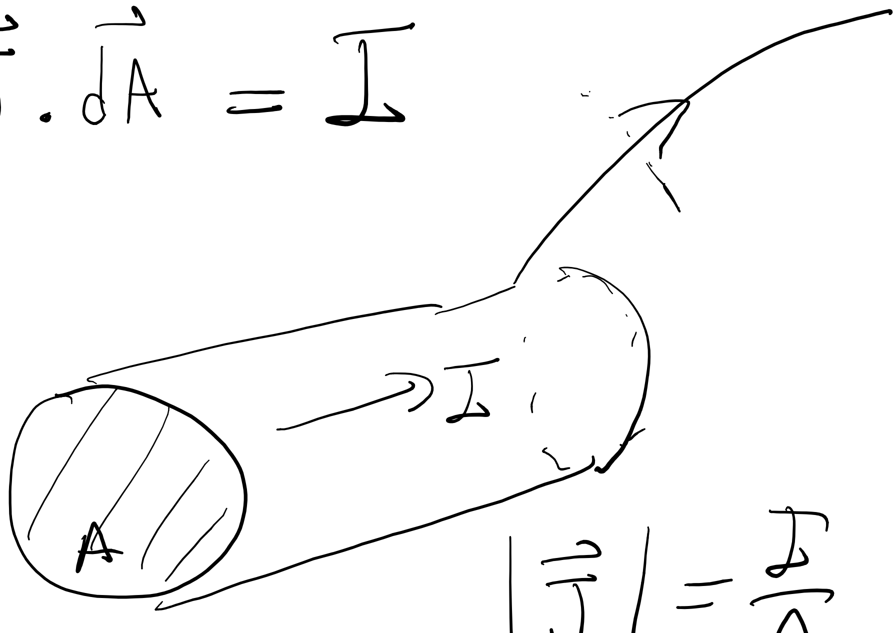
all of them move with velocity $\vec{V_D}$

all of them move with velocity v_D

$$\vec{J} = n(-e)\,\vec{V}_D \Rightarrow \boxed{\text{Current density}}$$



$$\int_{\text{cross section}} \vec{J}\cdot d\vec{A} = I$$



$$|\vec{J}| = \frac{I}{A}$$

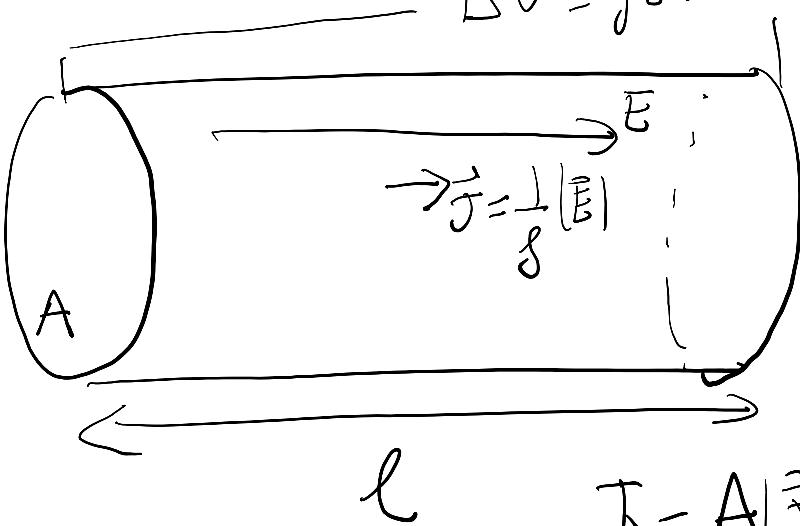
$$[\vec{J}] = A/m^2$$

resistivity

$$\rho\,\vec{J} = \vec{E}$$

$$\rho \vec{J} = \vec{E}$$

$$\Delta V = \int \vec{E} \cdot d\vec{l} = |\vec{E}| l$$



$$I = A|\vec{J}| = \frac{A}{\rho}|\vec{E}|$$

$$\frac{\Delta V}{I} = \frac{|\vec{E}| l}{\frac{A}{\rho}|\vec{E}|} = \boxed{\rho \frac{l}{A} = R}$$

material property

PHYSICS 102- Instructor: M. Özgür OKTEL- Spring 2016

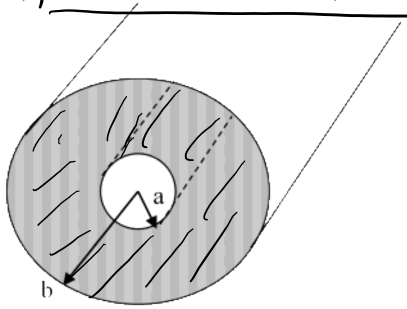
QUIZ-12

An infinite hallow wire is used to carry current. The inside of the wire (**r<a**) is empty, while the region between the inner and outer radii (**a<r<b**) is filled with a material of resistivity **ρ**.

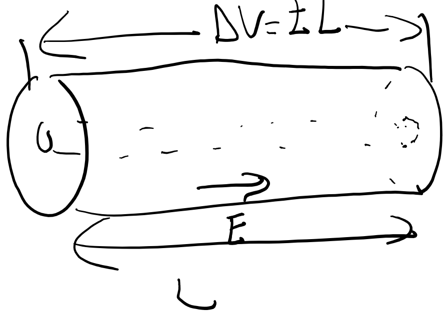
a) What is the resistance **R** of a section of the wire that has length **L**?

If the wire is carrying a total current of **I**, which is uniformly distributed,

b) What is the current density **J** within the wire?



$$|\vec{J}| = \frac{I}{A} = \frac{I}{\pi(b^2-a^2)}$$

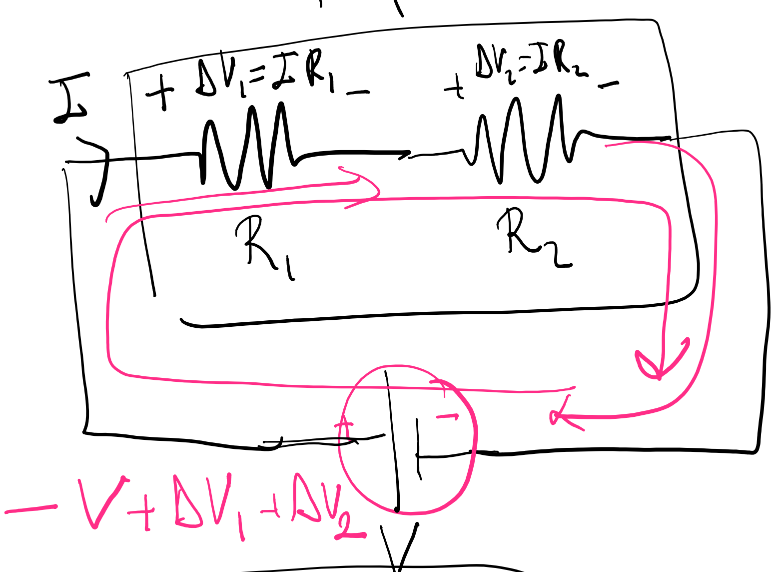


$$|\vec{J}| = \frac{1}{\rho}|\vec{E}|$$

$$I = A\frac{|\vec{E}|}{\rho} = \pi(b^2-a^2)\frac{|\vec{E}|}{\rho}$$

$$R = \frac{\Delta V}{I} = \frac{|\vec{E}|L}{\pi(b^2-a^2)\frac{|\vec{E}|}{\rho}} = \boxed{\rho\frac{L}{\pi(b^2-a^2)}}$$

Req



$$\boxed{V = \Delta V_1 + \Delta V_2}$$

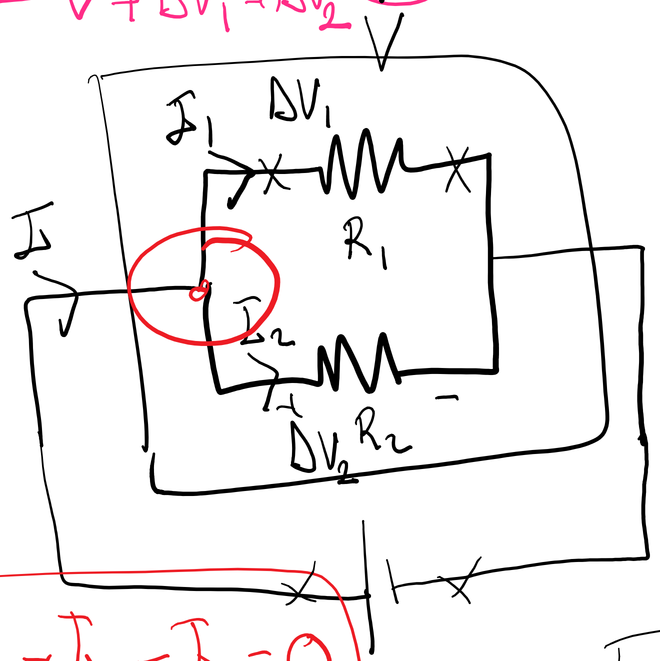
$$V = R_1 I + R_2 I$$

$$V = (R_1+R_2)\,I$$

$$\boxed{R_{eq} = R_1 + R_2}$$

$-V + \Delta V_1 + \Delta V_2$

Series



$$\Delta V_1 = V$$

$$I_1 R_1 = V \Rightarrow I_1 = \frac{V}{R_1}$$

$$\Delta V_2 = V$$

$$I_2 R_2 = V \Rightarrow I_2 = \frac{V}{R_2}$$

$$I - I_1 - I_2 = 0$$

$$\boxed{I = I_1 + I_2}$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V$$

$$V = \underbrace{\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}}_{} I$$

$$\boxed{R_{eq}^{-1} = R_1^{-1} + R_2^{-1}}$$

All circuits can be solved by the two following principles.

1°) Potential depends only on initial and final points so

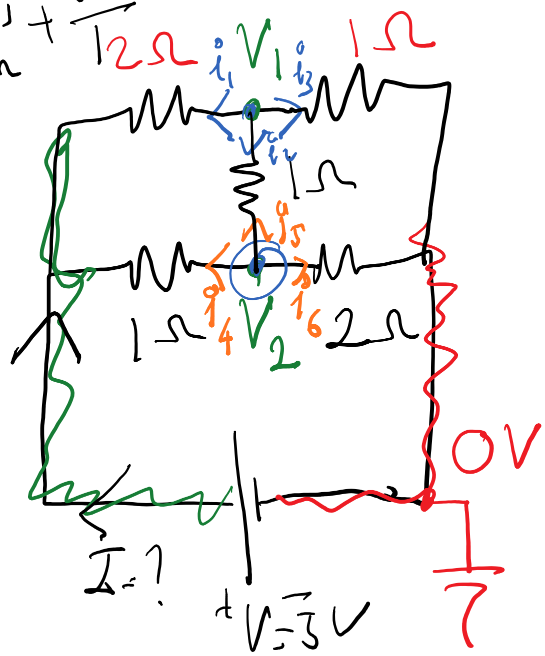
$$(\Delta V)_{loop} = 0$$

2°) At any node

Kirchoff's Laws

$$\sum (I_{incoming}) = 0$$

$$I = \frac{5-V_1}{2\Omega} + \frac{5-V_2}{1}$$



$$i_1 = \frac{V_1 - 5}{2} \qquad i_2 = \frac{V_1 - V_2}{1}$$

$$i_3 = \frac{V_1 - 0}{1}$$

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_1 - 5}{2} + \frac{V_1 - V_2}{1} + \frac{V_1}{1} = 0$$

$$V_1 - 5 + 2V_1 - 2V_2 + 2V_1 = 0$$

$$\boxed{5V_1 - 2V_2 = 5} \quad (1)$$

$$i_4 = \frac{V_2 - 5}{1} \qquad i_5 = \frac{V_2 - V_1}{1}$$

$$i_6 = \frac{V_2 - 0}{2} \qquad i_4 + i_5 + i_6 = 0$$

$$V_2 - 5 + V_2 - V_1 + \frac{V_2}{2} = 0$$

$$2V_2 - 10 + 2V_2 - 2V_1 + V_2 = 0$$

$$\boxed{5V_2 - 2V_1 = 10} \quad 2^o$$

$$2(5V_1 - 2V_2 = 5)$$
$$5(5V_2 - 2V_1 = 10)$$

$$\begin{array}{r} 10V_1 - 4V_2 = 10 \\ +\ 25V_2 - 10V_1 = 50 \\ \hline 21V_2 = 60 \end{array}$$

$$\boxed{V_2 = \frac{60}{21}\ V}$$

$$V_1 = \frac{5 + 2V_2}{5} = \frac{5 + \frac{120}{21}}{5}$$

$$= 1 + \frac{24}{21} = \boxed{\frac{45}{21}\ V = V_1}$$

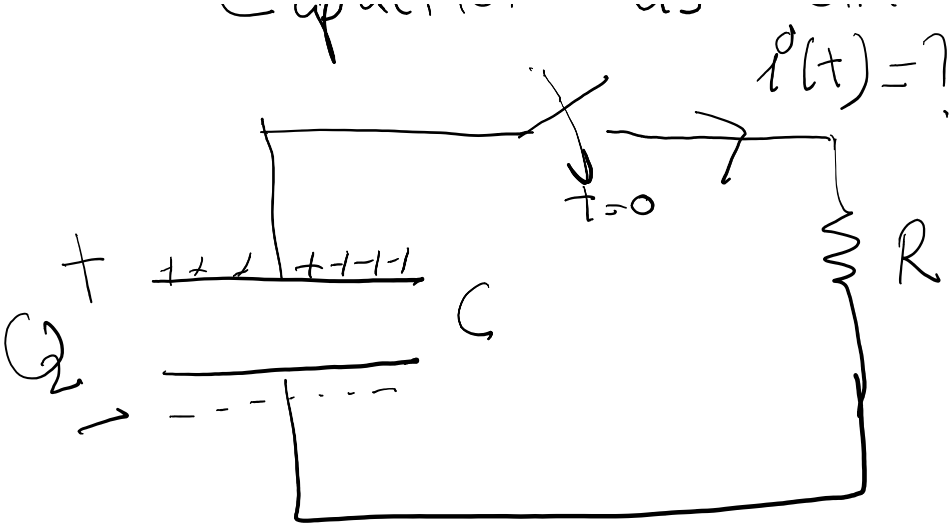
$$I = \frac{5 - V_1}{2} + \frac{5 - V_2}{1}$$

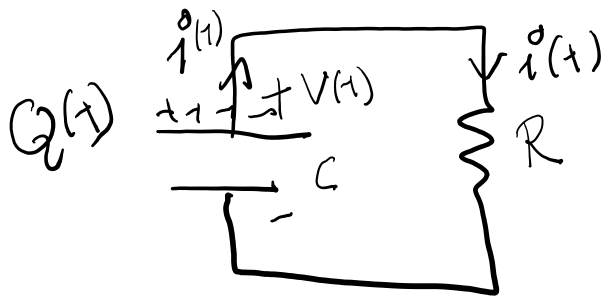
$$= \frac{5}{2} - \frac{45}{2 \times 21} + 5 - \frac{60}{21}$$

$$= \frac{15}{2} - \frac{45 + 120}{42} = \frac{15 \times 21 - 165}{42} \boxed{= 3.57\,A}$$

Capacitor as circuit element.

$i(t) = ?$





$$V(t) = \frac{Q(t)}{C}$$

$$i(t) = \frac{V(t)}{R}$$

$$i(t) = \frac{Q(t)}{RC}$$

$$i(t) = -\frac{dQ(t)}{dt}$$

$$-\frac{dQ(t)}{dt} = \frac{Q(t)}{RC}$$

$$\frac{dQ(t)}{dt} = -\frac{1}{RC}\,Q(t)$$

By inspection

$$Q(t) = A\,e^{\alpha t}$$

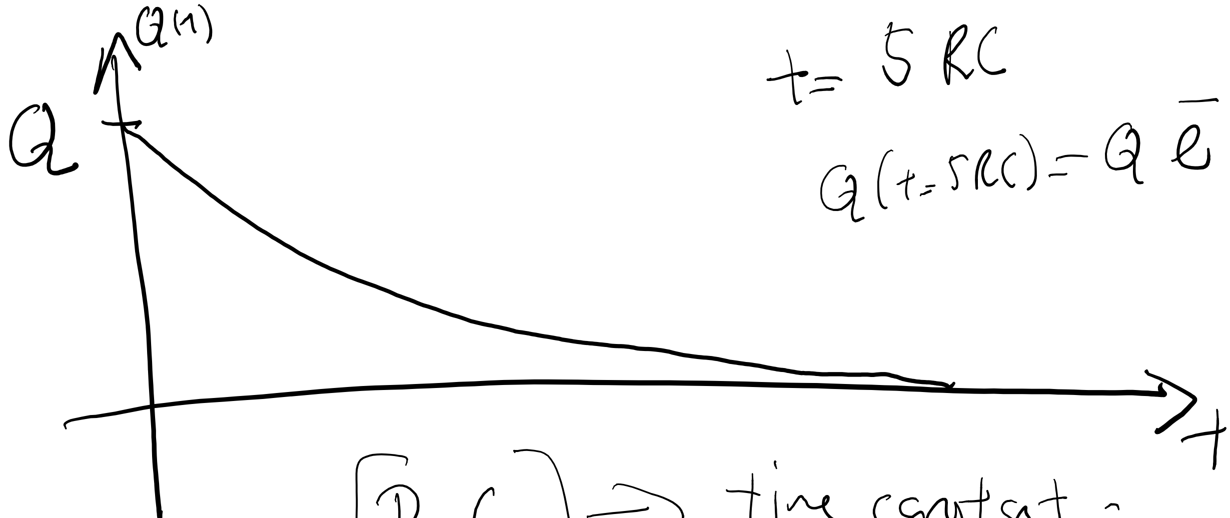
$$\frac{dQ(t)}{dt} = \alpha A\,e^{\alpha t}$$

$$\frac{}{dt}$$

$$\alpha \cancel{A} \cancel{e^{\alpha t}} = -\frac{1}{RC} \cancel{A} \cancel{e^{\alpha t}} \Rightarrow \boxed{\alpha = -\frac{1}{RC}}$$

$$Q(t=0) = Q = A \underbrace{e^{-\frac{1}{RC}0}}_{1} \Rightarrow A = Q$$

$$Q(t) = Q e^{-t/RC} \qquad e = 2.71828$$



$$t = 5RC$$

$$Q(t=5RC) = Q e^{-5}$$

$$[RC] \Rightarrow \text{time constant}$$

$$\tau = RC \qquad [\tau] = \boxed{\Omega F = \sec}$$

$$Q(t) = Q_0 e^{-t/RC}$$

$$1.6\ 10^{-19} = 1\ e^{-t/1}$$

QUIZ-13

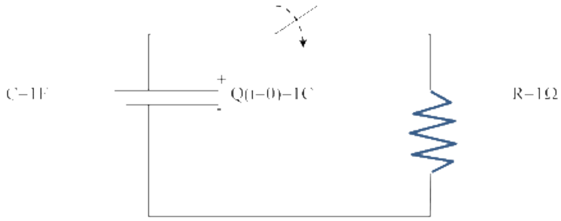
QUIZ-13

A capacitor C=1 F is charged with initial charge 1 Coulomb. This capacitor is discharged over a resistor R= 1 Ω. How long does it take for the charge of the capacitor to decrease to only a single electron. (Electron

QUIZ 15

A capacitor C=1 F is charged with initial charge 1 Coulomb. This capacitor is discharged over a resistor R= 1 Ω. How long does it take for the charge of the capacitor to decrease to only a single electron. (Electron charge is $e = 1.60\ 10^{-19} C$.

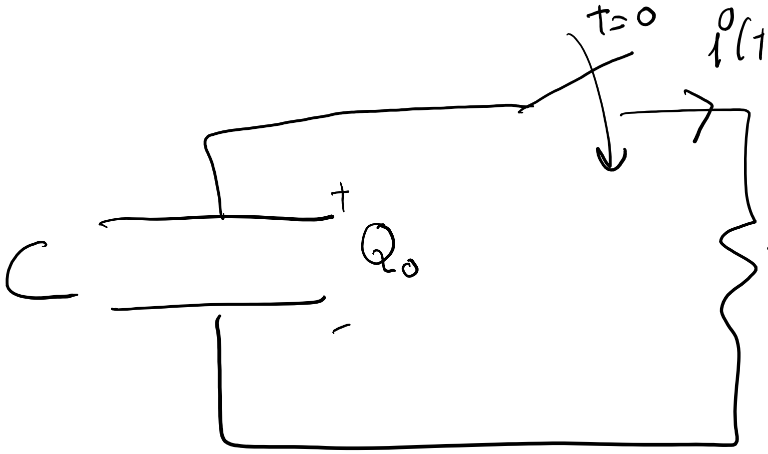
$$1.6\ 10^{-19} = 1\ e^{-t/1}$$



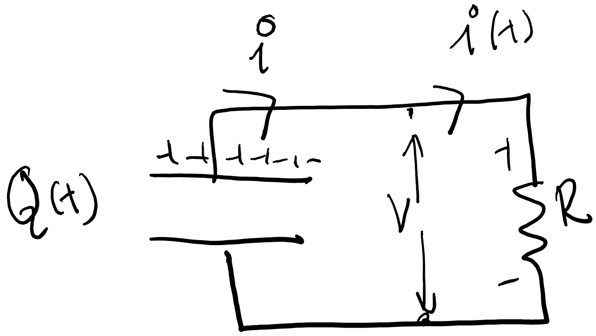
$$t = -\ln(1.6\ 10^{-19})\ \text{sec}$$

$$\boxed{t = 43.2 \text{ sec}}$$

$i(t) = ?$ $\quad Q(t) = ?$



Total power dissipated on the resistor.



$$V = \frac{Q(t)}{C} = i(t) R$$

$$\frac{1}{RC} Q(t) = i(t)$$

$$\frac{dQ}{dt} = -i$$

$$\frac{1}{RC} Q(t) = -\frac{d}{dt} Q(t)$$

$$\frac{1}{RC} Q = -\frac{dQ}{dt}$$

$$Q(t) = A\ e^{\alpha t}$$

$$\frac{dQ}{dt} = \alpha A\ e^{\alpha t}$$

Q_f

$$\int_0^T \frac{1}{RC}\, dt = -\int_{Q_{in}}^{Q_f} \frac{dQ}{Q}$$

$$\frac{1}{RC} T = -\ln \frac{Q_f}{Q_i}$$

$$Q_f = Q_i \, e^{-T/RC}$$

$$\frac{dQ}{dt} = \alpha A e^{\alpha t}$$

$$\frac{1}{RC} \cancel{A e^{\alpha t}} = -\alpha \cancel{A e^{\alpha t}}$$

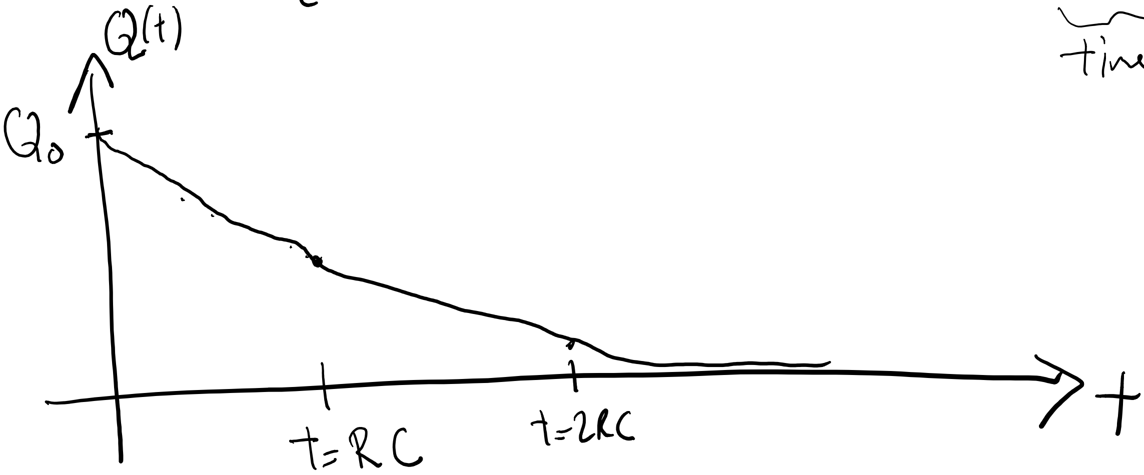
$$\alpha = -1/RC$$

$$Q(t) = A \, e^{-t/RC}$$

$$t=0 \quad Q(t=0) = Q_0$$

$$A \underbrace{e^{-0/RC}}_{1} = Q_0$$

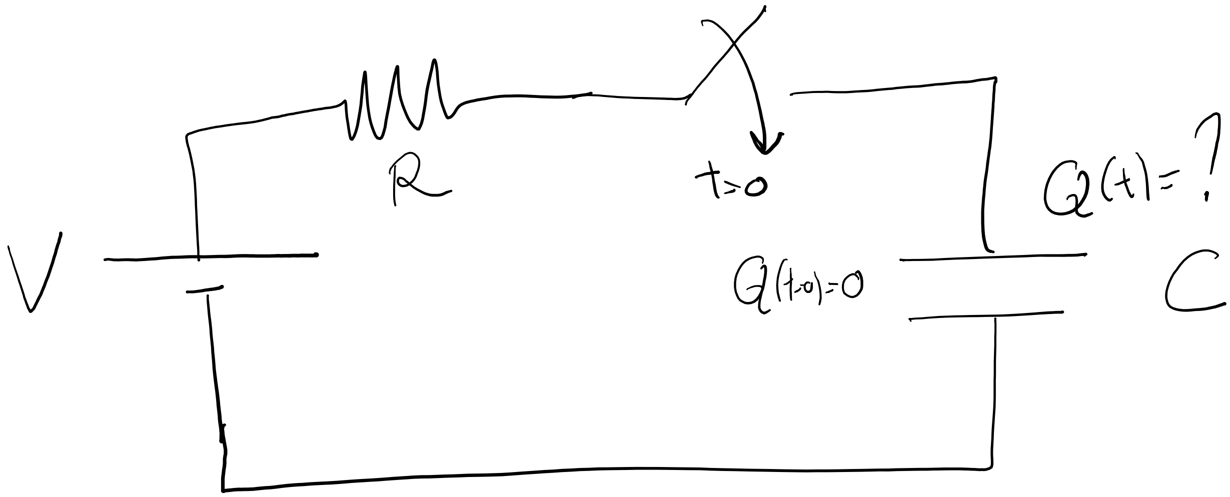
$$Q(t) = Q_0 \, e^{-t/RC} \qquad \underbrace{\tau = RC}_{\text{time constant}}$$

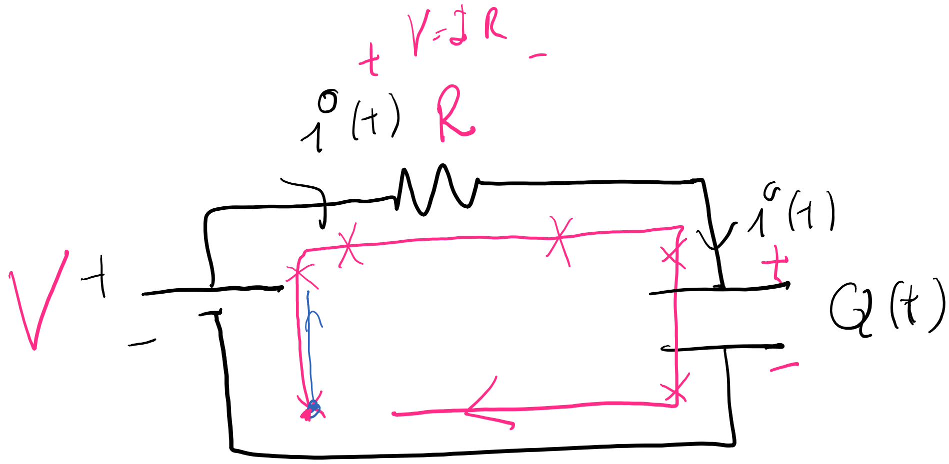


$$Q(t=\tau) = Q_0 \, e^{-1}$$

$$Q(t=2\tau) = Q_0 \, e^{-2}$$

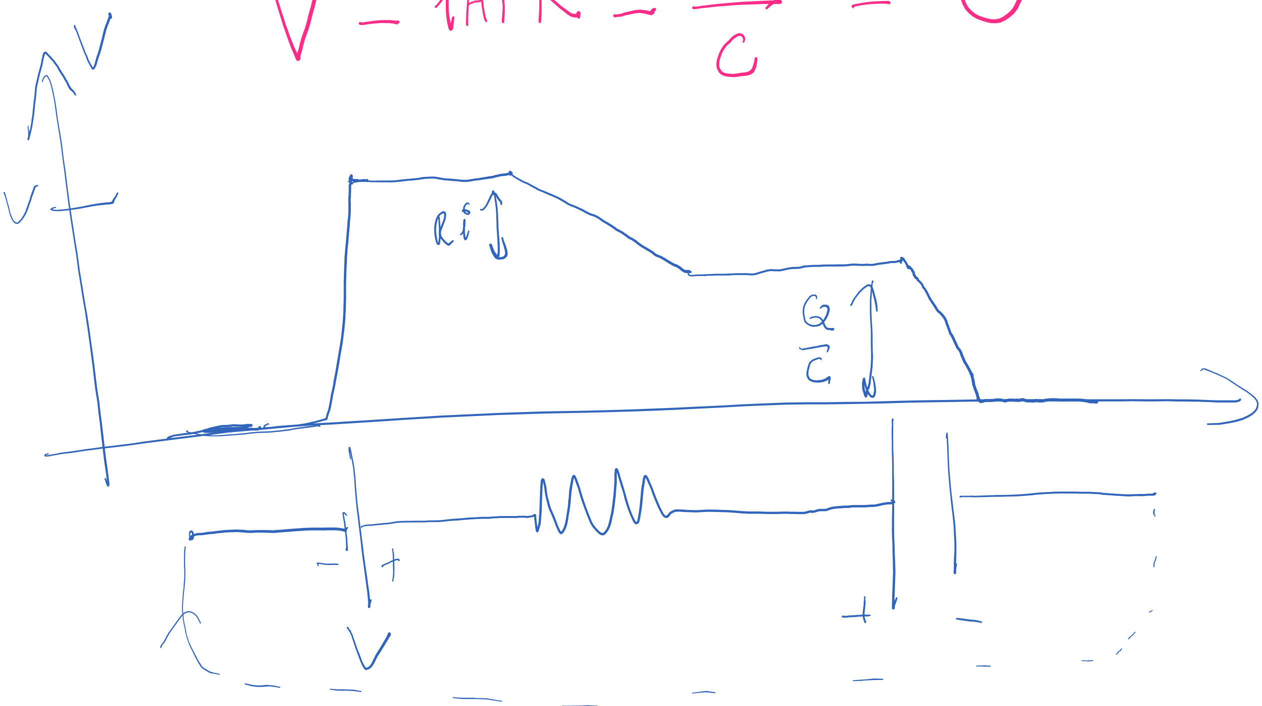
Charging a capacitor





$$i(t) = \frac{dQ}{dt}$$

$$V - i(t)R - \frac{Q(t)}{C} = 0$$



$$V - i(t)R - \frac{Q(t)}{C} = 0$$

$$V - R\frac{dQ(t)}{dt} - \frac{1}{C}Q(t) = 0$$

$$\frac{dQ(t)}{dt} = \left(\frac{V}{R} - \frac{1}{RC}Q(t)\right)$$

Define $q(t) = VC - Q(t) \Rightarrow Q(t) = VC - q(t)$

$$\boxed{\frac{dq}{dt} = -\frac{dQ}{dt}}$$

$$-\frac{dq}{dt} = \frac{1}{RC}\big(\underbrace{VC - Q(t)}_{q(t)}\big)$$

$$\boxed{\frac{dq}{dt} = -\frac{1}{RC}q(t)}$$

$$q(t) = q_0 e^{-t/RC}$$

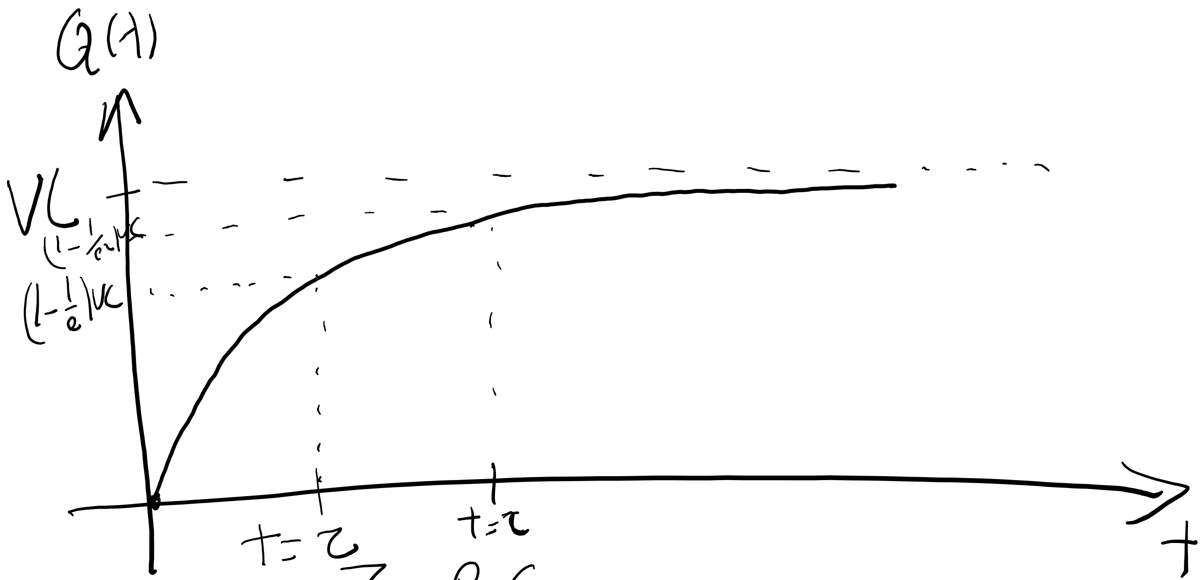
$Q(t=0) = 0$

$q_0 = VC - Q(t=0) = VC$

$$q(t) = VC\, e^{-t/RC}$$

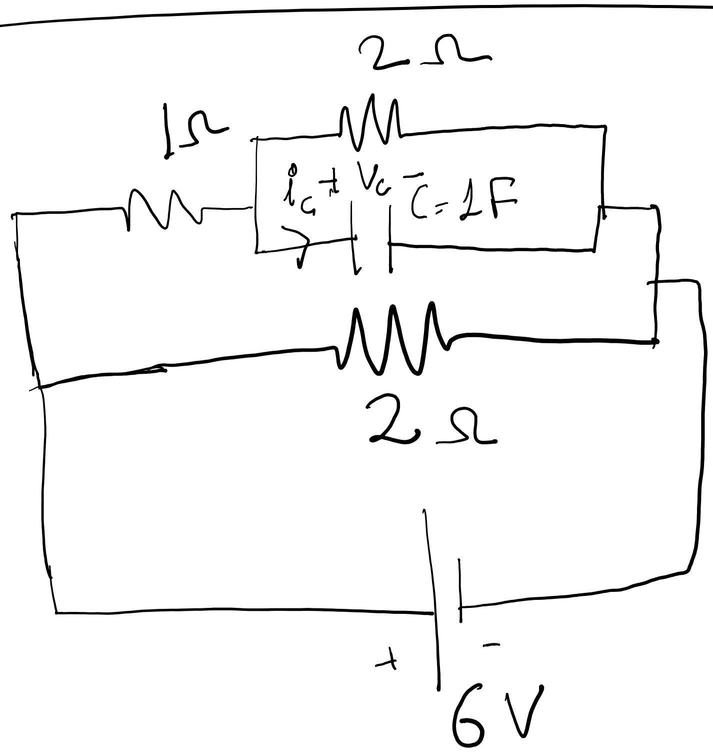
$$Q(t) = VC - VC e^{-t/RC}$$

$$Q(t) = VC\left(1 - e^{-t/RC}\right)$$



$\tau = RC$

EX



If we wait for very long time so that the system is in steady state what is the charge on the capacitor.

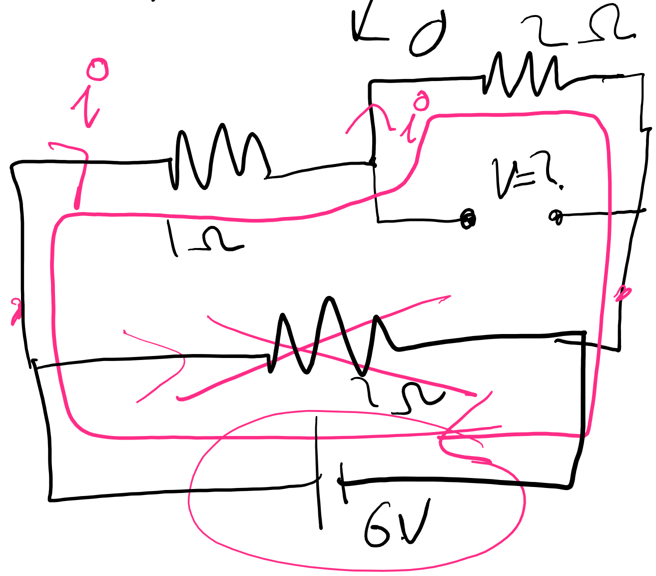
Steady state $\frac{dV}{dt} \to 0$ $\frac{dI}{dt} \to 0$

Steady state $\frac{dV}{dt} \to 0$ $\frac{dI}{dt} \to 0$

steady state

$$i_c = \frac{dQ}{dt} = C\frac{dV_c}{dt} \Longrightarrow i_c = 0$$

$\leftarrow 0$



$6V = i\,1\Omega + i\,2\Omega$

$6 = 3i$

$i = 2A$

$V = i\,2 = \boxed{4V}$

$Q = CV = \boxed{4C}$